Chapter 10 *8/25/22 copyright of Robert D. Klauber*

The Vacuum Revisited

"There might be more than you can see."
It's Not My Time
Three Doors Down

10.0 Background

10.0.1 Vacuum "Fluctuations"

Term "vacuum fluctuations" confusing as it has several meanings

The term "vacuum fluctuations" has several meanings in the literature, one of which is related to the higher order corrections of the last chapter. Other uses of it have little to do with them.

In my experience, although passed off as a seemingly simple concept in articles written for lay audiences, the term "vacuum fluctuations" is not only commonly misused in those articles, but from anything but a superficial perspective, invariably and inordinately confusing. At least it can be so for someone just getting grounded in QFT, who tries to relate such renditions of the term to the fundamentals of that theory.

I also feel obliged to pass on that I have found a number of established physicists seemingly at a loss to explain the so-called vacuum fluctuations to me in terms of those fundamentals. I write this chapter in hopes of clarifying this issue, as best I can, for newcomers and perhaps, for others.

10.0.2 Chapter Overview

Possible meanings for "vacuum fluctuations"

We will see how the term "vacuum fluctuations" can refer to any of the distinctly different

- evanescent particle pair creation and destruction of lay literature fame,
- ½ quanta expectation value for the vacuum state $|0\rangle$ (as in Chaps. 3, 4, 5),
- vacuum bubbles with three virtual particles (as in Chap. 8), and
- higher order correction virtual particles (as in Chap. 9).

Compare them to theory and experiment

Then, we will

- compare the above four cases theoretically, and
- compare them to experiment.

Examine QFT uncertainty and wave packets

Following that, we will

- review the uncertainty principle as applied to the first two cases, and
- analyze how wave packet theory in QFT relates to those cases.

10.1 Vacuum Fluctuations: The Theory

In discussing the theoretical side of vacuum fluctuations, we begin with "the story", the much circulated description of the vacuum, of which you have no doubt heard. We then review relevant parts of QFT, which we studied in earlier chapters, and compare those to the story.

The well told story: particle-antiparticle pairs popping in and out of the vacuum

10.1.1 The Story

A vacuum fluctuation is typically described as particle pair creation and destruction in the vacuum. A particle and its anti-particle pop into existence out of the vacuum, as shown in Fig. 10-1, and then, quickly (presumably before they can be measured) annihilate one another. Total charge is conserved since there was zero charge before the pair was created, total charge zero when both exist, and then zero charge once again after they mutually destruct.

Energy for the pair, which are both typically considered to have positive energy, is "borrowed" from the vacuum according to the uncertainty principle for energy and time. Similarly, 3-momenta of the two particles do not have to cancel one another, as total 3-momentum is also borrowed from the vacuum via the uncertainty principle for 3-momentum and space.[1]

Energy and momentum "borrowed" for short time via uncertainty principle

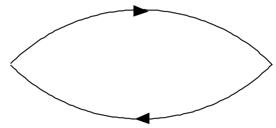


Figure 10-1. Pair Production Vacuum Fluctuation via "The Story"

The fluctuation does not exist long enough to be measured, so we never can detect it using our instruments. That is, for energy variation ΔE away from zero of significant enough magnitude to be measured, the duration of the existence of that energy (the time variation Δt) is unimaginably small, since $\hbar$ is so small (in everyday measuring units $\hbar = 1.0546 \times 10^{-34}$ joule-sec). This is the impact of the uncertainty relation for energy and time, the LH relation below.

$$\Delta E \Delta t \gtrsim \frac{\hbar}{2} \qquad \Delta p^{\underline{i}} \Delta x^{\underline{i}} \gtrsim \frac{\hbar}{2}. \tag{10-1}$$

Similarly, for 3-momentum variation of appreciable magnitude (RH relation above), the variation from zero of the length measurement of the particle in the direction of the 3-momentum is extremely small and typically below the threshold of detection via instruments. Also, a large 3-momentum variation entails a large energy variation, and thus an extremely short time interval for the existence of that variation, meaning it goes undetected. Because the particle/anti-particle pairs are not detectable, they are considered to be virtual, not real, particles.

Enormous number of pairs always being created and destroyed

Particle/anti-particle pairs of very high energy (and 3-momenta) exist for extremely fleeting moments of time (in regions of extremely tiny size). The vacuum is everywhere presumed to be a boiling cauldron of particle/anti-particle pairs, of all possible energies and momenta, popping continuously in and out of existence.

At Planck time and distance scale, particle energy so large, tiny black holes form

If this were true, then at very small time and distance scales, energy and 3-momenta values for the pairs would become so large as to exceed the mass-energy density needed to create black holes. Thus, microscopic black holes would be continually appearing and disappearing in the vacuum as a result of the creation and destruction of the most energetic pairs. One could imagine this as being like bubbling foam, and the phrase coined by John Wheeler[2], "quantum foam", has caught on.

The dimensions and energy levels of the quantum foam bubbles are those corresponding to mass-energy, time, and distance scales at which microscopic black holes would form. This scale is called the Planck scale, i.e., approximately

$$t_P = 5.39 \times 10^{-44}\text{ sec} \qquad m_P = 2.18 \times 10^{-8}\text{ kg} \qquad l_P = 1.62 \times 10^{-35}\text{ m}.$$

So many tiny black holes bubbling in and out → quantum foam

Renditions of the pair production/quantum foam story are often accompanied with comments that all of this is a result of QFT. So, let's review what we have learned so far in QFT about particle creation and destruction, and the properties of the vacuum.

10.1.2 Quantum Field Theory Phenomena that May be Relevant

Does the pair production story "jibe" with QFT?

The question we would like to answer is "where in QFT, if anywhere, can the vacuum fluctuation pair production phenomenon be found?" To start our quest for that answer, let's delineate the phenomena we have learned about that relate either to the vacuum or to virtual particle creation and destruction.

Free Fields Half Quanta in the Vacuum

1st candidate from QFT: ½ quanta of vacuum, the ZPE

In Chap. 3, Sects. 3.4.1 to 3.4.3 on pgs. 53-55, we found our theory predicts that for scalar fields, the vacuum is filled with free scalar particles and antiparticles, one for each possible energy level. Each particle's energy level is $\frac{1}{2}\hbar\omega_{\mathbf{k}}$ (= $\frac{1}{2}\omega_{\mathbf{k}}$ in natural units), i.e., they are half quanta in the sense of what we normally consider quanta to be. In principle, the fields range in energy from zero

[1] This process is not to be confused with strong field pair production, which is not a purely vacuum process, but entails a virtual photon from a strong e/m field turning into an electron/positron pair.

[2] J.A. Wheeler, On the Nature of Quantum Geometrodynamics, *Annal. Phys.* **2**, 604-614 (1957). For a lay person rendition, see J.A. Wheeler, *Geons, Black Holes, and Quantum Foam*, Norton (1998).

to infinity, although many consider the Planck energy to be a realistic upper limit cutoff. The energy from these half quanta is commonly called "zero-point energy" (ZPE).

In Chap. 4, we saw similar ½ quanta for fermions, but those had negative energies. In Chap. 5, we obtained similar results for photons as for scalars, i.e., an energy of $\frac{1}{2}\hbar\omega_{\mathbf{k}}$ for each **k** and *r* polarization state. These fermion and photon energies are also encompassed by the term ZPE.

ZPE fermions have negative energy

Interacting Fields Vacuum Bubbles

In Chap. 8, we saw how the interaction part of the theory predicts vacuum bubbles of a lepton, anti-lepton, and photon, such as that in Fig. 8-8, repeated below as Fig. 10-2.

2nd QFT candidate: three (virtual) particle vacuum bubbles

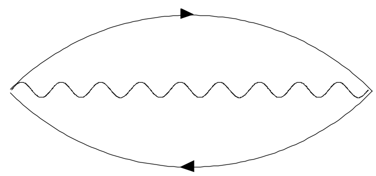


Figure 10-2. Vacuum Bubble for Interacting Fields

Interacting Fields Virtual Particle Higher Order Corrections

Any interaction has a simple, first order in α, version, and higher order corrections. For example, in Fig. 9-4, pg. 259, we show Feynman diagrams for the lowest order term (upper left, α order) and eleven second order (α^2 order) terms. In the second order diagrams, the extra virtual particles are sometimes called vacuum fluctuations. For examples, the photon loop (or closed fermion loop) in the middle of the $S^{(4)}_{B1-1}$ diagram, the positron loop (virtual positron and virtual photon) of $S^{(4)}_{B1-2}$, and the longer virtual photon in $S^{(4)}_{B1-11}$ would all, in this context, be considered vacuum fluctuations. Higher order (α^3, α^4, ..) corrections would result in a plethora of such virtual particles.

3rd QFT candidate: virtual particles from higher order corrections

Use of the Term "Vacuum Fluctuations"

The term "vacuum fluctuations" can be used to describe each of the above phenomena. That is, it can mean i) the story of pair production/annihilation and quantum foam, ii) half quanta fields, iii) three virtual particles vacuum bubbles, and iv) virtual particles in interactions. Our overriding question is whether i) is the same as any of ii), iii), or iv).

"Vacuum fluctuations" can mean any of above four cases

Comparing Each QFT Phenomenon with The Story

We will now take a separate section to compare each of the above phenomena found in QFT to the story and determine which, if any, of them corresponds to the vacuum pair production scenario of Sect. 10.1.1.

We will now compare them

Please note carefully that I have not seen elsewhere the material presented below in Sects. 10.1.3 to 10.1.6. Neither have I seen a complete summary of related experimental evidence as shown in Sect.10.2. The conclusions drawn in these sections are my own, arrived at after many years of pondering the vacuum fluctuation issue and searching for relevant analyses/reviews.

Given this, I ask two things. 1) Please consider that the views expressed are not embraced by many physicists (I don't believe they have been considered by many), and you must formulate your own position on the matter. 2) If any reader knows of other suitable summaries of this material, either agreeing or disagreeing with the material/position presented herein, please notify me via the website for this book. (See URL on pg. xvi, opposite pg. 1.)

10.1.3 Free Field Half Quanta (ZPE)

1st: ZPE vs the pair popping story

The reader should now re-read Sect. 3.4.3 Zero Point (Vacuum) Energy, pgs. 55-56, and Sect. 3.6.6 Normal Ordering, pgs. 60-61, to review key concepts regarding the free field ½ quanta.

First, we note that the ZPE quanta are derived from free field theory and therefore no interactions are involved. In the story of Fig. 10-1, however, a particle and antiparticle are created via a vertex interaction and then destroyed via an interaction at a second vertex. It does not seem, therefore, that the free ZPE fields can form particle/anti-particle vacuum bubbles because they don't interact with one another. They are free, not interacting, particles. There are no Feynman diagrams associated with them.

ZPE are from free field theory so shouldn't have interacting pairs

Second, we re-iterate what was noted in Sect. 3.4.3. In the QFT derivation of half quanta vacuum energy, those half quanta appear to simply be steadily "sitting" in the vacuum, and not "popping in and out" of it. There is no apparent mechanism whereby they exist part of the time, but not all of the time. In fact, the free Hamiltonian operating on the vacuum state (see (3-61), pg. 55) shows that the vacuum quanta are eigenstates of energy. Energy eigenstates have the same energy no matter when they are measured. They exist continually.

Via theory, ZPE seem steady and not "popping" in and out

Third, ZPE is not borrowed according to QFT, it just exists in the vacuum.

ZPE not borrowed, but simply "there"

Fourth, fermions have negative energy vacuum ½ quanta, as we saw in Chap. 4. There is no uncertainty principle for negative energy (although one could probably consider postulating one). And how would mini black holes form from negative energy? And if one posits (see pg. 61) we can simply use the infinite vacuum energy as a baseline and deal with $\Delta E = E - \infty$, then in interactions, we have one baseline ($+\infty$) for bosons and another ($-\infty$) for fermions[1]. Does that make sense?

Negative ZPE: no uncertainty principle and total vacuum energy < 0

Finally, there are more known fermions than bosons in nature, so according to QFT, the total ZPE should be negative, the opposite of that promoted in the pair popping story. In this light, tales of a vacuum filled with (positive) energy seem, to put it kindly, strange.

10.1.4 Interacting Fields Vacuum Bubbles

2nd: vac bubbles vs pair popping

In QED we only have vertices with three particles. There are no two particle vertices as proposed in the pair production scenario of Fig. 10-1.

No two particle vertices in standard QFT

The pair production scenario entails a temporary "borrowing" of energy (and momentum) from the vacuum via the uncertainty principle, meaning a net non-zero sum of the virtual pair particles' energies. But we learned in interaction theory that the sum of all incoming energy (and 3-momentum) at any vertex equals the total outgoing energy (and 3-momentum). In Fig. 10-1 there is zero incoming 4-momentum, so the total 4-momentum of the pair must also be zero. Thus, the story of vacuum pair production appears to be in conflict with QFT.

Vertex energy conservation → zero total energy in pair scenario, no borrowing

Quantum foam presumably results from large energies being found over short time and distance scales, resulting in the formation of myriads of tiny black holes. But, via QFT vertex conservation, there must be no net energy resulting from the pair production of Fig. 10-1, and so it does not seem it could be a source of the energy needed to form the black holes.

Zero total energy per pair → no mini black holes

Further, the three particle vertices of Fig. 10-2 result in no net energy for the three virtual particles, so they can not result in tiny black holes.

Nor can there be mini black holes from three-particle bubbles

Thus, the vacuum bubbles of Fig. 10-2 cannot, via standard QFT, play a role in the vacuum pair production story. Nor can they interact with other particles, since, they have no external legs.

10.1.5 Interacting Fields Virtual Particles

3rd: Higher order virtual particles vs pair popping

The virtual particles in higher order contributions to interactions, such as depicted in Fig. 9-4, pg. 259, have nothing to do with the vacuum *per se*, but with the particles (both real and virtual) off of which they "hang". They are manifestations springing from entities other than the vacuum.

Nevertheless, agreement between certain experiments (see below) and QFT predictions using higher order corrections (also called "radiative corrections") are sometimes cited as demonstrative of vacuum fluctuations. This would not seem appropriate.

Higher order correction virtual particles not found alone in vacuum, but with real particles

By any measure, virtual particles participating in interactions between real particles cannot be considered the mechanism for vacuum fluctuation pair production.

10.1.6 Conclusion: Theory and Vacuum Fluctuations

The story of vacuum pair production/destruction commonly called "vacuum fluctuations" is not consonant with the fundamental theory of QFT/QED, in particular, not with ZPE quanta, vacuum bubbles, nor higher order correction virtual particles.

There are no pair creation/ destruction Feynman diagrams

Bottom line: There are no Feynman diagrams like Fig. 10-1, and none for the ½ quanta.

10.2 Vacuum Fluctuations and Experiment

Vacuum pair production story vs experiment

In this section, we describe certain experiments that are used to justify the existence of vacuum fluctuations and how they relate to the theoretical side of QFT, as we understand it.

[1] Pointed out to the author in 2015 by Anna Pearson, then an Oxford University PhD candidate.

10.2.1 Casimir Plates

Two flat plates brought close together experience a small attractive force at very small separation distances. This effect was first predicted by Dutch physicists Hendrik B. G. Casimir and Dirk Polder in 1948. The attractive force has been attributed to ZPE, in heuristic and very simple terms, because the vacuum quantum waves outside the plates presumably exert greater force than the vacuum quantum waves between the plates. However, there are other interpretations.

> *... the Casimir effect is often invoked as decisive evidence that the zero point energies of quantum fields are "real". On the contrary, Casimir effects can be formulated and Casimir forces can be computed without reference to zero point energies.... The Casimir force is simply the (relativistic, retarded) van der Waals force between the metal plates.... So, the concept of zero point fluctuations is a heuristic and calculational aid in the description of the Casimir effect, but not a necessity.... No known phenomenon, including the Casimir effect, demonstrates that zero point energies are "real".*[1]

There are two key things to note.

Casimir experiment not proof ZPE quanta exist

1) While the Casimir effect can be calculated by assuming ZPE half quanta, the same result can also be calculated another way without using them at all. It thus does not prove their existence, contrary to what is often claimed.
2) In the Casimir calculation that does employ ZPE, the quanta are assumed to be continuously existing *standing waves*, *not particle pairs popping in and out of existence*.

Even if it were, no "popping" of particles needed

Conclusions: Casimir plate experiments do not provide proof of the existence of ZPE ½ quanta in any form. Even if one were, nevertheless, to consider such quanta responsible for the Casimir effect, there is no evidence those quanta are paired and evanescent (pop in and out of existence).

10.2.2 Lamb Shift

Lamb shift due to higher order corrections of real particle interactions

The Lamb shift is a small difference between the two energy levels $^2S_{1/2}$ and $^2P_{1/2}$ of the hydrogen atom, which according to RQM, should have the same energies. QFT, in its QED form, predicts this shift, and that prediction was one of the great early successes of the theory.

The Lamb shift calculation is long and difficult[2]. It is often described as taking vacuum fluctuations into account in order to obtain the correct result. However, in actuality, these "vacuum fluctuations" are really the radiative, or higher order, corrections (extra virtual particles in Feynman diagrams) of Sect. 10.1.5. These corrections to the Coulomb potential of the hydrogen atom (in diagrams, extra virtual photons, electrons, and positrons) yield the correct energy levels.

It has nothing to do with the vacuum

Conclusion: The Lamb shift does not prove vacuum pair production/destruction.

10.2.3 Anomalous Magnetic Moment of the Electron

Anomalous mag moment from higher order corrections of real interactions

The Dirac equation in RQM leads to a prediction of the magnetic dipole moment of the electron that is slightly different from that measured in experiment. The measured value was called the anomalous magnetic moment.

Using QED with radiative corrections, the anomalous magnetic moment value was accurately calculated[3]. This, too, is sometimes attributed to vacuum fluctuations. However, here again, it is higher order (radiative) corrections having nothing to do with the vacuum that are invoked.

It has nothing to do with the vacuum

Conclusion: The anomalous magnetic moment does not prove vacuum pair production/destruction.

10.2.4 The Fulling-Davies-Unruh Effect

If the vacuum is filled with ZPE, then as Stephen Fulling (1973), Paul Davies (1975) and Bill Unruh (1976) described, an accelerating observer would find the ZPE to look like black-body

---

[1] R. L. Jaffe, "Casimir Effect and the Quantum Vacuum", *Phys. Rev. D*72 021301(R) (2005) http://arxiv.org/abs/hep-th/0503158 .

[2] C. Itzykson and J.B. Zuber, *Quantum Field Theory* (McGraw-Hill 1985). Sect. 7-3-2, 358-365.

[3] See Chap. 16 of this book.

radiation, whereas the non-accelerating observer would not[1]. This effect became commonly known as the Unruh effect, though it is more appropriate to use all three names when referring to it.

Related to non-inertial frame vacuum, not elementary QFT

The Fulling-Davies-Unruh effect is not directly related to vacuum ½ quanta, vacuum bubbles, or radiative corrections, but to the difference between vacua in inertial and non-inertial frames[2]. The accelerated observer measuring the inertial vacuum detects particles not observed by the inertial observer. From this perspective, the effect seems quite unrelated to any concepts considered herein.

Controversial if Unruh effect has been measured

Some experimenters believe they have detected the Fulling–Davies–Unruh effect, but as of the date of this book, the claimed observations are controversial and under dispute. (See cited Wikipedia URL for the latest on this controversy.)

Even if measured, not proof of pair "popping"

Conclusions: There is no incontrovertible proof that Fulling–Davies–Unruh radiation exists. Even if it is confirmed, it would not prove pair popping or that ZPE quanta exist.

10.2.5 Measured Vacuum Energy

Measured vs theory ZPE differ by $>10^{120}$ (for bosons)

As noted in Chap. 3, ZPE calculations[3], assuming a Planck scale maximum allowable value (rather than infinity) for the ½ quanta, predict a positive vacuum energy density for bosons on the order of 10^{74} GeV^4 (natural units), whereas the observed value is $\leq 10^{-47}$ GeV. (See Appendix A.) This is the famous largest discrepancy between theory and experiment in the history of science.

For both bosons and fermions, theoretic total ZPE is negative

Usually going unmentioned in such discussions is that the total vacuum energy for all known bosons and fermions would be *negative* and of this order, as discussed at the end of Sect. 10.1.3

Conclusion: The observed vacuum energy density does not support the existence of ZPE or any other form of vacuum fluctuations.

10.2.6 Experimental Evidence Conclusion

No experimental evidence proving ZPE, let alone pair "popping"

At the time of this text version, there is no experimental evidence irrefutably demonstrating the existence of virtual particle/anti-particle pairs popping in and out of the vacuum and unrelated to interactions between real particles. Further, there is no such irrefutable evidence for ZPE ½ quanta in any form, including continuously existing waves. (See Appendix F, added in 2018 text version.)

Bottom line: No known experiment proves vacuum particle/anti-particle pair production.

10.3 Further Considerations of Uncertainty Principle

10.3.1 Uncertainty Principle and Commutation Relations

Non-relativistic Quantum Mechanics

Brief review of how non-commutation results in uncertainty

Recall from NRQM that the uncertainty principle is a direct result of non-commutation of certain operator pairs. In short, most elementary quantum mechanics courses prove that for any operators *P* and *Q*, the relation (10-2) below, where the < > brackets indicate expectation value and Δ indicates standard deviation, holds. Note that ΔP and ΔQ are not operators, so we can switch their order.

$$(\Delta P)(\Delta Q) \geq \tfrac{1}{2}\left| i\langle [P,Q]\rangle \right| . \qquad (10\text{-}2)$$

For position and momentum, we know that

$$\left[x^i, p^j\right] = i\hbar\delta_{ij} \quad \rightarrow \quad \left[p^j, x^i\right] = -i\hbar\delta_{ij} . \qquad (10\text{-}3)$$

Taking $Q = x^i$ and $P = p^j$, (10-3) into (10-2) yields the position/momentum uncertainty principle,

$$\left(\Delta p^j\right)\left(\Delta x^i\right) \geq \frac{\hbar}{2}\delta_{ij} \quad \text{equivalent to} \quad \left(\Delta x^i\right)\left(\Delta p^j\right) \geq \frac{\hbar}{2}\delta_{ij} . \qquad (10\text{-}4)$$

So non-commutation of operators means an uncertainty principle exists for those operators.

The LHS of (10-4) is generally equal to, or only a little larger than, the RHS. For Gaussian wave packets, for example, the RHS equals the LHS.

[1] See http://en.wikipedia.org/wiki/Unruh_effect.

[2] See V.F. Mukhanov and S. Winitzki, *Introduction to Quantum Effects in Gravity*, Cambridge (2010), Chap. 8.

[3] S. Weinberg, "The Cosmological Constant Problem", *Reviews of Modern Physics,* **61**, 1, 1-23 (Jan 1989). See also, Appendix A of this chapter.

Relativistic Quantum Mechanics

Similar logic results in a similar relation as (10-4) for RQM.

Quantum Field Theory

For 1st quantization (particles), we used (10-3). (See Chap. 1, pg. 4.) For 2nd quantization (fields),

$$\left[\phi_r\left(\mathbf{x},t\right),\pi_s\left(\mathbf{y},t\right)\right]=i\hbar\delta_{rs}\delta\left(\mathbf{x}-\mathbf{y}\right), \tag{10-5}$$

Non-commutation in field theory leads to field uncertainty

where ϕ_r represents a (bosonic) field, and π_s represents the field conjugate momentum. Substituting $Q = \phi_r$ and $P = \pi_s$ in (10-2), we get

$$\left(\Delta\phi_r\left(\mathbf{x},t\right)\right)\left(\Delta\pi_s\left(\mathbf{y},t\right)\right)\geq\frac{\hbar}{2}\delta_{rs}\delta\left(\mathbf{x}-\mathbf{y}\right), \tag{10-6}$$

an uncertainty principle for fields.

Non-commutation in field theory also leads to ZPE

Recall from Chap. 3 (or the Wholeness Chart at the end of Chap. 5) that (10-5) gives rise to the commutation relations for creation and destruction operators, such as $[a(\mathbf{k}),a^\dagger(\mathbf{k}')] = \delta_{\mathbf{k}\mathbf{k}'}$. Those commutation relations are used in the derivation of H_0 in terms of number operators (Chap. 3, (3-54) to (3-56), pgs. 54-55) and in so doing, give rise to the ½ quanta energy terms in H_0.

One reason ZPE sometimes said due to uncertainty

So, in QFT, the commutation relation (10-5) gives rise to i) the uncertainty principle for fields (10-6) and also to ii) the ½ quanta in the vacuum. This is one reason why the ½ quanta are often said to be the result of the quantum uncertainty principle.

10.3.2 Uncertainty Principle for Fields and Measurement

But QFT fields not measurable anyway, so uncertainty not relevant

As we noted at the end of Chap. 3 in Wholeness Chart 3-4, pg. 79 and also showed in Chap. 7 pgs. 189-190, fields are not observable. They have zero expectation values. The same is true of their conjugate momenta. This might lead us to believe that (10-6) is essentially meaningless with regard to measurement in the real world, since we can't measure the quantities it describes.

10.3.3 Uncertainty Principle for Particles and Measurement

Consider uncertainty in QFT for particles, not fields

Almost the entirety of this book, like almost all introductory texts on QFT, deals with particles in pure momentum eigenstates, represented by **k** or **p**. Recall we have noted that the 3-momentum eigenstates are good enough approximations to real particles (which are invariably wave packets and not momentum eigenstates) to yield highly accurate answers for scattering problems.

Pure **k** (or **p**) particle states extend across the entire region of 3D space in which measurements could take place (which could be the entire universe). Whenever a given such particle momentum (or energy) is measured, it will have the same value each time. That is, there is no variation in momentum (or energy).

In typical QFT, ZPE particles are pure **k** *states. No momentum or energy uncertainty, anyway.*

This applies not simply to real particles, but also to the presumed ½ quanta of the vacuum. Each such quantum has definite **k** value and is an eigenstate, not a general state (not a superposition of eigenstates.) Thus, there would be no variation in its energy or momentum upon measurement (assuming it could ever be measured.)

Further, both large and small |**k**| values would be comprised by waves that extend spatially to infinity. Each would have definite momentum and completely indefinite location. But microscopic black holes need highly energetic particles to be packed into small regions. The ½ quanta do not seem to satisfy this requirement.

10.3.4 Uncertainty Principle and Ground States

In NRQM, you probably saw how the ground state of a system such as the hydrogen atom, or the harmonic oscillator, could be deduced, approximately, from the uncertainty principle.[1] In such cases, the estimated ground state energy (lowest possible energy state of the system) was found to be non-zero. This seems to be the source for referencing the uncertainty principle in the pair popping and the vacuum ½ quanta cases.

However, the hydrogen atom and harmonic oscillator are bound state systems, i.e., particles experience a force via a potential. They are not free systems. Free systems in NRQM do not have to have a non-zero energy ground state. Free particle energy ground states can be zero (for zero

[1] See, for examples, R.G. Winter, *Quantum Physics* (Wadsworth, 1979), pgs. 18-19, or S. Gasiororwicz, *Quantum Physics* (Wiley, 1974), pgs. 37-40.

velocity, where we ignore the mass-energy equivalence in NRQM, and have wave functions of form e^{-ikx}, which have $\Delta\mathbf{x} \to \infty$). That is, the lowest energy eigenstate for free particles has zero energy.

Both the pair popping of lay literature and the ½ quanta of QFT represent free systems. If the uncertainty principle does not lead to a specific, discrete non-zero ground state for free systems of energy eigenstate particles in NRQM, then how can one use that particular argument to support non-zero ground state free particles in QFT?[1]

In fact, the usual treatment of uncertainty in state energy and momentum to determine the ground state gives one a zero ground state for free particles in any quantum theory, including QFT. Significantly, the vacuum states in NRQM and RQM have zero energy.

For a free particle wave packet (see Sect. 10.4 below for more), the uncertainty principle implies that any of a range of values for energy could be found upon measurement. But this range is continuous. That is, there is no specific, discrete ground state energy, such as $\frac{1}{2}\hbar\omega_{\mathbf{k}}$, for free wave packets. So, again, why should we expect the uncertainty principle to provide one in the pair popping scenario or QFT, if it doesn't in NRQM or RQM? Its use in the latter two theories is the supposed justification for applying it in the former one.

Specifically, the vacuum has $\Delta x^i \to \infty$. $(\Delta x^i)(\Delta p^i) \geq \hbar/2$, so we can have $(\Delta p^i) = 0$, and Δp^i = exactly 0. One might surmise from this that a zero energy vacuum does not violate uncertainty.

10.3.5 Uncertainty Principle Conclusions

Bottom line: In standard QFT, particularly with particles in **k** eigenstates, vacuum fluctuations do not appear to arise from an uncertainty principle for fields or particles.

10.4 Wave Packets

But wave packet approach to QFT would have uncertainty

One could counter much of the foregoing Sect. 10.3 with the argument that definite **k** states are not precise representatives of real world particles, which are really wave packets, and so our entire development of QFT is simply an approximation. That is, the ½ quanta are more probably wave packets of indefinite **k**, with an expectation (mean) value for **k** of $\bar{\mathbf{k}}$ and a standard deviation $\Delta\mathbf{k}$ about that mean. Then, all this talk of uncertainty in **k** and **x** would start to have meaning.

So, examine QFT for wave packets

To examine this, we would need to re-develop our entire theory for the continuous solutions, rather than discrete solutions, to the QFT wave equations. (See Chap. 3, (3-36) and (3-37) on pg. 50.) This would take an entire chapter, or more, and would lead us astray from more immediate goals. However, in Appendix C, I summarize important steps in this development and reference the book website, where detailed development of continuous solutions is presented. I do not recommend study of that appendix on one's first sojourn into QFT. I do recommend it once one has gained a solid footing in the theory of the discrete eigen solutions, which comprise almost the entire book.

Hamiltonian for discrete solution fields

The Hamiltonian operator for discrete solutions of the scalar field is

$$\begin{gathered} H_0^0 = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(N_a(\mathbf{k}) + \tfrac{1}{2} + N_b(\mathbf{k}) + \tfrac{1}{2} \right) \\ N_a(\mathbf{k}), N_b(\mathbf{k}) = \text{number of real particles, dimensionless} \\ \tfrac{1}{2} = \text{number of vacuum particles, dimensionless.} \end{gathered} \tag{10-7}$$

Hamiltonian for continuous solution fields, i.e., wave packet particles

The corresponding Hamiltonian operator for continuous solutions of the scalar field is (where we write $\iiint d^3k$ [which is the same as $\int d\mathbf{k}$] as $\int d^3k$ to save space yet make clear we mean triple integration over momentum space; and use $\delta(0)$ as a short form for $\delta^3(0)$)

$$\begin{gathered} H_0^0 = \int \omega_{\mathbf{k}} \left(\mathcal{N}_a(\mathbf{k}) + \tfrac{1}{2}\delta(0) + \mathcal{N}_b(\mathbf{k}) + \tfrac{1}{2}\delta(0) \right) d^3k \\ \mathcal{N}_a(\mathbf{k}), \mathcal{N}_b(\mathbf{k}) = (\text{num real particles, all } \mathbf{x} \text{ space}) / \text{unit } \mathbf{k} \text{ space vol, dimensions } 1/M^3 \\ \tfrac{1}{2} = (\text{num vacuum particles per } \mathbf{x} \text{ space vol}) / \text{unit } \mathbf{k} \text{ space vol, dimensions } 1/M^3 \\ \delta(0) = \text{infinite volume of universe} \end{gathered} \tag{10-8}$$

[1] Those wishing to counter with the argument that free states in QFT are harmonic oscillators should see Chap. 3, Sect. 3.12, pgs. 69-70.

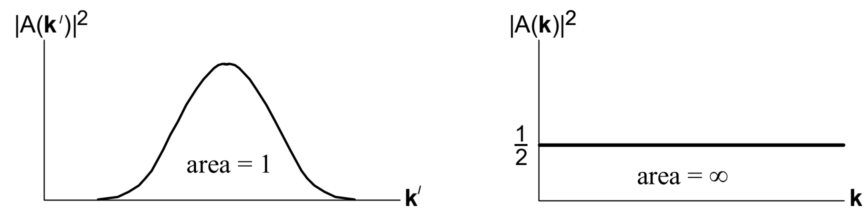
The units in the parenthetical part of the integrand of (10-8) are natural units with momentum cubed in the denominator. Momentum, in natural units, has the dimension of mass since velocity in those units is dimensionless. $\mathcal{N}_a(\mathbf{k})$ and $\mathcal{N}_b(\mathbf{k})$ for the continuous solutions are number density operators, rather than just number operators as in the discrete case.

The mathematical form in position space of a scalar wave packet ket of a particle (not anti-particle) in QFT is

Single particle QFT state in position space

$$|\phi\rangle = \left| \int \frac{A(\mathbf{k}')e^{-ik'x}}{\sqrt{(2\pi)^3}} d^3k' \right\rangle , \tag{10-9}$$

Where $A(\mathbf{k}')$ is a *coefficient* (*not an operator*) that defines the shape of the wave packet. Typically, $|A(\mathbf{k}')|$ has a Gaussian shape as in Fig. 10-3a



a) Typical shape of $|A(\mathbf{k}')|^2$ for particle wave b) Shape of $|A(\mathbf{k})|^2$ for vacuum

Figure 10-3. Shape of $|A(\mathbf{k}')|^2$ Coefficient in k Space

$|A(\mathbf{k}')|^2$ represents the wave packet density per unit volume in $\mathbf{k}'$ space at a given $\mathbf{k}'$ for a single particle wave packet. By doing Prob. 1, you can show that for $|\phi\rangle$ having unit norm, then

Coefficient property for unit norm state

$$\int |A(\mathbf{k}')|^2 d^3k' = 1 . \tag{10-10}$$

Wholeness Chart 10-2 in Appendix C, by comparison with discrete solutions, may make this a bit clearer.

The expectation value of the number density operator (10-8) for the wave packet state (10-9) is

Number density expectation value for single particle wave packet state

$$\langle\phi|\mathcal{N}_a(\mathbf{k})|\phi\rangle = \left\langle \int \frac{A(\mathbf{k}'')e^{-ik''x}}{\sqrt{(2\pi)^3}} d^3k'' \right| \mathcal{N}_a(\mathbf{k}) \left| \int \frac{A(\mathbf{k}')e^{-ik'x}}{\sqrt{(2\pi)^3}} d^3k' \right\rangle = |A(\mathbf{k})|^2 \tag{10-11}$$

At each $\mathbf{k}'$ inside the ket, the number operator $\mathcal{N}_a(\mathbf{k})$ will pick up the number density of particles at that value of $\mathbf{k} = \mathbf{k}'$. Had our ket state comprised two wave packet particles of exactly the same particle state (same $A(\mathbf{k}')$ distribution function), we would have obtained

Number density expectation value for two particle wave packet state

$$\langle 2\phi|\mathcal{N}_a(\mathbf{k})|2\phi\rangle = 2|A(\mathbf{k})|^2 \tag{10-12}$$

However, for simplicity, we are not going to consider multiparticle states anymore, just single particle ones.

We now ask "what is the expectation value of energy for the single particle wave packet state of (10-11)?" The answer is

Energy expectation value of single particle wave packet state

$$\bar{E} = \langle\phi|H_0^0|\phi\rangle = \left\langle \int \frac{A(\mathbf{k}'')e^{-ik''x}}{\sqrt{(2\pi)^3}} d^3k'' \right| \int \omega_{\mathbf{k}} \left(\mathcal{N}_a(\mathbf{k}) + \mathcal{N}_b(\mathbf{k}) + \delta(0)\right) d^3k \left| \int \frac{A(\mathbf{k}')e^{-ik'x}}{\sqrt{(2\pi)^3}} d^3k' \right\rangle$$

$$= \int \omega_{\mathbf{k}} \left(|A(\mathbf{k})|^2 + \delta(0) \right) d^3k = \int \omega_{\mathbf{k}} \left(|A(\mathbf{k})|^2 + \tfrac{1}{2}\delta(0) + \tfrac{1}{2}\delta(0) \right) d^3k \tag{10-13}$$

Infinite vacuum energy from an integral

$$= \underbrace{\int |A(\mathbf{k})|^2 \omega_{\mathbf{k}} d^3k}_{\substack{\text{real particle expectation} \\ \text{energy } = \bar{\omega}}} + \underbrace{\delta(0) \int \tfrac{1}{2}\omega_{\mathbf{k}} d^3k}_{\substack{\text{vacuum particle} \\ \text{expectation energy}}} + \underbrace{\delta(0) \int \tfrac{1}{2}\omega_{\mathbf{k}} d^3k}_{\substack{\text{vacuum anti-particle} \\ \text{expectation energy}}} .$$

We get two parts to our answer. One is the expectation value of the wave packet particle energy $\bar{\omega}$. The other is the energy of the vacuum, infinite and reminiscent of the discrete solutions states summation, which we repeat below for a single particle state for comparison.

Compare to energy expectation value of single particle eigenstate

$$\bar{E} = \langle \phi_{\mathbf{k}'} | H_0^0 | \phi_{\mathbf{k}'} \rangle = \langle \phi_{\mathbf{k}'} | \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(N_a(\mathbf{k}) + \tfrac{1}{2} + N_b(\mathbf{k}) + \tfrac{1}{2} \right) | \phi_{\mathbf{k}'} \rangle = \langle \phi_{\mathbf{k}'} | \omega_{\mathbf{k}'} + \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(\tfrac{1}{2} + \tfrac{1}{2} \right) | \phi_{\mathbf{k}'} \rangle$$

$$= \left(\omega_{\mathbf{k}'} + \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(\tfrac{1}{2} + \tfrac{1}{2} \right) \right) \langle \phi_{\mathbf{k}'} || \phi_{\mathbf{k}'} \rangle = \underbrace{\omega_{\mathbf{k}'}}_{\substack{\text{real} \\ \text{particle} \\ \text{energy}}} + \underbrace{\sum_{\mathbf{k}} \frac{1}{2} \omega_{\mathbf{k}}}_{\substack{\text{vacuum} \\ \text{particle} \\ \text{energy}}} + \underbrace{\sum_{\mathbf{k}} \frac{1}{2} \omega_{\mathbf{k}}}_{\substack{\text{vacuum} \\ \text{anti-particle} \\ \text{energy}}} . \qquad (10\text{-}14)$$

Infinite vacuum energy from a sum

10.4.1 An Important Point

Note in (10-13) that the particle state, being a wave packet, has an envelope $A(\mathbf{k})$, which is effectively only non-zero over a limited range of $\mathbf{k}$ values. And the integral (10-10) means its amplitude even over that range is finite.

On the other hand, the two vacuum energy contributions of (10-13) (last two terms) have no such limiting envelope. In fact, the amplitude of the effective coefficient for each of those terms is $1/\sqrt{2}$ per unit $\mathbf{x}$ space volume, over the entire range of $\mathbf{k}$. For the vacuum, we have $A_\infty(\mathbf{k}) = 1/\sqrt{2}$ in the next to last term of (10-13), and coefficient $B_\infty(\mathbf{k}) = 1/\sqrt{2}$ in the last term. See Fig. 10-3b.

Vacuum energy in continuous case is single particle wave packet per unit volume with amplitude constant = $1/\sqrt{2}$ for all $\mathbf{k}$ (similar for anti-particle)

So, according to (10-13), we can think of the vacuum as two single particle wave packets (particle and anti-particle) in every $\mathbf{x}$ space unit volume, each with constant amplitude = $1/\sqrt{2}$ over all $\mathbf{k}$ in $\mathbf{k}$ space. But such a constant amplitude cannot be normalized like (10-10), and each vacuum particle then has a numerical probability of being measured of infinity. (Recall, $|A(\mathbf{k})|^2$ is probability density per unit $\mathbf{k}$.) This is clearly mathematically inconsistent.

In other words, there appears to be no interpretation whereby the vacuum is comprised, in a meaningful way, of individual wave packet particle states. Without wave packet states, we have no uncertainty principle, no greater energy/momentum expectation value for shorter time and distance scales.

And this does not appear to make sense

10.4.2 If the Vacuum Had Wave Packet ½ Quanta

If the vacuum were filled with ½ quanta wave packets having $\bar{\omega}_{\mathbf{k}}/2$ energy expectation values, then each such wave packet would need its own envelope $A_j(\mathbf{k})$, where j labels the wave packet. In other words, our expected energy would, instead of (10-13), look like

If vacuum ZPE had many wave packets, math form of energy expectation would look different

$$\bar{E} = \langle \phi | H_0^0 | \phi \rangle = \underbrace{\int |A(\mathbf{k})|^2 \omega_{\mathbf{k}} d^3k}_{\substack{\text{real particle} \\ \text{expectation energy} = \bar{\omega}}} + \sum_j^{\infty} \underbrace{\tfrac{1}{2} \int |A_j(\mathbf{k})|^2 \omega_{\mathbf{k}} d^3k}_{\substack{\text{vacuum expectation energy} \\ \bar{\omega}_j/2 \text{ for } j\text{th particle}}} + \sum_j^{\infty} \underbrace{\tfrac{1}{2} \int |A_j(\mathbf{k})|^2 \omega_{\mathbf{k}} d^3k}_{\substack{\text{vacuum expectation energy} \\ \bar{\omega}_j/2 \text{ for } j\text{th anti-particle}}} . \qquad (10\text{-}15)$$

The vacuum contribution, for wave packet ½ quanta, would look like the RHS of (10-15), rather than the RHS of the last row of (10-13). If we have ½ quanta wave packets in the vacuum, then each such ½ quantum needs a wave packet form, and that means an envelope $A_j(\mathbf{k}) \neq$ constant for all $\mathbf{k}$. It would need $A_j(\mathbf{k})$ obeying (10-10).

10.4.3 Wave Packet Conclusions:

QFT seems to predict a single, weird looking wave packet in each unit volume of the vacuum, not many usual type packets

1. QFT does not appear to predict separate wave packets in the vacuum, but a seeming single particle packet (and a similar single anti-particle packet) per unit **x** space volume of envelope $A_j(\mathbf{k}) = 1/\sqrt{2}$.
2. Thus, the uncertainty principles for energy/time and momentum/position do not appear applicable to the vacuum as described by standard QFT.

Bottom line: QFT wave packet analysis does not appear to support vacuum fluctuations.

10.5 Further Considerations

10.5.1 Vacuum Fluctuations May Yet Exist

Vacuum fluctuations may still exit.

Of course, all of the above does not mean that vacuum fluctuations do not exist. The uncertainty principle leads one to think that they may well proliferate in the vacuum. Many of the world's top physicists are, in fact, convinced that they do in some form. But, the precise mechanism by which it would occur, if it does, is not obvious in standard QFT.

10.5.2 More Advanced Theories

Vacuum fluctuations may be valid part of advanced theories

As of the date of this text, we have no viable theory of quantum gravity. Perhaps, in that theory, when it is finally developed, there will be vacuum fluctuations, driven by the uncertainty principle. Further, superstring (more properly M-theory) theorists regularly consider vacuum fluctuations of the strings. The competing theory of loop quantum gravity considers spacetime comprised of small "quanta" of geometry linked in ways that lead to microscopic behavior similar to that of spacetime foam. So, perhaps one of these theories, or a third known as "twistor theory", will one day put a firm foundation under vacuum fluctuations.

Bottom line: It is possible more advanced theories can prescribe vacuum fluctuations.

10.6 Chapter Summary

Wholeness Chart 10-1, along with the following bottom line statement, summarize the present chapter. See also, Appendix F, which was added to the 2018 version of the text.

Bottom line for this chapter: According to standard QFT, only three particle virtual bubbles can truly be called "vacuum fluctuations", and they have zero net energy so cannot contribute to vacuum energy.

Only vacuum bubbles are truly vacuum fluctuations, but their energy = 0

10.7 Addenda

10.7.1 Hidden in the Theory is a Way for ½ Quanta to Disappear

Author's research: If include alternative solutions to field equations in QFT, ZPE disappear

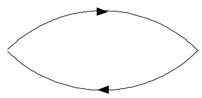
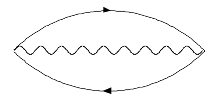
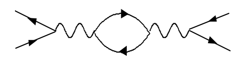
I noted in Chap. 3 (footnote on pg. 50) that there are little recognized alternative solutions to the QFT field equations that are not used in the standard renditions of the theory. The traditional solutions have the familiar $\pm\, i\,(\omega_{\mathbf{k}}\, t - \mathbf{k}\cdot\mathbf{x})$ form in the exponent. The alternative solutions to the field equations have form $\pm\, i(\omega_{\mathbf{k}}\, t + \mathbf{k}\cdot\mathbf{x})$. I call these supplemental solutions.

When the supplemental solutions are included in the theory (see footnote citation), one finds boson energy terms $-\omega_{\mathbf{k}}/2$ arising in the vacuum that cancel the contributions from the traditional solutions, leaving a net vacuum expectation energy of zero. A similar cancellation effect occurs for fermions. This, of course, is closer to the observed value.

10.7.2 Caveats Again

As I said earlier, much in this chapter comprises my own ruminations on the vacuum fluctuations subject. The reader should consider this in drawing her/his own conclusions.

Wholeness Chart 10-1. Comparison of Vacuum Fluctuation Scenarios

	Pair Production	**½ Quanta, Zero Point Energy (ZPE)**	**Virtual Bubbles**	**Radiative Corrections**
Basic Description	Particle and anti-particle pairs continually popping in and out of vacuum	Quanta of $\frac{1}{2}\omega_{\mathbf{k}}$ sitting in vacuum, particles and anti-particles	Three virtual particles arise from and dissolve into vacuum	Higher order virtual particle corrections to lowest order interaction
Chapter in this book	This chapter.	Chaps. 3,4,5	Chap. 8	Chap. 9
How Proposed to Arise?	Uncertainty principle for states	2^{nd} quantization	Interaction terms in Hamiltonian	Interaction terms in Hamiltonian
Typical Feynman diagram		None		
Does such a Feynman diagram exist in QFT?	No	N/A	Yes	Yes
Free or Interaction Theory?	Not in QFT, free or interacting	Free	Interacting	Interacting
Interacts with rest of universe?	Yes, it is claimed	No (part of free theory)	No	Yes
Positive, negative, or zero vacuum energy?	Positive, borrowed via uncertainty principle	Bosons positive, fermions negative, total negative	Zero. Negative virtual energy cancels positive	No contribution to vacuum energy
Mathematics behind it	Non-commutation $\mathbf{p}_{\mathbf{x}}$ and $\mathbf{x}$ $\rightarrow$ uncertainty in $\mathbf{p}_{\mathbf{x}}$ and $\mathbf{x}$	Non-commutation π and $\phi \rightarrow \frac{1}{2}\omega_{\mathbf{k}}$ in vacuum for all $\mathbf{k}$	Interaction term $e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$ in $\mathcal{H}$	Same as at left
Uncertainty? Measure?	Can measure $\mathbf{p}_{\mathbf{x}}$, $\mathbf{x}$ in principle	Can't measure π, ϕ even in principle	Can't measure	Measure indirectly
Exist continually or evanescently?	Evanescent	Continually exist (no mechanism for otherwise)	Evanescent	Evanescent
Arise in vacuum alone?	Yes	Yes	Yes	No
Experiments prove?				
Casimir plates	No, despite claims	No	No	Possibly
Lamb shift	No	No	No	Yes
Anomalous magnetic moment	No	No	No	Yes
Fulling-Davis-Unruh	No	Inconclusive	No	No
Yields known vacuum energy?	No	No	Close, at the least	N/A
Wave packet theory supports?	No	Ill-defined. Does not appear supported by QFT math.	Yes	Yes

10.8 Appendix A: Theoretical Value for Vacuum Energy Density

10.8.1 The Cut-off Method

Given (10-14), the vacuum energy density of the ZPE in a rectangular solid shape volume V is

$$\bar{\rho}_v = \frac{\bar{E}_v}{V} = \frac{1}{V}\langle 0|H_0^0|0\rangle = \frac{1}{V}\langle 0|\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(\tfrac{1}{2}+\tfrac{1}{2}\right)|0\rangle = \frac{1}{V}\sum_{\mathbf{k}}\omega_{\mathbf{k}} = \frac{1}{l_1 l_2 l_3}\sum_{\mathbf{k}}\omega_{\mathbf{k}} = \sum_n \frac{\omega_{\mathbf{k}_n}}{l_1 l_2 l_3}, \qquad (10\text{-}16)$$

where l_i is the length of the ith side of volume V inside of which the particle/waves are found, and we have used slightly different labeling in the last expression to suit our immediate needs. Boundary conditions on V give the wavelength of the nth wave (n = 1, 2,..) in the x^1 direction as $\lambda_{n\,1} = l_1/n$, with similar relations for the other two directions. So, the wave number $k_{n\,1}$ of the nth wave is

$$k_{n\,1} = \frac{2\pi}{\lambda_{n\,1}} = \frac{2\pi n}{l_1}. \qquad (10\text{-}17)$$

Defining $\Delta k_1 = k_{(n+1)\,1} - k_{n\,1}$ we have, from (10-17),

$$\Delta k_1 = \frac{2\pi(n+1)}{l_1} - \frac{2\pi\, n}{l_1} = \frac{2\pi}{l_1} \quad \text{so that} \quad \frac{1}{l_1} = \frac{\Delta k_1}{2\pi}.. \qquad (10\text{-}18)$$

Similar results hold for l_2 and l_3. Thus, (10-16) (with the relativistic expression for energy) is

$$\bar{\rho}_v = \sum_n \frac{\omega_{\mathbf{k}_n}}{l_1 l_2 l_3} = \sum_n \omega_{\mathbf{k}_n}\frac{\Delta k_1}{2\pi}\frac{\Delta k_2}{2\pi}\frac{\Delta k_3}{2\pi} = \sum_n \sqrt{m^2+\mathbf{k}_n{}^2}\,\frac{\Delta k_1}{2\pi}\frac{\Delta k_2}{2\pi}\frac{\Delta k_3}{2\pi}. \qquad (10\text{-}19)$$

For l_1 very large, from (10-18), $\Delta k_1 \to dk_1$, with similar expressions for large l_2 and l_3. In the limit of large l_i (large volume V), (10-19) becomes

$$\bar{\rho}_v = \int_{-\infty}^{\infty}\sqrt{m^2+\mathbf{k}^2}\,\frac{dk_1}{2\pi}\frac{dk_2}{2\pi}\frac{dk_3}{2\pi} = \int_{-\infty}^{\infty}\frac{1}{(2\pi)^3}\sqrt{m^2+\mathbf{k}^2}\,d^3k \qquad (10\text{-}20)$$

Using (9-9) on pg. 260, we find this becomes

$$\bar{\rho}_v = \int_0^{\infty}\frac{1}{(2\pi)^3}\sqrt{m^2+k^2}\,4\pi k^2 dk = \frac{1}{2\pi^2}\int_0^{\infty}\sqrt{m^2+k^2}\,k^2 dk. \qquad (10\text{-}21)$$

This is obviously infinite, unless we take an upper limit cutoff, typically considered the Planck scale mass (energy). This is because a particle with energy of the Planck mass is assumed to have an associated Compton wavelength (size of the particle) so small that its associated mass-energy forms a microscopic black hole. Such particles would instantaneously collapse, so smaller size (larger energy) particles may not be able to exist in our universe. Given such logic, with Λ = Planck mass >> m, we find (10-21) is

$$\bar{\rho}_v = \frac{1}{2\pi^2}\int_0^{\Lambda}\sqrt{m^2+k^2}\,k^2 dk \approx \frac{1}{2\pi^2}\int_0^{\Lambda}k^3 dk = \frac{\Lambda^4}{8\pi^2} \qquad (10\text{-}22)$$

In natural units, the Planck mass ≈ 1.22×10^{19} GeV. Thus, we get the theoretical value

$$\bar{\rho}_v \approx \frac{\left(1.22\times10^{19}\right)^4}{8\pi^2}\text{GeV}^4 = 2.80\times10^{74}\ \text{GeV}^4. \qquad (10\text{-}23)$$

Note length in natural units is GeV^{-1}, so an energy per unit volume would be measured in GeV^4. The experimental value for the upper limit on energy density of the vacuum is

$$\bar{\rho}_v \le 10^{-47}\ \text{GeV}^4, \qquad (10\text{-}24)$$

a discrepancy between theory and experiment by a factor of more than 10^{120}. Yikes.

Note that in this scenario, the vacuum particles are complex sinusoidal waves extending across the universe from one end to the other (just like the real particles in this scenario).

10.8.2 Other Methods of Calculating Vacuum Energy

As pointed out by J. Martin [Everything you always wanted to know about the cosmological constant problem (but were afraid to ask), *C. R. Physique*, **13**, 566–665 (2012)], the cutoff method is not Lorentz invariant, since the energy Λ is different in different frames, and therefore, though simple in concept, is not valid. Martin uses a Lorentz invariant evaluation of (10-20) and arrives at a vacuum energy density "only" 10^{55} times greater than that observed.

10.9 Appendix B: Symmetry Breaking, Mass Terms, and Vacuum Pairs

This appendix is not for newcomers to QFT, but for veterans familiar with electroweak interactions and symmetry breaking.

In QED, mass terms are part of the free Hamiltonian. So, their effect shows up when we determine the energy expectation value for the Hamiltonian for any given state, including the vacuum state (VEV of the Hamiltonian). Thus, the mass terms end up contributing as part of the ½ energy quanta of the vacuum and, as we saw, do not have Feynman diagram type interactions.

In electro-weak theory with symmetry breaking, however, mass terms arise from coupling of the Bose and Fermi fields with the Higgs field. When the Higgs field symmetry breaks, the Higgs field gets a VEV, and massless fields get masses. Since these mass terms, such as

$$m\bar{\psi}\psi \tag{10-25}$$

arise in the interaction (not the free) part of $\mathcal{H}$ and $\mathcal{L}$, they result in interactions of the type shown in the Feynman diagrams of Fig. 10-4.

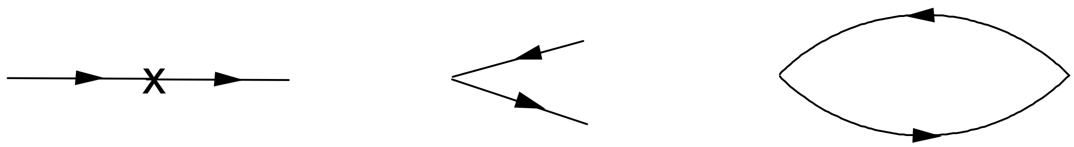


Figure 10-4. Three Types of Interactions from Symmetry Breaking Mass Terms

<u>For Fig. 10-4b</u>

One could then posit that we do indeed have pairs of particles "popping out" of the vacuum, as in Fig. 10-4b, which represents the first order term in the amplitude. Similarly, there would be destructions of fermion and anti-fermion pairs as well (not shown). Key points to be made in this regard are

- The energies and momenta of the particles in Fig. 10-4b must still sum to zero, so there is no vacuum energy contribution.
- The pairs of Fig. 10-4b are real, not virtual. Real particles cannot have negative energy. But since total energy should sum to zero, one of the particles must have negative energy. Hence, we can conclude that Fig. 10-4b does not represent a real physical process and cannot occur.
- The particle pairs so produced presumably would not yield the Casimir plate quantitative result. The ½ quanta generating that result had different mathematical factors than would result from the particles of Fig. 10-4b.
- The pairs do not arise in the vacuum alone, but as a result of the Higgs field. In essence, Fig. 10-4b really has a Higgs field source (which might be visualized as a pre-symmetry breaking Higgs particle coming in from the left.) These pairs are *not* pure vacuum pairs.
- At high energy, particles are massless and terms of form (10-25) do not exist. That is, they are replaced by a Higgs field interacting with the fermion and anti-fermion as discussed in the prior point. In other words, there are no interactions like that of Fig. 10-4b at high energy, so no tiny black holes (posited to result from high energy vacuum particles) could form.
- The probability of interaction occurring (pair formation) is not a function of e/m coupling α, nor weak coupling, nor strong coupling, but of the Higgs coupling.

<u>For Fig. 10-4c</u>

Using the Dyson-Wick expansion for the amplitude, we would find second order terms of form

$$m^2 \int d^4x_1 d^4x_2 N\left\{ \left(\bar{\psi}\psi\right)_{x_1} \left(\bar{\psi}\psi\right)_{x_2} \right\} \tag{10-26}$$

represented by Fig. 10-4c.

- The energies and momenta of the particles in Fig. 10-4c must still sum to zero, so the total loop energy is zero, and there is no vacuum energy contribution.
- One of the virtual particles must have negative energy. If one insisted on applying an uncertainty principle, then it must be applied for negative energy as well. Hence, any "borrowed energy" fluctuations about zero employed to produce Fig. 10-4c must be both positive and negative. The sum of all such fluctuations must be zero, and that means zero vacuum energy.
- The last four bullets for Fig. 10-4b apply here as well.

These points lead to the conclusions that neither of the pair productions of Fig. 10-4b and c is the vacuum pair popping production commonly referred to in the literature, and they make no contribution to vacuum energy.

For Higgs Vacuum Energy Remnant

To be complete in our discussion of vacuum energy, we need to consider the vacuum energy generated by the spontaneous breaking of the electroweak symmetry via the Higgs mechanism. That process leaves an energy density remnant in the vacuum, known as the Higgs condensate energy density.

But the Higgs condensate energy simply sits (essentially statically) in the vacuum unconnected to other particles. It is not related to particle-antiparticle pair popping in and out of existence via the uncertainty principle.

10.10 Appendix C: Comparison of QFT for Discrete vs Continuous Solutions

The overview in Wholeness Chart 10-2 (below and on the following pages) is presented without explanatory text (which can be found at the website for this book listed on pg. xvi, opposite pg. 1). Extensive study of it may be warranted for QFT veterans but is not recommended for newcomers.

Wholeness Chart 10-2. Discrete vs Continuous Versions of QFT
(Only Scalars Shown)

	Discrete	**Continuous**
Field Equations Solutions	$\phi(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}}\left(a(\mathbf{k})e^{-ikx} + b^{\dagger}(\mathbf{k})e^{ikx}\right)$ $\phi^{\dagger}(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}}\left(b(\mathbf{k})e^{-ikx} + a^{\dagger}(\mathbf{k})e^{ikx}\right)$	$\phi(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\,\omega_{\mathbf{k}}}}\left(a(\mathbf{k})e^{-ikx} + b^{\dagger}(\mathbf{k})e^{ikx}\right)$ $\phi^{\dagger}(x) = \int \frac{d^3k}{\sqrt{2(2\pi)^3\,\omega_{\mathbf{k}}}}\left(b(\mathbf{k})e^{-ikx} + a^{\dagger}(\mathbf{k})e^{ikx}\right)$
Coefficient commutators	$\left[a(\mathbf{k}),a^{\dagger}(\mathbf{k}')\right]=\left[b(\mathbf{k}),b^{\dagger}(\mathbf{k}')\right]=\delta_{\mathbf{k}\mathbf{k}'}$	$\left[a(\mathbf{k}),a^{\dagger}(\mathbf{k}')\right]=\left[b(\mathbf{k}),b^{\dagger}(\mathbf{k}')\right]=\delta(\mathbf{k}-\mathbf{k}')$
$\mathcal{H}_0^0$	$\pi_0^0\dot{\phi} + \pi_0^{0\dagger}\dot{\phi}^{\dagger} - \mathcal{L}_0^0 = \left(\dot{\phi}\dot{\phi}^{\dagger} + \nabla\phi^{\dagger}\cdot\nabla\phi + \mu^2\phi^{\dagger}\phi\right)$	as at left, and in terms of number operators $\int \omega_{\mathbf{k}}\left(\frac{\mathcal{N}_a(\mathbf{k})}{V} + \frac{1}{2} + \frac{\mathcal{N}_b(\mathbf{k})}{V} + \frac{1}{2}\right)d^3k \quad V \to \delta(0)$
H_0^0	$\sum_{\mathbf{k}} \omega_{\mathbf{k}}\left(N_a(\mathbf{k}) + \frac{1}{2} + N_b(\mathbf{k}) + \frac{1}{2}\right)$ $N_a(\mathbf{k}) = a^{\dagger}(\mathbf{k})\,a(\mathbf{k})$, $N_b(\mathbf{k}) = b^{\dagger}(\mathbf{k})\,b(\mathbf{k})$	$\int \omega_{\mathbf{k}}\left(\mathcal{N}_a(\mathbf{k}) + \frac{1}{2}\delta(0) + \mathcal{N}_b(\mathbf{k}) + \frac{1}{2}\delta(0)\right)d^3k$ $\mathcal{N}_a(\mathbf{k}) = a^{\dagger}(\mathbf{k})\,a(\mathbf{k})$, $\mathcal{N}_b(\mathbf{k}) = b^{\dagger}(\mathbf{k})\,b(\mathbf{k})$
Operator Units	$N_a(\mathbf{k})$, number of real particles, unitless, M^0 ½ , number of vacuum particles, unitless $a(\mathbf{k})$, $a^{\dagger}(\mathbf{k})$, unitless Similar for $N_b(\mathbf{k})$, $b(\mathbf{k})$, $b^{\dagger}(\mathbf{k})$,	$\mathcal{N}_a(\mathbf{k})$, (num real particles)/(**k** space vol), M^{-3} ½ , (num vac particles)/(**k** vol) /(**x** vol), M^{-6} $a(\mathbf{k})$, $a^{\dagger}(\mathbf{k})$, $M^{-3/2}$ Similar for $\mathcal{N}_b(\mathbf{k})$, $b(\mathbf{k})$, $b^{\dagger}(\mathbf{k})$,

Single Particle (No Anti-particle) State Relations		
Eigenstate Creation	$a^\dagger(\mathbf{k})\lvert 0\rangle = \lvert\phi_{\mathbf{k}}\rangle = \left\lvert\frac{e^{-ikx}}{\sqrt{V}}\right\rangle$ Eigenstate at one point in **k** space, spread over volume V in **x** space.	$a^\dagger(\mathbf{k})\lvert 0\rangle = \lvert\phi(\mathbf{k})\rangle = \left\lvert\frac{e^{-ikx}}{\sqrt{(2\pi)^3}}\right\rangle$ Eigenstate at one point in **k** space, spread over universe in **x** space. $V \to \infty$
General State Creation, *C* is General State Creation Operator	$\lvert\phi\rangle = C\lvert 0\rangle = \sum_{\mathbf{k}} A_{\mathbf{k}} a^\dagger(\mathbf{k})\lvert 0\rangle$ $= \left\lvert\frac{1}{\sqrt{V}}\left(A_{\mathbf{k}_1}e^{-ik_1x} + A_{\mathbf{k}_2}e^{-ik_2x} + ..\right)\right\rangle = \left\lvert\sum_{\mathbf{k}} A_{\mathbf{k}}\frac{e^{-ikx}}{\sqrt{V}}\right\rangle$ Coefficient $A_{\mathbf{k}}$ unitless. State units $l^{-3/2} = M^{3/2}$	$\lvert\phi\rangle = C\lvert 0\rangle = \int A(\mathbf{k})a^\dagger(\mathbf{k})d^3k\lvert 0\rangle$ $= \left\lvert\int A(\mathbf{k})\frac{e^{-ikx}}{\sqrt{(2\pi)^3}}d^3k\right\rangle$ Coefficient $A(\mathbf{k})$ units $l^{3/2} = M^{-3/2}$. State units $l^{-3/2} = M^{3/2}$
Creation Operator *C*	$C = \sum_{\mathbf{k}} A_{\mathbf{k}} a^\dagger(\mathbf{k})$	$C = \int A(\mathbf{k})a^\dagger(\mathbf{k})d^3k$
State Norms	$\langle\phi\vert\phi\rangle = 1$	$\langle\phi\vert\phi\rangle = 1$
Coefficient Properties	$\sum_{\mathbf{k}} \lvert A_{\mathbf{k}}\rvert^2 = 1$	$\int\lvert A(\mathbf{k})\rvert^2 d^3k = 1$
For an Eigenstate	Only one $A_{\mathbf{k}}$, with $\lvert A_{\mathbf{k}}\rvert = 1$. $\lvert A_{\mathbf{k}}\rvert/\sqrt{V} \to 1/\sqrt{V}$	Not very meaningful.
***N_a* (k) Acting on General State**	$N_a(\mathbf{k})\left\lvert\sum_{\mathbf{k}'} A_{\mathbf{k}'}\frac{e^{-ik'x}}{\sqrt{V}}\right\rangle$ $= A_{\mathbf{k}}\left\lvert\frac{e^{-ikx}}{\sqrt{V}}\right\rangle$	$\mathcal{N}_a(\mathbf{k})\left\lvert\int A(\mathbf{k}')\frac{e^{-ik'x}}{\sqrt{(2\pi)^3}}d^3k'\right\rangle$ $= A(\mathbf{k})\left\lvert\frac{e^{-ikx}}{\sqrt{(2\pi)^3}}\right\rangle$
Eigenstate Energy Expectation Value	$\bar{E} = \langle\phi_{\mathbf{k}'}\rvert\sum_{\mathbf{k}}\omega_{\mathbf{k}}\left(N_a(\mathbf{k}) + \tfrac{1}{2} + N_b(\mathbf{k}) + \tfrac{1}{2}\right)\lvert\phi_{\mathbf{k}'}\rangle$ $= \left(\omega_{\mathbf{k}'} + \tfrac{1}{2}\sum_{\mathbf{k}}\omega_{\mathbf{k}} + \tfrac{1}{2}\sum_{\mathbf{k}}\omega_{\mathbf{k}}\right)\langle\phi_{\mathbf{k}'}\Vert\phi_{\mathbf{k}'}\rangle$ $= \omega_{\mathbf{k}'} + \sum_{\mathbf{k}}\omega_{\mathbf{k}}$	$\bar{E} = \langle\phi(\mathbf{k}')\rvert\int\omega_{\mathbf{k}}(\mathcal{N}_a(\mathbf{k}) + \tfrac{1}{2}\delta(0)$ $+ \mathcal{N}_b(\mathbf{k}) + \tfrac{1}{2}\delta(0))d^3k\lvert\phi(\mathbf{k}')\rangle$ $= \int\omega_{\mathbf{k}}\left(\delta(\mathbf{k}-\mathbf{k}') + \tfrac{1}{2}\delta(0) + \tfrac{1}{2}\delta(0)\right)d^3k\langle\phi\Vert\phi\rangle$ $= \omega_{\mathbf{k}'} + \delta(0)\int\omega_{\mathbf{k}}d^3k$
General State Energy Expectation Value	$\bar{E} = \left\langle\sum_{\mathbf{k}'} A_{\mathbf{k}'}\frac{e^{-ik'x}}{\sqrt{V}}\right\rvert\sum_{\mathbf{k}}\omega_{\mathbf{k}}(N_a(\mathbf{k}) + \tfrac{1}{2}$ $+ N_b(\mathbf{k}) + \tfrac{1}{2})\left\lvert\sum_{\mathbf{k}'} A_{\mathbf{k}'}\frac{e^{-ik'x}}{\sqrt{V}}\right\rangle$ $= \left(\sum_{\mathbf{k}'}\omega_{\mathbf{k}'}A^\dagger_{\mathbf{k}'}A_{\mathbf{k}'}\delta_{\mathbf{k}\mathbf{k}'} + \tfrac{1}{2}\sum_{\mathbf{k}}\omega_{\mathbf{k}} + \tfrac{1}{2}\sum_{\mathbf{k}}\omega_{\mathbf{k}}\right)\langle\phi\vert\phi\rangle$ $= \sum_{\mathbf{k}}\lvert A_{\mathbf{k}}\rvert^2\omega_{\mathbf{k}} + \sum_{\mathbf{k}}\omega_{\mathbf{k}} = \bar{\omega} + \sum_{\mathbf{k}}\omega_{\mathbf{k}}$	$\bar{E} = \left\langle\int A(\mathbf{k}')\frac{e^{-ik'x}}{\sqrt{(2\pi)^3}}d^3k'\right\rvert\int\omega_{\mathbf{k}}(\mathcal{N}_a(\mathbf{k}) + \tfrac{1}{2}\delta(0)$ $+ \mathcal{N}_b(\mathbf{k}) + \tfrac{1}{2}\delta(0))d^3k\left\lvert\int A(\mathbf{k}')\frac{e^{-ik'x}}{\sqrt{(2\pi)^3}}d^3k'\right\rangle$ $= \int\omega_{\mathbf{k}}\left(\lvert A(\mathbf{k}')\rvert^2\delta(\mathbf{k}-\mathbf{k}') + \delta(0)\right)d^3k\langle\phi\Vert\phi\rangle$ $= \int\lvert A(\mathbf{k})\rvert^2\omega_{\mathbf{k}}d^3k + \delta(0)\int\omega_{\mathbf{k}}d^3k = \bar{\omega} + \delta(0)\int\omega_{\mathbf{k}}d^3k$

Multi-particle (without Anti-particles) State Relations		
Multi Eigen Particles Creation	$a^\dagger(\mathbf{k}_1)a^\dagger(\mathbf{k}_2)...\|0\rangle = \|\phi_{\mathbf{k}_1},\phi_{\mathbf{k}_2},..\rangle$	$a^\dagger(\mathbf{k}_1)a^\dagger(\mathbf{k}_2)...\|0\rangle = \|\phi(\mathbf{k}_1),\phi(\mathbf{k}_2),..\rangle$
Multi General Particles Creation, C_q is qth Particle Creation Operator	$\|\phi_q,\phi_r,...\rangle = (C_q C_r..)\|0\rangle =$ $\left(\sum_{\mathbf{k}} \frac{A_{q\mathbf{k}}}{\sqrt{n_{q\mathbf{k}}+1}} a^\dagger(\mathbf{k})\right)\left(\sum_{\mathbf{k}} \frac{A_{r\mathbf{k}}}{\sqrt{n_{r\mathbf{k}}+1}} a^\dagger(\mathbf{k})\right)(...)\|0\rangle$ $= \left\|\sum_{\mathbf{k}} A_{q\mathbf{k}} \frac{e^{-ikx_q}}{\sqrt{V}}, \sum_{\mathbf{k}} A_{r\mathbf{k}} \frac{e^{-ikx_r}}{\sqrt{V}},..\right\rangle$ where $n_{q\mathbf{k}}$ (or $n_{r\mathbf{k}}$,...) = number of **k** momentum particles in ket when C_q (or C_r,..) operates	$\|\phi_q,\phi_r,...\rangle = (C_q C_r..)\|0\rangle =$ $\left(\int \frac{A_q(\mathbf{k})}{\sqrt{n_{q\mathbf{k}}+1}} a^\dagger(\mathbf{k})d^3k\right)\left(\int \frac{A_r(\mathbf{k})}{\sqrt{n_{r\mathbf{k}}+1}} a^\dagger(\mathbf{k})d^3k\right)(..)\|0\rangle$ $= \left\|\int A_q(\mathbf{k})\frac{e^{-ikx_q}}{\sqrt{(2\pi)^3}}d^3k, \int A_r(\mathbf{k})\frac{e^{-ikx_r}}{\sqrt{(2\pi)^3}}d^3k,..\right\rangle$
Normalized Creation Operator C_q	$C_q = \sum_{\mathbf{k}} \frac{A_{q\mathbf{k}}}{\sqrt{n_{q\mathbf{k}}+1}} a^\dagger(\mathbf{k})$	$C_q = \int \frac{A_q(\mathbf{k})}{\sqrt{n_{q\mathbf{k}}+1}} a^\dagger(\mathbf{k})d^3k$
State Norms	$\langle\phi_q,\phi_r,...\|\phi_q,\phi_r,...\rangle = 1$	$\langle\phi_q,\phi_r,...\|\phi_q,\phi_r,...\rangle = 1$
Coefficient Properties	$\sum_{\mathbf{k}} \|A_{q\mathbf{k}}\|^2 = 1, \quad \sum_{\mathbf{k}} \|A_{r\mathbf{k}}\|^2 = 1,$ etc.	$\int \|A_q(\mathbf{k})\|^2 d^3k = 1, \quad \int \|A_r(\mathbf{k})\|^2 d^3k = 1,$ etc.
***N_a* (k) Acting on Multi General Particles State**	$N_a(\mathbf{k})\|\phi_q,2\phi_r,..\rangle$ $= N_a(\mathbf{k})\left\|\sum_{\mathbf{k}'} A_{q\mathbf{k}'} \frac{e^{-ik'x_q}}{\sqrt{V}}, 2\sum_{\mathbf{k}''} A_{r\mathbf{k}''} \frac{e^{-ik''x_r}}{\sqrt{V}},..\right\rangle$ $= A_{q\mathbf{k}}\|\phi_{q\mathbf{k}},2\phi_r,..\rangle + 2A_{r\mathbf{k}}\|\phi_q,2\phi_{r\mathbf{k}},..\rangle + ...$	$\mathcal{N}_a(\mathbf{k})\|\phi_q,2\phi_r,..\rangle = \mathcal{N}_a(\mathbf{k})\times$ $\left\|\int A_q(\mathbf{k}')\frac{e^{-ik'x_q}}{\sqrt{(2\pi)^3}}d^3k', 2\int A_r(\mathbf{k}'')\frac{e^{-ik''x_r}}{\sqrt{(2\pi)^3}}d^3k'',..\right\rangle$ $= A_q(\mathbf{k})\|\phi_{q\mathbf{k}},2\phi_r,..\rangle + 2A_r(\mathbf{k})\|\phi_q,2\phi_{r\mathbf{k}},..\rangle$
Multi Eigen Particles Energy Expectation Value	$\bar{E} = \langle\phi_{\mathbf{k}_1},2\phi_{\mathbf{k}_2},..\|\sum_{\mathbf{k}} \omega_{\mathbf{k}}(N_a(\mathbf{k}) + \tfrac{1}{2} +$ $N_b(\mathbf{k}) + \tfrac{1}{2})\|\phi_{\mathbf{k}_1},2\phi_{\mathbf{k}_2},..\rangle$ $= (\omega_{\mathbf{k}_1} + 2\omega_{\mathbf{k}_2} + ..$ $+ \tfrac{1}{2}\sum_{\mathbf{k}} \omega_{\mathbf{k}} + \tfrac{1}{2}\sum_{\mathbf{k}} \omega_{\mathbf{k}})\langle\phi_{\mathbf{k}_1},2\phi_{\mathbf{k}_2},..\|\phi_{\mathbf{k}_1},2\phi_{\mathbf{k}_2},..\rangle$ $= \omega_{\mathbf{k}_1} + 2\omega_{\mathbf{k}_2} + ... + \sum_{\mathbf{k}} \omega_{\mathbf{k}}$	$\bar{E} = \langle\phi(\mathbf{k}_1),2\phi(\mathbf{k}_2),..\|\int \omega_{\mathbf{k}}(\mathcal{N}_a(\mathbf{k}) + \tfrac{1}{2}\delta(0)$ $+ \mathcal{N}_b(\mathbf{k}) + \tfrac{1}{2}\delta(0))d^3k\|\phi(\mathbf{k}_1),2\phi(\mathbf{k}_2),..\rangle$ $= (\omega_{\mathbf{k}_1} + 2\omega_{\mathbf{k}_2} + ... + \delta(0)\int \omega_{\mathbf{k}} d^3k)\times$ $\langle\phi(\mathbf{k}_1),2\phi(\mathbf{k}_2),.\|\phi(\mathbf{k}_1),2\phi(\mathbf{k}_2),.\rangle$ $= \omega_{\mathbf{k}_1} + 2\omega_{\mathbf{k}_2} + ... + \delta(0)\int \omega_{\mathbf{k}} d^3k$
Multi General Particles Energy Expectation Value	$\bar{E} = \langle\phi_q,2\phi_r,..\|\sum_{\mathbf{k}} \omega_{\mathbf{k}}(N_a(\mathbf{k}) + \tfrac{1}{2}$ $+ N_b(\mathbf{k}) + \tfrac{1}{2})\|\phi_q,2\phi_r,..\rangle$ $= (\sum_{\mathbf{k}} \|A_{q\mathbf{k}}\|^2 \omega_{\mathbf{k}} + 2\sum_{\mathbf{k}} \|A_{r\mathbf{k}}\|^2 \omega_{\mathbf{k}} + ...$ $+ \tfrac{1}{2}\sum_{\mathbf{k}} \omega_{\mathbf{k}} + \tfrac{1}{2}\sum_{\mathbf{k}} \omega_{\mathbf{k}})\langle\phi_q,2\phi_r,..\|\phi_q,2\phi_r,..\rangle$ $= \bar{\omega}_q + 2\bar{\omega}_r + ... + \sum_{\mathbf{k}} \omega_{\mathbf{k}}$	$\bar{E} = \langle\phi_q,2\phi_r,..\|\int \omega_{\mathbf{k}}(\mathcal{N}_a(\mathbf{k}) + \tfrac{1}{2}\delta(0)$ $+ \mathcal{N}_b(\mathbf{k}) + \tfrac{1}{2}\delta(0))d^3k\|\phi_q,2\phi_r,..\rangle$ $= (\int \omega_{\mathbf{k}} \|A_q(\mathbf{k})\|^2 d^3k + 2\int \omega_{\mathbf{k}} \|A_r(\mathbf{k})\|^2 d^3k + ...$ $+ \delta(0)\int \omega_{\mathbf{k}} d^3k)\langle\phi_q,2\phi_r,..\|\phi_q,2\phi_r,..\rangle$ $= \bar{\omega}_q + 2\bar{\omega}_r + ... + \delta(0)\int \omega_{\mathbf{k}} d^3k$

Note		In the energy expectation derivation for the continuous case, one finds a Dirac delta function squared in the vacuum energy part. This is undefined mathematically. By some perspectives, its evaluation leaves a vacuum term of energy $\omega(\mathbf{k}=0)$ which equals μ (one particle mass). An alternative perspective is shown above.

10.11 Appendix D: Free Fields and "Pair Popping" Re-visited

One possible issue some might raise with this chapter needs to be addressed. That is, by using the Interaction Picture (I.P.), where operator fields are free and particle behavior is described by the interaction Hamiltonian, are we somehow obscuring some physics? Recall we derived Feynman diagrams from only the interaction term in the Hamiltonian. If we included the free Hamiltonian in such a derivation, would we possibly find Feynman diagrams producing and destroying particle/antiparticle pairs?

Specifically, $\mathcal{H}_0$ has creation and destruction operators paired together in the same terms. For scalar fields, in the Heisenberg Picture (H.P.), we found

$$\mathcal{H}_0^0 = \pi_0^0\dot{\phi} + \pi_0^{0\,\dagger}\dot{\phi}^\dagger - \mathcal{L}_0^0 = \left(\dot{\phi}\dot{\phi}^\dagger + \nabla\phi^\dagger\cdot\nabla\phi + \mu^2\phi^\dagger\phi\right) = \left(\pi_0^{0\,\dagger}\pi_0^0 + \nabla\phi^\dagger\cdot\nabla\phi + \mu^2\phi^\dagger\phi\right) \quad (10\text{-}27)$$

$$\text{with } \phi(x) = \sum_{\mathbf{k}}\frac{1}{\sqrt{2V\omega_{\mathbf{k}}}}\left(a(\mathbf{k})e^{-ikx} + b^\dagger(\mathbf{k})e^{ikx}\right). \quad (10\text{-}28)$$

So, we might end up with a creation ($a^\dagger(\mathbf{k})$ or $b^\dagger(\mathbf{k})$) and a destruction ($a(\mathbf{k})$ or $b(\mathbf{k})$) operator in the same term in H_0^0. In particular, if we had terms containing factors of $a^\dagger(\mathbf{k})b^\dagger(\mathbf{k})$ or $a^\dagger(\mathbf{k})b^\dagger(-\mathbf{k})$, we might expect creation of a particle and an antiparticle at the same event in the vacuum.

To see how states might change, let's use the Schrödinger picture (S.P.), in which operators do not change in time, but states do. In the S.P. for free fields, scalar states are governed by

$$i\frac{d}{dt}\left|\Phi\right\rangle = H_0^0\left|\Phi\right\rangle, \quad (10\text{-}29)$$

where $|\Phi\rangle$ is in general a multi-scalar particle state and H_0^0 is the same in the H.P. as the S.P. (See Wholeness Chart 2-4, pg. 28.) Using H_0^0, specifically the RHS of (10-27), along with (10-28) and its associated quantities, we can follow the same steps as we did in Chap. 8 to determine the evolution of the state $|\Phi\rangle$. (See Wholeness Chart 8-4, pgs. 248-251.) This leads us to

$$\left|\Phi(t_f)\right\rangle = S\left|\Phi(t_i)\right\rangle = e^{-i\int_{-\infty}^{\infty}H_0^0 dt'}\left|\Phi(t_i)\right\rangle \qquad S_{fi} = \left\langle\Phi(t_f)\right|S\left|\Phi(t_i)\right\rangle = \left\langle f\right|S\left|i\right\rangle. \quad (10\text{-}30)$$

With the Dyson expansion, we find

$$S = \underbrace{I}_{S^{(0)}} \underbrace{-i\int_{-\infty}^{\infty}\mathcal{H}_0^0(x_1)d^4x_1}_{S^{(1)}} \underbrace{-\frac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}T\left\{\mathcal{H}_0^0(x_1)\mathcal{H}_0^0(x_2)\right\}d^4x_1d^4x_2}_{S^{(2)}} + \ldots\ldots \quad (10\text{-}31)$$

For the $S^{(1)}$ term, the integration over all space was carried out in Chap. 3, Sect. 3.4.1, pgs. 53-54. In that derivation, we saw all terms containing factors $a^\dagger(\mathbf{k})b^\dagger(-\mathbf{k})$, $a^\dagger(\mathbf{k})b^\dagger(\mathbf{k})$, $a(\mathbf{k})b(-\mathbf{k})$, and $a(\mathbf{k})b(\mathbf{k})$ dropped out. That is, no terms remain that create a particle/anti-particle pair at the same event in the vacuum. Ditto for destruction of such a pair.

For the $S^{(2)}$ term, at any given point in time, we can integrate $H_0^0(x_1)$ over $d^3\mathbf{x_1}$ without regard to the integration of $H_0^0(x_2)$ over $d^3\mathbf{x_2}$. For that integration, we would get the same result as for $S^{(1)}$, i.e., no terms with factors creating or destroying a particle/anti-particle pair. The same result would hold for $S^{(n)}$ for any n.

The transition amplitude S_{fi} would, therefore, not contain any terms creating/destroying such pairs. And so, we would have no Feynman diagrams representing such a thing.

Further, by reviewing the above cited section of Chap. 3, one can see that the ½ quanta terms, commonly considered "vacuum fluctuations" come from the $a(\mathbf{k})a^\dagger(\mathbf{k})$ and $b(\mathbf{k})b^\dagger(\mathbf{k})$ terms and the coefficient commutation relations. Even if we chose to use these terms directly, without employing the commutation relations, the $a(\mathbf{k})a^\dagger(\mathbf{k})$ term is not coupled to the $b(\mathbf{k})b^\dagger(\mathbf{k})$term so both terms together would not represent a vertex in a Feynman diagram. In that interpretation, one might think of the $a(\mathbf{k})a^\dagger(\mathbf{k})$ as representing creation of a particle and destruction of the same particle at the same event, i.e., nothing would happen as time evolves. No evanescence. No pair popping.

In summary, for free fields

- Terms in the free Hamiltonian density containing two creation operators that might create a particle/antiparticle pair at an event drop out of the full (not density) Hamiltonian.
- The only terms surviving in the full Hamiltonian have creation and destruction operators paired. These would create and destroy the same particle at the same event, i.e., nothing would effectively happen.

We conclude that the free field components of the Hamiltonian do not lead to particle/antiparticle pairs popping in and out of the vacuum.

10.12 Appendix E: Considerations for Finite Volume Interactions

All of the foregoing material in this chapter related to "standard" QFT, in which fields/particles are considered to extend over infinite volume V and infinite time T. That assumption, as we will see in Part 4 of this book, leads to accurate real-world predictions for real world fields/particles of finite extensions in V and T.

In developing our theory, this assumption gave rise to Dirac delta functions (see (8-30), pg. 222) because we integrated over unbounded space and time. These Dirac delta functions, arising in each transition amplitude, led to strict conservation of 4-momentum at every vertex. Had V and T been finite instead of unbounded, integration would not have led to Dirac delta functions, and so one might question if, with finite V and T, the resulting relation would lead to uncertainty in outgoing 4-momentum. Presumably, for large V and T, the relation would approximate a Dirac delta function implying approximate, but not exact, conservation of 4-momenta. And thus, smaller V and T would mean 4-momenta would be less constrained to be conserved.

This would give rise to an uncertainty in outgoing 4-momentum at any vertex for which the fields did not have infinite extension in V and T. Smaller V and T means greater uncertainty in 3-momentum and energy, respectively, and this correlates with the familiar uncertainty principle.

To examine this more closely, consider the Dirac delta function shown in (8-30), pg. 222, where $k = P_f$ is the 4-momentum leaving the vertex and $P_i = p_1 + p_2$ is the incoming 4-momentum,

$$(2\pi)^4\,\delta^{(4)}\left(P_f - P_i\right) = \int_{-\infty}^{\infty} e^{i\left(P_f - P_i\right)x_2}\, d^4x_2\,. \tag{10-32}$$

Now consider the RHS of (10-32) integrated over finite, instead of infinite, V and T, where, to keep things simple, we use the 1D correlate of the 4D integral, and represent that with the symbol I,

$$\int_{-V/2,-T/2}^{V/2,T/2} e^{i\left(P_f - P_i\right)x_2}\, d^4x_2 \xrightarrow{\;1D\;} \int_{-L/2}^{L/2} e^{i\left(P_f - P_i\right)x_2}\, dx_2 = I\left(P_f - P_i\right) \tag{10-33}$$

The integral is easy to evaluate, and I is found to be

$$I\left(P_f - P_i\right) = \left.\frac{e^{i\left(P_f - P_i\right)x_2}}{i\left(P_f - P_i\right)}\right|_{-L/2}^{L/2} = \left.\frac{\cos\left(P_f - P_i\right)x_2 + i\sin\left(P_f - P_i\right)x_2}{i\left(P_f - P_i\right)}\right|_{-L/2}^{L/2} = 2\frac{\sin\left(\left(P_f - P_i\right)\frac{L}{2}\right)}{\left(P_f - P_i\right)}. \tag{10-34}$$

In the development of NRQM, RQM, and QFT (see (3-24) to (3-25), pg. 46 and Sect. 3.4.1, pgs. 53-54), we typically assume

$$P_i = \frac{2\pi n_i}{L} \quad P_f = \frac{2\pi n_f}{L} \quad \rightarrow \quad P_f - P_i = \frac{2\pi\left(n_f - n_i\right)}{L} \qquad n_i, n_f \text{ integers}\,, \tag{10-35}$$

because (10-35) results in orthogonal functions of e^{iPx} and zero values for quantities like probability density in NRQM for particles at $L/2$ and $-L/2$, as well as certain terms in the probability of RQM and in the Hamiltonian of QFT that must be zero. (See above references.)

<u>n_i and n_f as integers</u>

For (10-35) in (10-34), we find

$$I\left(P_f - P_i\right) = 2\frac{\sin\left(\frac{2\pi\left(n_f - n_i\right)}{L}\frac{L}{2}\right)}{\frac{2\pi\left(n_f - n_i\right)}{L}} = L\frac{\sin\left(\pi\left(n_f - n_i\right)\right)}{\pi\left(n_f - n_i\right)} . \qquad (10\text{-}36)$$

Due to the numerator, this is zero except for $n_f = n_i$. Then

$$I\left(P_f - P_i\right)_{n_f = n_i} = \lim_{n_f \to n_i}\left(L\frac{\sin\left(\pi\left(n_f - n_i\right)\right)}{\pi\left(n_f - n_i\right)}\right) = L\frac{\pi\left(n_f - n_i\right)}{\pi\left(n_f - n_i\right)} = L \qquad (10\text{-}37)$$

So, I is zero, except when $n_f = n_i$, i.e., when $P_f = P_i$. That behaves like a Dirac delta function for argument $P_f \neq P_i$. However when $P_f = P_i$., I is not ∞, as a delta function is, as long as L is finite.

Looking again at our transition amplitude calculation in (8-30), pg. 222, we see that the finite L (V there for 3D case; V and T, for 4D) will still leave us with a zero value unless $P_f = P_i$ ($k = p_1 + p_2$ there.) The value of the transition amplitude will change because we now have L (V for 3D, VT for 4D) finite when P_f=P_i, but other values for P_f are prohibited (have zero probability of occurring.)[1]

Bottom line: For n_f and n_i as integers, and finite volume and time, we still must have strict 4-momentum conservation at a vertex. That is, there is no uncertainty principle at play giving rise to evanescent energy and 3-momentum "popping in and out" of the vacuum.

<u>n_f and n_i as non-integers</u>

If, however, n_f and n_i could be non-integers, then I of (10-36) can have non zero values when $n_f \neq n_i$ (and thus when $P_f \neq P_i$). Analogous results hold for 4D, so for finite V and T, we could have non-zero probability (due to a non-zero value in the RHS of (10-32)) for $P_f \neq P_i$ and not have strict conservation of 4-momentum[2].

Bottom line: For n_f and n_i as non-integers, and finite volume and time, we do not have strict 4-momentum conservation at a vertex. That is, there would be an uncertainty principle of sorts at play, which could give rise to evanescent energy and 3-momentum "popping in and out" of the vacuum. For infinite volume and time, strict conservation exists.

<u>Impact of n_f and n_i as non-integers on various kinds of "vacuum fluctuations"</u>

If non-integer values for n_f and n_i manifest in nature, then the following may be surmised for each type of "vacuum fluctuation" in QFT.

"Pair Popping"

The functional form of the transition amplitude and thus questions involving the Dirac delta function found therein are not relevant to the pair popping story, as there are no transition amplitudes having vertices with only two (not three, as for vacuum bubbles) particles. (See "Virtual Bubbles" section below.)

Zero Point Energy

The non-integer n_f and n_i condition would not modify anything we have said herein about the ZPE ½ quanta, as they represent free fields, with no vertices, i.e., no interactions. However, it does relate to virtual vacuum bubbles and radiative corrections, which are manifestations of interacting fields. (See Wholeness Chart 10-1, pg. 278, and below.)

---

1 In the limit where $L \to \infty$, (10-37) becomes $2\pi\,\delta(P_f - P_i)$. When $V \to \infty$, we get the 3D Dirac delta function, and for $T, V \to \infty$, the 4D relation.

2 Additional analysis, which we mention but not do here, leads to the conclusion that for n_i and n_f as non-integers, we do get a Dirac delta function in (10-33) (and (10-32) as $L \to \infty$ ($V,T \to \infty$).

Virtual Bubbles

A 3-particle virtual bubble has zero initial 4-momentum, but as noted above for finite V and T, it could then, after the first vertex, have a non-zero total 4-momentum (solely for non-integer n_f and n_i). And this then starts to look like the pair popping scenario (even though there are three, not two particles.)

However, we have seen that negative energy virtual particles are as likely as positive energy ones. So, the sum total energy of the bubble could be positive or negative. The sum over large numbers of such bubbles would be effectively zero energy. In other words, even for small values of V and T, there would be no net global contribution to the energy of the vacuum from virtual bubbles. It is conceivable, however, that tiny black holes could exist for positive energy bubbles, and possibly "white holes", we could call them, for the negative ones. We could have quantum foam, but zero total vacuum energy.

Radiative Corrections

As noted, radiative corrections do not arise alone in the vacuum and make no direct contribution to vacuum energy. This is true for finite, or infinite, V and T. Additionally, variations in energy from uncertainty at each vertex would go in both directions (positive and negative) and cancel globally, over many interactions.

BUT remember

Integer values for n_f and n_i in (10-35) seem to be required by nature. If this were not true, we would not have orthogonal functions as our solutions to the RQM/QFT wave equations and certain derivations, such as that for the number operator form of the Hamiltonian, would no longer be valid.

Bottom line:

Thus, vacuum energy, carried by particles popping in and out of the vacuum (for virtual 3 particle bubbles), appears inconsistent with the rest of our theory. To my knowledge, this issue (regarding non-integer n_f and n_i in transition amplitudes) has not been explored in great depth and might make a good research topic for someone. If any reader does pursue this, please apprise me of the results (via the website for this book, the address of which is found on pg. xvi, opposite pg. 1.)

10.13 Appendix F: Vacuum Fluctuations Update (*Added in 2018 Text Revision*)

There are several experimental results and two theoretical papers related to vacuum fluctuations, as well as another well-known phenomenon often linked to the vacuum, that were not originally covered in this chapter. One of the experiments (actually a cosmological observation) was done in 2012, and I was unaware of it when I wrote the book (2013, 1st edition). The others and the theoretical articles have only been made public in the three years before the 2018 revision of the text. I review these in chronological order below and supply links to the original articles. I then briefly address spontaneous radiation emission, often linked in the past to vacuum fluctuations.

Note that this appendix is posted on the book website (see pg. xvi, opposite pg. 1 for URL) with live links for websites cited below.

10.13.1 Cosmological Observations of Photons and Neutrinos (2012-2018)

Tiny scattering effects from Planck-scale quantum foam on photons propagating over billions of light-years should be cumulative and lead to detectable dispersion of those photons when they arrive on Earth. Lack of such dispersion would support the notion that vacuum fluctuations do not exist. A 2012 analysis of gamma ray bursts (GRBs) by Nemiroff et al[1] implied no Planck-scale quantum foam. Popular accounts include "Cosmic race ends in a tie" by R. Cowen, *Nature*. (10 January 2012) and "Spacetime: A smoother brew than we knew" (January 2013) [2]. Other research, such as that by Vasileiou et al[3] also indicate smooth spacetime at the Planck-scale.

---

[1] R. J. Nemiroff, R. Connolly, J. Holmes, and A. B. Kostinski1, "Bounds on Spectral Dispersion from Fermi-detected Gamma Ray Bursts" *Phys. Rev. Lett.* **108** (23): 231103 (2012). https://arxiv.org/abs/1109.5191.

[2] www.nature.com/news/cosmic-race-ends-in-a-tie-1.9768; https://phys.org/news/2013-01-spacetime-smoother-brew-knew.html .

[3] Vasileiou, V., Granot, J., Piran, T. and Amelino-Camelia, G., "A Planck-scale limit on spacetime fuzziness and stochastic Lorentz invariance violation", *Nat. Phys. Lett.* **11**, 344-346, April 2015 www.nature.com/articles/nphys3270 .

However, later work by Xu and Ma[1] and Amelino-Camelia et al[2] seem to contradict this, as they suggest evidence that cosmological photons and neutrinos may disperse. However, none of these results, either for or against spacetime foam, is statistically ironclad.

10.13.2 Usual Analysis of Casimir Plate Effect May be Faulty (*2016 - 2017*)

Nikolić[3] notes, among other points, that typical analyses of the Casimir effect use a Hamiltonian that has implicit dependence on matter fields and illegitimately treat it as if the dependence were explicit. He contends the true origin of the Casimir force is the van der Waals force.

10.13.3 Vacuum Fluctuations Experiment (*2017*)

An experimental group at the University of Konstanz[4] claimed the first direct detection of ZPE fluctuations in a laboratory experiment. Their technical article is quite difficult for a non-specialist in nonlinear optics to understand, so I have written a pedagogic introduction[5] to their work on the book website. Note that in that article I question whether ZPE fluctuations have really been detected and provide reasons why they may not have been. The result is controversial.

10.13.4 Vacuum Fluctuation Experiment (2021) – added in September 2022 revision

Since 2017, some other researchers have claimed detection of ZPE, but for those unfamiliar with their apparatuses and complex techniques, it is difficult to decipher what they have done, and the reported results are controversial. It is noteworthy that experiments at Fermilab have been reported as showing no vacuum fluctuations at the Planck scale[6].

10.13.5 Spontaneous Emission

As early as 1913, A. Einstein and O. Stern[7] noted that a zero-point energy term had to be added to the classical theory to obtain the Planck radiation spectrum formula. Subsequent research, cited and summarized by P. W. Milonni[8], extended that perspective to spontaneous emission of radiation from an atom. It appeared that a vacuum contribution was needed to help "jiggle" an orbiting electron and "stimulate" it to jump down an energy level, thereby emitting e/m radiation.

However, Milonni, probably the leading expert on the subject, has noted that, similar to the Casimir plates case, there are different ways to carry out the calculations, and in at least one of them, no vacuum contribution is needed. He says (Milonni 1988), " .. the effects usually attributed to vacuum field fluctuations may instead be attributed to radiation reaction."

He goes on to say

---

[1] Xu, H. and Ma, B.Q., "Light Speed Variation from GRB 160509A", *Phys. Lett. B* **760** (2016) 602 https://arxiv.org/abs/1607.08043

[2] Amelino-Camelia, G., D'Amico, G., Rosati, G. and Loret, N., "In vacuo-dispersion features for GRB neutrinos and photons", *Nat. Astron*. 1, 0139 (2017) https://arxiv.org/abs/1612.02765.

[3] H. Nikolić, "Proof that Casimir forces do not originate from vacuum energy", *Phys. Lett. B* **761** (2016) 197-202. https://arxiv.org/abs/1605.04143, and "Is zero-point energy physical? A toy model for Casimir-like effect", *Ann. of Phys*, **383** (2017) 181-195 https://arxiv.org/abs/1702.03291.

[4] Riek, C., Sulzer, P., Seeger, M., Moskalenko, A.S., Burkard, G., Seletskiy, D.V., and Leitenstorfer, A.. "Subcycle quantum electrodynamics", *Nature* **541**, 376-379 (19 Jan 2017) https://arxiv.org/abs/1611.06773. Popular accounts include "Traffic Jam in Empty Space" http://www.uni-konstanz.de/en/university/news-and-media/current-announcements/news/news-in-detail/verkehrsstau-im-nichts/ and "Physicists observe weird quantum fluctuations of empty space – maybe" www.sciencemag.org/news/2015/10/physicists-observe-weird-quantum-fluctuations-empty-space-maybe.

[5] Klauber, R., "Vacuum Fluctuations Detection: A Pedagogic Overview of the University of Konstanz Experiment" (Nov 2017) www.quantumfieldtheory.info/pedagog_U_Konstanz.pdf .

[6] Hogan, H., "Random twists of place: How quite is quantum space-time at the Planck scale?", https://news.fnal.gov/2021/02/

random-twists-of-place-how-quiet-is-quantum-space-time-at-the-planck-scale/; Richardson, J. W. et al, "Interferometric Constraints on Spacelike Coherent Rotational Fluctuations", https://arxiv.org/abs/2012.06939

[7] Einstein, A., and Stern, O. *Ann. Phys.* **40**, 551.

[8] Milonni. P./W., "Different Ways of Looking at the Electromagnetic Vacuum", *Physica Scripta*, T21, 102-109 (1988) and *The Quantum Vacuum: An Introduction to Quantum Electrodynamics*, (Academic Press, 1994).

> *"..radiation reaction nevertheless offers a valid basis for understanding spontaneous emission, provided the radiation reaction field is handled properly as a quantum-mechanical operator.*
>
> *.. It was shown in the case of spontaneous emission that the physical interpretation suggested by quantum electrodynamics is more or less a consequence of the way we choose to order commuting (underlining added) atomic and field operators.*
>
> *.. The level shifts and widths can be attributed exclusively to radiation reaction ..., or to linear combinations of the two.*
>
> *.. There is no ordering that attributes the radiative decay of a level entirely to the vacuum field.*
>
> *..Furthermore this picture (of the vacuum contribution) offers no explanation as to why there is no spontaneous absorption (underlining added) from the vacuum field."*

Note that it is the order of operators that commute which changes the relative contributions of the ZPE and radiation reaction. In all the work we have done, the order of commuting operators is unimportant. It is the order of non-commuting operators that impacts our results, and about which we need to take special care. Here, Milonni tells us, the order of *commuting* operators affects the degree to which we can attribute spontaneous emission to ZPE or radiation reaction effects. For a certain order, there is no vacuum contribution. For another order, the ZPE quanta play a part, and the radiation reaction plays a part. There is no ordering for which the effect is entirely attributable to the vacuum. For all orderings, the final result is the same. But the attribution of cause varies.

Hence, like we have seen in other cases, most notably the Casimir effect, the experimentally verified result can be determined theoretically without recourse to vacuum fluctuations.

Still further, if the vacuum plays a role in spontaneous emission, why is there no spontaneous absorption by it?

10.13.6 ZPE and Experimental Measurement

If ZPE fluctuations really impact the physical world, we should be able to detect them directly. Yet, a detector picks up the non-vacuum contribution, but nothing from the vacuum.

As noted by Jaynes[1]

> *"It seems to me that, if you say radiation is "real," you ought to mean by that, that it can be detected by a real detector. But an optical pyrometer sees only the Planck term, and not the zero-point term, in black body-radiation.*
>
> *It is a supple ontology which supposes that vacuum fluctuations are just real enough to shift the hydrogen 2s level by 4 microvolts; but not real enough to be seen by our eyes, although in the optical band they correspond to a flux of over 100 kilowatts/cm^2. Nevertheless, the dark-adapted eye, looking for example at a faint star, can see real radiation of the order of 10^{-15} watts/cm^2."*

10.14 Problem

1. Show that for the single particle state $|\phi\rangle$, which can be expressed in function form as $\phi_{state} = \int \frac{A(\mathbf{k}')e^{-ik'x}}{\sqrt{(2\pi)^3}}\, d^3k'$ having unit norm, i.e. $\langle\phi|\phi\rangle = 1$, that $\int \left|A(\mathbf{k}')\right|^2 d^3k' = 1$. Hint: The bra $\langle\phi|$ can be expressed in function form as $\phi^{\dagger}_{state} = \int \frac{A^{\dagger}(\mathbf{k}'')e^{ik''x}}{\sqrt{(2\pi)^3}}\, d^3k''$, and the norm implies integration of the bra times the ket over **x**, with $\int e^{i(\mathbf{k}''-\mathbf{k}')\cdot\mathbf{x}}\, d^3x = (2\pi)^3\, \delta^{(3)}(\mathbf{k}''-\mathbf{k}')$.

[1] Jaynes, E. T., *Coherence and Quantum Optics IV*, edited by L. Mandel and E. Wolf (Plenum Press, New York, 1978), http://bayes.wustl.edu/etj/articles/electrodynamics.today.pdf , pgs. 5-6.