Supplemental Solutions Background

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Abstract

Derivations are presented of key relationships in the articles "Mechanism for Zero-Point Energy" and "A Symmetry for Resolution of the Gauge Hierarchy Problem without SUSY, Null Zero Point Energy, and Null Higgs Condensate Energy" by Robert D. Klauber.

1 Scalar Supplemental Fields

1.1 Supplemental Solutions to the Field Equations

Restating relations in Klauber[1][2], the traditional solutions to the Klein-Gordon equation for a complex scalar field are

$$\phi = \phi^+ + \phi^- = \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k} \right)^{1/2} \left\{ a\left(\mathbf{k}\right) e^{-ikx} + b^\dagger\left(\mathbf{k}\right) e^{ikx} \right\}, \tag{1}$$

and the supplemental solutions are

$$\begin{aligned} \underline{\phi} = \underline{\phi}^+ + \underline{\phi}^- &= \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k} \right)^{1/2} \left\{ \underline{a}\left(\mathbf{k}\right) e^{i\left(\omega_{\mathbf{k}} t + \mathbf{k}\cdot\mathbf{x}\right)} + \underline{b}^\dagger\left(\mathbf{k}\right) e^{-i\left(\omega_{\mathbf{k}} t + \mathbf{k}\cdot\mathbf{x}\right)} \right\} \\ &= \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k} \right)^{1/2} \left\{ \underline{a}\left(\mathbf{k}\right) e^{i\underline{kx}} + \underline{b}^\dagger\left(\mathbf{k}\right) e^{-i\underline{kx}} \right\}. \end{aligned} \tag{2}$$

1.2 Supplemental Coefficient Commutation Relations

To derive the commutation relations for the supplemental coefficients, one can follow the steps for derivation of the traditional coefficient commutation relations of Klauber[3], pgs. 52-53. Simply change the sign on $\omega_{\mathbf{k}}$ everywhere (except in the normalization constants) from (3-42) to (3-47) and put underbars on all operators. The result is

$$\left[\underline{a}\left(\mathbf{k}\right), \underline{a}^\dagger\left(\mathbf{k}'\right)\right] = \left[\underline{b}\left(\mathbf{k}\right), \underline{b}^\dagger\left(\mathbf{k}'\right)\right] = -\delta_{\mathbf{k}\mathbf{k}'} \ . \tag{3}$$

1.3 The Supplemental Hamiltonian

To derive the Hamiltonian for scalar supplemental fields, one can follow the steps for derivation of the traditional Hamiltonian of Ref. 3, pgs. 53-54. Simply change the sign on $\omega_{\mathbf{k}}$ everywhere from (3-48) to (3-56) and put underbars on all operators. The result is (where the superscript refers to spin zero and the subscript to free field)

$$\underline{H}_0^0 = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left(\underline{a}^\dagger\left(\mathbf{k}\right) \underline{a}\left(\mathbf{k}\right) - \tfrac{1}{2} + \underline{b}^\dagger\left(\mathbf{k}\right) \underline{b}\left(\mathbf{k}\right) - \tfrac{1}{2} \right). \tag{4}$$

1.4 Number Operators

Interpreting (4) in similar manner to H_0^0 in traditional theory (Ref. 3, pgs. 54 to 55), one finds the number operators defined, and acting on states, as in (5) and (6). Note the number eigenvalues for supplemental states are negative.

$$\underline{N}_a(\mathbf{k}) = \underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k}) \qquad \underline{N}_b(\mathbf{k}) = \underline{b}^\dagger(\mathbf{k})\underline{b}(\mathbf{k}) \tag{5}$$

$$\begin{aligned} \underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k})|\underline{n}_\mathbf{k}\rangle &= \underline{N}_a(\mathbf{k})|\underline{n}_\mathbf{k}\rangle = \underline{n}_\mathbf{k}|\underline{n}_\mathbf{k}\rangle = -|\underline{n}_\mathbf{k}||\underline{n}_\mathbf{k}\rangle \qquad & \underline{n}_\mathbf{k} \le 0 \\ \underline{b}^\dagger(\mathbf{k})\underline{b}(\mathbf{k})|\underline{n}_\mathbf{k}\rangle &= \underline{N}_b(\mathbf{k})|\overline{\underline{n}}_\mathbf{k}\rangle = \overline{\underline{n}}_\mathbf{k}|\overline{\underline{n}}_\mathbf{k}\rangle = -|\overline{\underline{n}}_\mathbf{k}||\overline{\underline{n}}_\mathbf{k}\rangle \qquad & \overline{\underline{n}}_\mathbf{k} \le 0 \end{aligned} \tag{6}$$

1.5 Raising and Lowering Operators

Consider the effect of $\underline{a}(\mathbf{k})$ acting on a supplemental state, in terms of what number of particles is in the resultant state. Use (5) and (3) to find

$$\underline{N}_a(\mathbf{k})\underline{a}(\mathbf{k})|\underline{n}_\mathbf{k}\rangle = \left(\underline{a}(\mathbf{k})\underline{a}^\dagger(\mathbf{k}) + 1\right)\underline{a}(\mathbf{k})|\underline{n}_\mathbf{k}\rangle = \underline{a}(\mathbf{k})\left(\underline{N}_a(\mathbf{k}) + 1\right)|\underline{n}_\mathbf{k}\rangle = \left(\underline{n}_\mathbf{k} + 1\right)\underline{a}(\mathbf{k})|\underline{n}_\mathbf{k}\rangle. \tag{7}$$

Thus, $a(\mathbf{k})$ raises the supplemental particle number by one. If $\underline{n}_\mathbf{k}$ is negative, this effectively *reduces* the number of supplemental particles by one.

Similarly, for $a^\dagger(\mathbf{k})$,

$$\begin{aligned} \underline{N}_a(\mathbf{k})\underline{a}^\dagger(\mathbf{k})|\underline{n}_\mathbf{k}\rangle &= \left(\underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k})\right)\underline{a}^\dagger(\mathbf{k})|\underline{n}_\mathbf{k}\rangle = \underline{a}^\dagger(\mathbf{k})\left(\underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k}) - 1\right)|\underline{n}_\mathbf{k}\rangle \\ &= \underline{a}^\dagger(\mathbf{k})\left(\underline{N}_a(\mathbf{k}) - 1\right)|\underline{n}_\mathbf{k}\rangle = \left(\underline{n}_\mathbf{k} - 1\right)\underline{a}^\dagger(\mathbf{k})|\underline{n}_\mathbf{k}\rangle \end{aligned}, \tag{8}$$

and $a^\dagger(\mathbf{k})$ lowers the supplemental particle number by one (effectively *increasing* the number of supplemental particles for negative $\underline{n}_\mathbf{k}$).

1.6 Supplemental Commutators and the Indefinite Metric

If one proceeds to develop a quantum field theory of supplemental particles in the usual way, (3) leads to an indefinite metric in the Fock space of states, and negative norms for certain states[4][5][6][7], as shown below.

To find the numerical coefficient A in

$$\underline{a}^\dagger(\mathbf{k})|\underline{n}_\mathbf{k}\rangle = \underline{A}|\underline{n}_\mathbf{k} - 1\rangle\ , \tag{9}$$

consider the norm of that state, as we might expect it to be calculated,

$$\langle\underline{n}_\mathbf{k}|\ |\ \underline{a}(\mathbf{k})\underline{a}^\dagger(\mathbf{k})|\underline{n}_\mathbf{k}\rangle = \langle\underline{n}_\mathbf{k} - 1|\underline{A}^\dagger\underline{A}|\underline{n}_\mathbf{k} - 1\rangle = \underline{A}^\dagger\underline{A}, \tag{10}$$

where we have assumed $|\underline{n}_\mathbf{k} - 1\rangle$ has positive unit norm, and thus (10) must be positive. However, (10) can be re-expressed using (3) as

$$\langle\underline{n}_\mathbf{k}|\underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k}) - 1|\underline{n}_\mathbf{k}\rangle = \left(\underline{n}_\mathbf{k} - 1\right)\langle\underline{n}_\mathbf{k}|\underline{n}_\mathbf{k}\rangle, \tag{11}$$

where the factor in parentheses on the RH is negative. The only way (10) and (11) can be equal, which they must be, is if the norm of $|\underline{n}_\mathbf{k}\rangle$ is negative. Thus, we find that the sign of the norms alternates between positive and negative values as one increases the number of supplemental particles of the same $\mathbf{k}$. And for appropriate normalization, we have

$$\langle\underline{n}_\mathbf{k}|\underline{n}_\mathbf{k}\rangle = (-1)^{\underline{n}_\mathbf{k}} \qquad \underline{n}_\mathbf{k} = 0, -1, -2, \ldots . \tag{12}$$

The metric $g^{Fock}_{\mathbf{k}\,\underline{mn}}$ for Fock space, defined by

$$\langle \underline{m}_{\mathbf{k}} \big| = \left(g^{Fock}_{\mathbf{k}\ \underline{mn}} \big| \underline{n}_{\mathbf{k}} \rangle \right)^{\dagger}, \tag{13}$$

can be expressed in matrix notation for the subspace comprising one type of supplemental particle such as a supplemental electron (of one particular value of 3-momentum) as

$$g^{Fock}_{\mathbf{k}\ \underline{mn}} = \begin{bmatrix} 1 & 0 & 0 & 0 & .. \\ 0 & -1 & 0 & 0 & .. \\ 0 & 0 & 1 & 0 & .. \\ 0 & 0 & 0 & -1 & .. \\ .. & .. & .. & .. & ... \end{bmatrix}, \tag{14}$$

which is definitely indefinite. This result differs from the identity matrix of the traditional particle Fock space metric and caused Pauli[7] to conclude that fields with commutation relations such as (3) were impossible.

We can re-express the norm (12) of a supplemental state as

$$\langle \underline{n}_{\mathbf{k}} \big| \underline{n}_{\mathbf{k}} \rangle = \left(\left[\underline{a}(\mathbf{k}), \underline{a}^{\dagger}(\mathbf{k}') \right] \right)^{\underline{n}_{\mathbf{k}}} \tag{15}$$

which again, is a very general relationship applicable to both traditional and supplemental particle states, except that for traditional particles, the commutator term in parentheses is +1 and the particle numbers are positive. So, we can re-cast the inner product, in general,, where a bar through a quantity indicates it can be either traditional or supplemental, as

$$\langle \cancel{n}_{\mathbf{k}} \big| \cancel{n}_{\mathbf{k}} \rangle = \left(\left[\cancel{a}(\mathbf{k}), \cancel{a}^{\dagger}(\mathbf{k}') \right] \right)^{\cancel{n}_{\mathbf{k}}} . \tag{16}$$

As shown in Ref. 2, we can resolve the issue of metric indefiniteness by defining expectation values for supplemental particle eigenstates of a given operator $\mathcal{O}$, where we incorporate (12), as

$$\overline{\mathcal{O}} \equiv (-1)^{\underline{n}_{\mathbf{k}}} \langle \underline{n}_{\mathbf{k}} \big| \mathcal{O} \big| \underline{n}_{\mathbf{k}} \rangle = (-1)^{\underline{n}_{\mathbf{k}}} \mathcal{O}_{\mathbf{k}} \langle \underline{n}_{\mathbf{k}} \big|\big| \underline{n}_{\mathbf{k}} \rangle = (-1)^{\underline{n}_{\mathbf{k}}} \mathcal{O}_{\mathbf{k}} (-1)^{\underline{n}_{\mathbf{k}}} = \mathcal{O}_{\mathbf{k}} (-1)^{2\underline{n}_{\mathbf{k}}} = \mathcal{O}_{\mathbf{k}} . \tag{17}$$

Correspondingly, the magnitude of a supplemental state is defined via

$$\text{square of magnitude of state with } \underline{n}_{\mathbf{k}} \text{ supplemental particles} \equiv (-1)^{\underline{n}_{\mathbf{k}}} \langle \underline{n}_{\mathbf{k}} \big| \underline{n}_{\mathbf{k}} \rangle \ . \tag{18}$$

We can generalize (17) and (18), to

$$\begin{gathered} \overline{\mathcal{O}} \equiv \left(\left[\cancel{a}(\mathbf{k}), \cancel{a}^{\dagger}(\mathbf{k}') \right] \right)^{\cancel{n}_{\mathbf{k}}} \langle \cancel{n}_{\mathbf{k}} \big| \mathcal{O} \big| \cancel{n}_{\mathbf{k}} \rangle = \cancel{\mathcal{O}}_{\mathbf{k}} \\ \text{square of magnitude of state} \equiv \left(\left[\cancel{a}(\mathbf{k}), \cancel{a}^{\dagger}(\mathbf{k}') \right] \right)^{\cancel{n}_{\mathbf{k}}} \langle \cancel{n}_{\mathbf{k}} \big| \cancel{n}_{\mathbf{k}} \rangle . \end{gathered} \ . \tag{19}$$

1.7 Raising and Lowering Coefficients

Employing (17) and (18) with (9), instead of (10), we have an expectation value

$$(-1)^{\underline{n}_{\mathbf{k}}} \langle \underline{n}_{\mathbf{k}} \big| \,|\, \underline{a}(\mathbf{k}) \underline{a}^{\dagger}(\mathbf{k}) \big| \underline{n}_{\mathbf{k}} \rangle = (-1)^{\underline{n}_{\mathbf{k}}-1} \langle \underline{n}_{\mathbf{k}} - 1 \big| \underline{A}^{\dagger} \underline{A} \big| \underline{n}_{\mathbf{k}} - 1 \rangle = (-1)^{2\underline{n}_{\mathbf{k}}-2} \underline{A}^{\dagger} \underline{A} = \underline{A}^{\dagger} \underline{A} . \tag{20}$$

Similarly, from the commutation relations (3) in the LHS of (20), using (17), we get (21) instead of (11).

$$\begin{aligned} (-1)^{\underline{n}_{\mathbf{k}}} \langle \underline{n}_{\mathbf{k}} \big| \underline{a}^{\dagger}(\mathbf{k}) \underline{a}(\mathbf{k}) - 1 \big| \underline{n}_{\mathbf{k}} \rangle &= (-1)^{\underline{n}_{\mathbf{k}}} \langle \underline{n}_{\mathbf{k}} \big| \underline{N}(\mathbf{k}) \big| \underline{n}_{\mathbf{k}} \rangle + (-1)^{\underline{n}_{\mathbf{k}}} \langle \underline{n}_{\mathbf{k}} \big| (-1) \big| \underline{n}_{\mathbf{k}} \rangle \\ &= (-1)^{\underline{n}_{\mathbf{k}}} \underline{n}_{\mathbf{k}} \langle \underline{n}_{\mathbf{k}} \big| \underline{n}_{\mathbf{k}} \rangle - (-1)^{\underline{n}_{\mathbf{k}}} \langle \underline{n}_{\mathbf{k}} \big| \underline{n}_{\mathbf{k}} \rangle \\ &= (-1)^{\underline{n}_{\mathbf{k}}} \underline{n}_{\mathbf{k}} (-1)^{\underline{n}_{\mathbf{k}}} - (-1)^{\underline{n}_{\mathbf{k}}} (-1)^{\underline{n}_{\mathbf{k}}} = (-1)^{2\underline{n}_{\mathbf{k}}} (\underline{n}_{\mathbf{k}} - 1) = \underline{n}_{\mathbf{k}} - 1 . \end{aligned} \tag{21}$$

(20) and (21) are equal, so, from the RHS of both,

$$\underline{A} = \sqrt{\underline{n}_{\mathbf{k}} - 1}\,, \tag{22}$$

making (9)

$$\underline{a}^{\dagger}(\mathbf{k})\left|\underline{n}_{\mathbf{k}}\right\rangle = \sqrt{\underline{n}_{\mathbf{k}} - 1}\left|\underline{n}_{\mathbf{k}} - 1\right\rangle\,, \tag{23}$$

or restated as in Ref. 2,

$$\underline{a}^{\dagger}(\mathbf{k})\left|\underline{n}_{\mathbf{k}} + 1\right\rangle = \sqrt{\underline{n}_{\mathbf{k}}}\left|\underline{n}_{\mathbf{k}}\right\rangle\,. \tag{24}$$

Similarly, for

$$\underline{a}(\mathbf{k})\left|\underline{n}_{\mathbf{k}}\right\rangle = \underline{A}'\left|\underline{n}_{\mathbf{k}} + 1\right\rangle\,, \tag{25}$$

$$(-1)^{\underline{n}_{\mathbf{k}}}\left\langle\underline{n}_{\mathbf{k}}\right|\,|\,\underline{a}^{\dagger}(\mathbf{k})\underline{a}(\mathbf{k})\left|\underline{n}_{\mathbf{k}}\right\rangle = (-1)^{\underline{n}_{\mathbf{k}}}\left\langle\underline{n}_{\mathbf{k}}\right|\,|\,\underline{N}(\mathbf{k})\left|\underline{n}_{\mathbf{k}}\right\rangle = (-1)^{\underline{n}_{\mathbf{k}}}\,\underline{n}_{\mathbf{k}}\left\langle\underline{n}_{\mathbf{k}}\middle|\underline{n}_{\mathbf{k}}\right\rangle = (-1)^{2\underline{n}_{\mathbf{k}}}\,\underline{n}_{\mathbf{k}} = \underline{n}_{\mathbf{k}}\,, \tag{26}$$

which can be expressed somewhat differently as

$$\begin{aligned}(-1)^{\underline{n}_{\mathbf{k}}}\left\langle\underline{n}_{\mathbf{k}}\right|\,|\,\underline{a}^{\dagger}(\mathbf{k})\underline{a}(\mathbf{k})\left|\underline{n}_{\mathbf{k}}\right\rangle &= (-1)^{\underline{n}_{\mathbf{k}}+1}\left\langle\underline{n}_{\mathbf{k}} + 1\right|\underline{A}'^{\dagger}\underline{A}'\left|\underline{n}_{\mathbf{k}} + 1\right\rangle \\ &= (-1)^{\underline{n}_{\mathbf{k}}+1}\,\underline{A}'^{\dagger}\underline{A}'\left\langle\underline{n}_{\mathbf{k}} + 1\middle|\underline{n}_{\mathbf{k}} + 1\right\rangle = (-1)^{2\underline{n}_{\mathbf{k}}+2}\,\underline{A}'^{\dagger}\underline{A}' = \underline{A}'^{\dagger}\underline{A}'\,.\end{aligned}\,, \tag{27}$$

We find, from the RHS of (26) and (27),

$$\underline{A}' = \sqrt{\underline{n}_{\mathbf{k}}}\,, \tag{28}$$

making (25)

$$\underline{a}(\mathbf{k})\left|\underline{n}_{\mathbf{k}}\right\rangle = \sqrt{\underline{n}_{\mathbf{k}}}\left|\underline{n}_{\mathbf{k}} + 1\right\rangle\,. \tag{29}$$

1.8 Four Currents, Probability Density, and Particle Velocity

1.8.1 Background for Traditional Particles

In non-relativistic quantum mechanics (NRQM), norms of single particle states represent probabilities and always equal positive unity (for proper normalization.) Extrapolating this to the indefinite metric reflected in the norms (12), one might interpret this as meaning supplemental particle states could have either positive or negative probability, with the obvious theory devastating implications. In QFT, however, supplemental particle states have definite positive probabilities, even though their norms do not.

From Ref. 3, pgs. 46 and 62, the conserved scalar four current operator is

$$j^{\mu} = i\left(\phi^{,\mu}\phi^{\dagger} - \phi^{\dagger,\mu}\phi\right). \tag{30}$$

The probability density operator is (30) with $\mu = 0$,

$$\rho_{oper} = j^{0} = i\left(\frac{\partial\phi}{\partial t}\phi^{\dagger} - \frac{\partial\phi^{\dagger}}{\partial t}\phi\right) = \frac{1}{V}\sum_{\mathbf{k}}\left(a^{\dagger}(\mathbf{k})a(\mathbf{k}) - b^{\dagger}(\mathbf{k})b(\mathbf{k})\right) = \frac{1}{V}\sum_{\mathbf{k}}\left(N_{a}(\mathbf{k}) - N_{b}(\mathbf{k})\right), \tag{31}$$

where the last two relations are for plane waves. For a multi-particle state, the expectation value for probability density is

$$\begin{aligned}\overline{\rho} &= \left\langle n_1, n_2, n_3, \ldots\right|\rho^{oper}\left|n_1, n_2, n_3, \ldots\right\rangle \\ &= \left\langle n_1, n_2, n_3, \ldots\right|\frac{1}{V}\sum_{\mathbf{k}}\left(N_a(\mathbf{k}) - N_b(\mathbf{k})\right)\left|n_1, n_2, n_3, \ldots\right\rangle = \frac{n_1 + n_2 + n_2 + \ldots}{V}\,,\end{aligned} \tag{32}$$

which is really a particle number density. For a single particle state, (32) becomes the expectation value of the probability density. That is,

$$\overline{\rho} = \left\langle n_{\mathbf{k}} = 1 \middle| \rho \middle| n_{\mathbf{k}} = 1 \right\rangle = \left\langle n_{\mathbf{k}} = 1 \middle| \frac{1}{V} \sum_{\mathbf{k}'} \left(N_a\left(\mathbf{k}'\right) - N_b\left(\mathbf{k}'\right) \right) \middle| n_{\mathbf{k}} = 1 \right\rangle = \left\langle n_{\mathbf{k}} = 1 \middle| \frac{1}{V} \middle| n_{\mathbf{k}} = 1 \right\rangle = \frac{1}{V} . \quad (33)$$

Note that for anti-particles (b type particles) the probability density is negative, which led to the interpretation of (31) as a charge density operator. In other words, the sign is conventional, but we can still interpret the absolute value of (33), or its counterpart for an antiparticle, as the expectation value for probability density.

From (30) with $\mu = i$ =1, 2, 3, the probability 3-current operator is

$$\begin{aligned} j^i_{oper} &= i\left(\phi^{,i}\phi^\dagger - \phi^{\dagger,i}\phi\right) = -i\left(\phi_{,i}\phi^\dagger - \phi^\dagger{}_{,i}\phi\right) = \sum_{\mathbf{k}'} \frac{k^i}{\omega_{\mathbf{k}} V}\left(N_a\left(\mathbf{k}'\right) - N_b\left(\mathbf{k}'\right) \right) \\ &\rightarrow \mathbf{j}_{oper} = \sum_{\mathbf{k}'} \frac{\mathbf{k}'}{\omega_{\mathbf{k}} V}\left(N_a\left(\mathbf{k}'\right) - N_b\left(\mathbf{k}'\right) \right) \end{aligned} \quad (34)$$

with expectation value for a single particle state

$$\overline{\mathbf{j}} = \left\langle n_{\mathbf{k}} = 1 \middle| \sum_{\mathbf{k}'} \frac{\mathbf{k}'}{\omega_{\mathbf{k}} V}\left(N_a\left(\mathbf{k}'\right) - N_b\left(\mathbf{k}'\right) \right) \middle| n_{\mathbf{k}} = 1 \right\rangle = \frac{\mathbf{k}}{\omega_{\mathbf{k}} V} . \quad (35)$$

We can simplify and restate (35) in terms of the current density operator acting on a single particle state as

$$\mathbf{j}_{oper} \left| n_{\mathbf{k}} = 1 \right\rangle = \frac{\mathbf{k}}{\omega_{\mathbf{k}} V} \left| n_{\mathbf{k}} = 1 \right\rangle . \quad (36)$$

Note that $\overline{\mathbf{j}}$ represents the movement of the probability density of the particle. That is, it is proportional to the expected velocity of the particle, i.e., effectively, the classical particle velocity. A shown in (35), and as it must be, it is in the same direction as 3-momentum $\mathbf{k}$.

1.8.2 Supplemental Particles

In similar fashion, for supplemental fields, with a similar form for the Lagrangian, we get a relation similar to (30), i.e.,

$$\underline{j}^\mu = i\left(\underline{\phi}^{,\mu}\underline{\phi}^\dagger - \underline{\phi}^{\dagger,\mu}\underline{\phi}\right) . \quad (37)$$

and, similar to (31), except for the sign change due to the time derivative,

$$\underline{\rho}_{oper} = \underline{j}^0 = i\left(\frac{\partial \underline{\phi}}{\partial t}\underline{\phi}^\dagger - \frac{\partial \underline{\phi}^\dagger}{\partial t}\underline{\phi} \right) = -\frac{1}{V}\sum_{\mathbf{k}}\left(\underline{a}^\dagger\left(\mathbf{k}\right)\underline{a}\left(\mathbf{k}\right) - \underline{b}^\dagger\left(\mathbf{k}\right)\underline{b}\left(\mathbf{k}\right) \right) = -\frac{1}{V}\sum_{\mathbf{k}}\left(\underline{N}_a\left(\mathbf{k}\right) - \underline{N}_b\left(\mathbf{k}\right) \right) . \quad (38)$$

Thus,

$$\begin{aligned} \underline{\overline{\rho}} &= \left(-1\right)^{\underline{n}_1 + \underline{n}_2 + \underline{n}_3 + ..} \left\langle \underline{n}_1, \underline{n}_2, \underline{n}_3, ... \middle| \underline{\rho}^{oper} \middle| \underline{n}_1, \underline{n}_2, \underline{n}_3, ... \right\rangle \\ &= \left(-1\right)^{\underline{n}_1 + \underline{n}_2 + \underline{n}_3 + ..} \left\langle \underline{n}_1, \underline{n}_2, \underline{n}_3, ... \middle| -\frac{1}{V}\sum_{\mathbf{k}}\left(\underline{N}_a\left(\mathbf{k}\right) - \underline{N}_b\left(\mathbf{k}\right) \right) \middle| \underline{n}_1, \underline{n}_2, \underline{n}_3, ... \right\rangle \\ &= -\frac{\underline{n}_1 + \underline{n}_2 + \underline{n}_2 + ...}{V} = \frac{\left|\underline{n}_1\right| + \left|\underline{n}_2\right| + \left|\underline{n}_2\right| + ...}{V} , \end{aligned} \quad (39)$$

and for a single supplemental particle,

$$\underline{\overline{\rho}} = \left(-1\right)^{\underline{n}_{\mathbf{k}} = -1} \left\langle \underline{n}_{\mathbf{k}} = -1 \middle| \rho \middle| \underline{n}_{\mathbf{k}} = -1 \right\rangle = \frac{1}{V} . \quad (40)$$

Hence, the probability density expectation for a single supplemental particle (40) is positive, and identical to that of a single traditional particle (33). If one interprets (33) and (40) as charge densities, then one can conclude that the supplemental $\underline{a}$ type particles have the same charge as the traditional a type particles.

Relations similar to (34) and (36) for supplemental particles are

$$\underline{\mathbf{j}}_{oper} = \sum_{\mathbf{k}} \frac{\mathbf{k}}{\omega_{\mathbf{k}} V} \left(\underline{N}_a(\mathbf{k}) - \underline{N}_b(\mathbf{k}) \right) \tag{41}$$

$$\underline{\mathbf{j}}_{oper} \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle = \sum_{\mathbf{k}'} \frac{\mathbf{k}'}{\omega_{\mathbf{k}} V} \left(\underline{N}_a(\mathbf{k}') - \underline{N}_b(\mathbf{k}') \right) \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle = (-1) \frac{\mathbf{k}}{\omega_{\mathbf{k}} V} \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle = \frac{-\mathbf{k}}{\omega_{\mathbf{k}} V} \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle \tag{42}$$

The key thing to note with (42) is that particle velocity is in the opposite direction of 3-momentum $\mathbf{k}$, in stark contrast to the situation for traditional particles.

1.9 3-Momentum

Consider the physical 3-momentum operator, which can be found from the conjugate momentum density (see Ref. 3, pg. 64) via

$$P^{i\,oper} = \int \wp^{i\,oper} d^3x = -\int \pi_r \frac{\partial \phi^r}{\partial x^i} d^3x = -\int \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \frac{\partial \phi}{\partial x^i} + \frac{\partial \mathcal{L}}{\partial \dot{\phi}^\dagger} \frac{\partial \phi^\dagger}{\partial x^i} \right) d^3x \,. \tag{43}$$

For momentum density in the x direction ($k = 1$) and the form of the solutions to the field equation employed above, the operator form (ignoring anti-particles) becomes

$$P^{1\,oper} = \sum_{\mathbf{k}} k^1 \left(a^\dagger(\mathbf{k}) a(\mathbf{k}) - b^\dagger(\mathbf{k}) b(\mathbf{k}) \right) \quad \rightarrow \quad \mathbf{P}^{oper} = \sum_{\mathbf{k}} \mathbf{k} \left(N_a(\mathbf{k}) + N_b(\mathbf{k}) \right). \tag{44}$$

For supplemental solutions, (43) will change signs due to the time derivative in (43), and the parallel form of (44) becomes

$$P^{1\,oper} = -\sum_{\mathbf{k}} k^1 \left(\underline{a}^\dagger(\mathbf{k}) \underline{a}(\mathbf{k}) - \underline{b}^\dagger(\mathbf{k}) \underline{b}(\mathbf{k}) \right) \quad \rightarrow \quad \underline{\mathbf{P}}^{oper} = -\sum_{\mathbf{k}} \mathbf{k} \left(\underline{N}_a(\mathbf{k}) + \underline{N}_b(\mathbf{k}) \right). \tag{45}$$

Hence,

$$\underline{\mathbf{P}}^{oper} \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle = -\sum_{\mathbf{k}'} \mathbf{k}' \left(\underline{N}_a(\mathbf{k}') + \underline{N}_b(\mathbf{k}') \right) \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle = -\mathbf{k}(-1) \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle = \mathbf{k} \left| \underline{n}_{\mathbf{k}} = -1 \right\rangle \,. \tag{46}$$

We get a second sign change from the traditional particle momentum due to the negative value of the supplemental particle number. The result is momentum in the same direction as the traditional particle.

Since the supplemental particle velocity determined by probability current direction in (42) is reversed from that of the traditional particle, yet the 3 momentum direction remains the same, we are led to the remarkable result that supplemental particle momentum is in the opposite direction of its travel.

While this may at first seem to be disastrous, it is important to recognize that supplemental particles are presumed to exist only virtually. Certainly, no real particles manifest this quality, just as they do not manifest negative energy. However, in simple 1D scattering of a positively charged particle with a negatively charged particle, the virtual particle must, of necessity, carry momentum in the opposite direction of its travel, in order for the two real particles to exchange momentum in a manner that allows them to attract one another. Thus, this seemingly bizarre property of supplemental particles is already realized in our extant theory of virtual particles.

1.10 Pressure

Pressure in the x direction is the T_{11} component of the stress-energy tensor. It can be shown[8], that for traditional real scalar particles, the pressure operator is

$$T_{11}^{oper} = \left(\partial_1 \phi \right)^2 + \tfrac{1}{2} \left(\left(\dot{\phi} \right)^2 - \left(\nabla \phi \right)^2 - m^2 \phi^2 \right) = \frac{1}{V} \sum_{\mathbf{k}} \frac{\left(k_1 \right)^2}{\omega_{\mathbf{k}}} \left\{ a^\dagger(\mathbf{k}) a(\mathbf{k}) + \tfrac{1}{2} + b^\dagger(\mathbf{k}) b(\mathbf{k}) + \tfrac{1}{2} \right\}, \tag{47}$$

where we have assumed the same plane wave form as earlier for eigensolutions to the field equation. The comparable value for supplemental particles is

$$\underline{T}_{11}^{oper} = \left(\partial_1 \underline{\phi}\right)^2 + \tfrac{1}{2}\left(\left(\dot{\underline{\phi}}\right)^2 - \left(\nabla \underline{\phi}\right)^2 - m^2 \underline{\phi}^2\right) = \frac{1}{V}\sum_{\mathbf{k}} \frac{\left(k_1\right)^2}{\omega_{\mathbf{k}}}\left\{\underline{a}^{\dagger}\left(\mathbf{k}\right)\underline{a}\left(\mathbf{k}\right) - \tfrac{1}{2} + \underline{b}^{\dagger}\left(\mathbf{k}\right)\underline{b}\left(\mathbf{k}\right) - \tfrac{1}{2}\right\}, \quad (48)$$

where, as with energy, we see that the ½ quanta terms drop out if one includes supplemental particles into the total vacuum pressure calculation. By comparing (47) and (48) in the manner done above for 3-momentum, we can see that in the expectation value for any state, the pressure contribution from supplemental particles will have opposite sign from that of their traditional siblings. If traditional particles exert positive pressure (compression), their supplemental counterparts will exert negative pressure (tension.)

Thus, where we ignore the ½ terms,

$$\underline{T}_{11}^{oper}\left|\underline{n}_{\mathbf{k}}\right\rangle = \frac{1}{V}\sum_{\mathbf{k}} \frac{\left(k_1\right)^2}{\omega_{\mathbf{k}}} \underline{N}_a\left(\mathbf{k}\right)\left|\underline{n}_{\mathbf{k}}\right\rangle = \frac{\left(k_1\right)^2}{\omega_{\mathbf{k}} V} \underline{n}_{\mathbf{k}}\left|\underline{n}_{\mathbf{k}}\right\rangle = -\frac{\left(k_1\right)^2}{\omega_{\mathbf{k}} V}\left|\underline{n}_{\mathbf{k}}\right|\left|\underline{n}_{\mathbf{k}}\right\rangle . \quad (49)$$

1.11 Propagators

For an extensive, step-by-step derivation of what is expressed succinctly below, see Klauber[9].

The Feynman propagator Δ_F for a traditional complex scalar field is defined by (see Ref. 3, pg. 73)

$$\begin{aligned} &\text{for } t_y < t_x \quad i\Delta_F\left(x-y\right) = \left\langle 0\left|\left[\phi^{+}\left(x\right), \phi^{\dagger -}\left(y\right)\right]\right|0\right\rangle \\ &\text{for } t_x < t_y \quad i\Delta_F\left(x-y\right) = \left\langle 0\left|\left[\phi^{\dagger +}\left(y\right), \phi^{-}\left(x\right)\right]\right|0\right\rangle . \end{aligned} \quad (50)$$

When the integration implicit in (50) (for fields expressed as integrals, not sums, over $\mathbf{k}$) is carried out, one finds a factor of the coefficient commutator

$$\left[a\left(\mathbf{k}\right), a^{\dagger}\left(\mathbf{k}'\right)\right] = \delta_{\mathbf{k}\mathbf{k}'} \quad (51)$$

in front of each term. The final result is

$$\Delta_F\left(x-y\right) = \frac{1}{\left(2\pi\right)^4}\int \frac{d^4k e^{-ik\left(x-y\right)}}{k^2 - \mu^2 + i\varepsilon} \quad (52)$$

$$\Delta_F\left(k\right) = \frac{1}{k^2 - \mu^2 + i\varepsilon} \quad (53)$$

For a supplemental complex scalar field, the same procedure yields a coefficient commutator factor of (3) i.e, having the opposite sign from (51). All other steps in the derivation of the propagator parallel that of the traditional real scalar particle. Thus,

$$\underline{\Delta}_F\left(x-y\right) = \frac{-1}{\left(2\pi\right)^4}\int \frac{d^4k e^{-i\underline{k}\left(\underline{x}-\underline{y}\right)}}{k^2 - \mu^2 + i\varepsilon} = \frac{-1}{\left(2\pi\right)^4}\int \frac{d^4k e^{-ik\left(x-y\right)}}{k^2 - \mu^2 + i\varepsilon} \quad (54)$$

where the last part is true because we integrate over all positive and negative $\omega_{\mathbf{k}}$ and $\mathbf{k}$. Hence,

$$\Delta_F\left(k\right) = -\underline{\Delta}_F\left(k\right), \quad (55)$$

and the supplemental propagator has the same form as, but opposite sign from, its traditional counterpart.

The results of (54) and (55) generalize to fields of spin ½ and 1.

2 Spinor Supplemental Fields

2.1 Overview of Dirac Equation Development and Its Different Forms

Dirac began the derivation of his famous equation by postulating a relativistic Schrödinger equation that was first order in the Hamiltonian operator H, rather than second order as in the Klein-Gordon equation. See Ref. 3, pgs 86-89. It was a matrix equation with form

$$i\frac{\partial}{\partial t}\psi = H\psi = \left(\boldsymbol{\alpha}\bullet\mathbf{p} + \beta m\right)\psi \tag{56}$$

where

$$\beta = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \quad \alpha_1 = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{bmatrix} \quad \alpha_2 = \begin{bmatrix} & & & -i \\ & & i & \\ & -i & & \\ i & & & \end{bmatrix} \quad \alpha_3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ 1 & & & \\ & -1 & & \end{bmatrix} \tag{57}$$

and ψ is a four component column matrix,

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}. \tag{58}$$

Pre-multiplying (56) by β of (57), and defining the Dirac gamma matrices γ^μ as

$$\gamma^0 = \beta \qquad \gamma^1 = \beta\alpha_1 \qquad \gamma^2 = \beta\alpha_2 \qquad \gamma^3 = \beta\alpha_3 \tag{59}$$

such that

$$\gamma^0 = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix} \quad \gamma^1 = \begin{bmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{bmatrix} \quad \gamma^2 = \begin{bmatrix} & & & -i \\ & & i & \\ & i & & \\ -i & & & \end{bmatrix} \quad \gamma^3 = \begin{bmatrix} & & 1 & \\ & & & -1 \\ -1 & & & \\ & 1 & & \end{bmatrix}, \tag{60}$$

turned (56) into the familiar form for the Dirac equation we use today,

$$\left(i\gamma^\mu\partial_\mu - m\right)\psi = 0\,. \tag{61}$$

(61), like (56), is a 4X4 matrix equation which can be expressed as

$$i\left(\begin{bmatrix} \partial_0 & 0 & \partial_3 & \partial_1 - i\partial_2 \\ 0 & \partial_0 & \partial_1 + i\partial_2 & -\partial_3 \\ -\partial_3 & -\partial_1 + i\partial_2 & -\partial_0 & 0 \\ -\partial_1 - i\partial_2 & \partial_3 & 0 & -\partial_0 \end{bmatrix}\right)\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = m\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}. \tag{62}$$

Note that (56) and (61) (fully written out as (62)) are simply different forms of the Dirac equation. We use (61) almost exclusively in physics because it is easier in almost all applications.

2.2 A Simplified Special Case

2.2.1 Traditional Plus Supplemental Solutions Arise

However, it will help us to first make a point as simply as possible if, instead of (61), we look at (56). To make things even simpler, we will take a special case where $\mathbf{p} = 0$, so (56) becomes

$$i\frac{\partial}{\partial t}\psi = \beta m\psi \qquad \rightarrow \qquad i\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix} = m\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{bmatrix}\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{bmatrix}. \tag{63}$$

Now, in finding solutions to the Dirac equation, one typically assumes the forms

$$\psi = \psi_{\mathbf{p}} = A_{\mathbf{p}} u(\mathbf{p}) e^{-ipx} = A_{\mathbf{p}} u(\mathbf{p}) e^{-i\left(E_{\mathbf{k}} t - \mathbf{p}\cdot\mathbf{x}\right)} \quad \text{or} \quad \psi' = \psi'_{\mathbf{p}} = B_{\mathbf{p}} v(\mathbf{p}) e^{+ipx} = B_{\mathbf{p}} v(\mathbf{p}) e^{+i\left(E_{\mathbf{k}} t - \mathbf{p}\cdot\mathbf{x}\right)}, \tag{64}$$

where $u_r(\mathbf{p})$ and $v_r(\mathbf{p})$ are four component column vectors. With the first of (64) into (63), we get

$$\begin{bmatrix} E_{\mathbf{p}} & & & \\ & E_{\mathbf{p}} & & \\ & & E_{\mathbf{p}} & \\ & & & E_{\mathbf{p}} \end{bmatrix} \psi_{\mathbf{p}} = \begin{bmatrix} m & & & \\ & m & & \\ & & -m & \\ & & & -m \end{bmatrix} \psi_{\mathbf{p}} = \underbrace{E_{\mathbf{p}} \psi_{\mathbf{p}}}_{\substack{\text{Re-expressing} \\ \text{LHS to emphasize} \\ \text{eigenvalue nature}}} . \tag{65}$$

What we have done is taken the eigenvalue problem of (56) in the simplest possible form, i.e., where $\mathbf{p} = 0$, in order to make a point.

Our eigenvalues for $E_{\mathbf{p}}$, according to (65), are

$$E_{\mathbf{p}}^{(1)} = m \qquad E_{\mathbf{p}}^{(2)} = m \qquad E_{\mathbf{p}}^{(3)} = -m \qquad E_{\mathbf{p}}^{(4)} = -m . \tag{66}$$

with the corresponding eigenvectors

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad u_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{67}$$

Thus, from (64), the four solutions to (56) [and (61)] for this special case are (where we carry the $\mathbf{p}$ term in the exponent for illustration, even though for this special case $\mathbf{p} = 0$)

$$\begin{aligned} \psi_{\mathbf{p}}^{(1)} &= A_{\mathbf{p}} u_1(\mathbf{p}) e^{-i\left(E_{\mathbf{k}}^{(1)} t - \mathbf{p}\cdot\mathbf{x}\right)} & \qquad \psi_{\mathbf{p}}^{(2)} &= A_{\mathbf{p}} u_2(\mathbf{p}) e^{-i\left(E_{\mathbf{k}}^{(2)} t - \mathbf{p}\cdot\mathbf{x}\right)} \\ \psi_{\mathbf{p}}^{(3)} &= A_{\mathbf{p}} u_3(\mathbf{p}) e^{-i\left(E_{\mathbf{k}}^{(3)} t - \mathbf{p}\cdot\mathbf{x}\right)} & \qquad \psi_{\mathbf{p}}^{(4)} &= A_{\mathbf{p}} u_4(\mathbf{p}) e^{-i\left(E_{\mathbf{k}}^{(4)} t - \mathbf{p}\cdot\mathbf{x}\right)} . \end{aligned} \tag{68}$$

Or, with (66),

$$\begin{aligned} \psi_{\mathbf{p}}^{(1)} &= A_{\mathbf{p}} u_1(\mathbf{p}) e^{-imt + i\mathbf{p}\cdot\mathbf{x}} = A_{\mathbf{p}} u_1(\mathbf{p}) e^{-i\left|E_{\mathbf{p}}^{(1)}\right| t + i\mathbf{p}\cdot\mathbf{x}} & \qquad \psi_{\mathbf{p}}^{(2)} &= A_{\mathbf{p}} u_2(\mathbf{p}) e^{-imt + i\mathbf{p}\cdot\mathbf{x}} = A_{\mathbf{p}} u_2(\mathbf{p}) e^{-i\left|E_{\mathbf{p}}^{(2)}\right| t + i\mathbf{p}\cdot\mathbf{x}} \\ \psi_{\mathbf{p}}^{(3)} &= A_{\mathbf{p}} u_3(\mathbf{p}) e^{+imt + i\mathbf{p}\cdot\mathbf{x}} = A_{\mathbf{p}} u_3(\mathbf{p}) e^{+i\left|E_{\mathbf{p}}^{(3)}\right| t + i\mathbf{p}\cdot\mathbf{x}} & \qquad \psi_{\mathbf{p}}^{(4)} &= A_{\mathbf{p}} u_4(\mathbf{p}) e^{+imt + i\mathbf{p}\cdot\mathbf{x}} = A_{\mathbf{p}} u_4(\mathbf{p}) e^{+i\left|E_{\mathbf{p}}^{(4)}\right| t + i\mathbf{p}\cdot\mathbf{x}} \end{aligned} . \tag{69}$$

The main point is that two of the four solutions (the lower row of (69)) have negative energy. This is clear from the form of their exponents in (69), or more directly from (66). These solutions are typically ignored in traditional QFT. Only the two solutions in the upper row of (69) are included. But the other two are clearly (from (67), at the least) independent solutions.

A similar analysis can be done for the RHS solution forms of (64), with similar results. That is,

$$\begin{aligned} \psi_{\mathbf{p}}^{\prime(1)} &= B_{\mathbf{p}} v_1(\mathbf{p}) e^{+imt - i\mathbf{p}\cdot\mathbf{x}} = B_{\mathbf{p}} v_1(\mathbf{p}) e^{+i\left|E_{\mathbf{p}}^{(1)}\right| t - i\mathbf{p}\cdot\mathbf{x}} & \qquad \psi_{\mathbf{p}}^{\prime(2)} &= B_{\mathbf{p}} v_2(\mathbf{p}) e^{+imt - i\mathbf{p}\cdot\mathbf{x}} = B_{\mathbf{p}} v_2(\mathbf{p}) e^{+i\left|E_{\mathbf{p}}^{(2)}\right| t - i\mathbf{p}\cdot\mathbf{x}} \\ \psi_{\mathbf{p}}^{\prime(3)} &= B_{\mathbf{p}} v_3(\mathbf{p}) e^{-imt - i\mathbf{p}\cdot\mathbf{x}} = B_{\mathbf{p}} v_3(\mathbf{p}) e^{-i\left|E_{\mathbf{p}}^{(3)}\right| t - i\mathbf{p}\cdot\mathbf{x}} & \qquad \psi_{\mathbf{p}}^{\prime(4)} &= B_{\mathbf{p}} v_4(\mathbf{p}) e^{-imt - i\mathbf{p}\cdot\mathbf{x}} = B_{\mathbf{p}} v_4(\mathbf{p}) e^{-i\left|E_{\mathbf{p}}^{(4)}\right| t - i\mathbf{p}\cdot\mathbf{x}} , \end{aligned} \tag{70}$$

where

$$v_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad v_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} . \tag{71}$$

Here, again traditional QFT only retains the two positive energy solutions, even though all four are independent in form. Thus, the four spinor (column vectors) one usually only deals with in QFT are u_1, u_2, v_1, and v_2. As was done with scalars in Ref. 2, we deem the other solutions "supplemental solutions".

2.2.2 Mathematical Equivalence of Traditional and Supplemental Solutions

Note, however, that if we sum solutions over all values of **k** to get the general solution, then, for example,

$$\begin{aligned} &\psi'^{(3)}_{-\mathbf{p}} = B_{-\mathbf{p}} v_3\left(-\mathbf{p}\right) e^{-imt+i\mathbf{p}\cdot\mathbf{x}} = B_{-\mathbf{p}} v_3\left(-\mathbf{p}\right) e^{-i\left|E^{(3)}_{\mathbf{p}}\right|t+i\mathbf{p}\cdot\mathbf{x}} \text{ is mathematically} \\ &\text{equal to} \quad \psi^{(2)}_{\mathbf{p}} = A_{\mathbf{p}} u_2\left(\mathbf{p}\right) e^{-imt+i\mathbf{p}\cdot\mathbf{x}} = A_{\mathbf{p}} u_2\left(\mathbf{p}\right) e^{-i\left|E^{(2)}_{\mathbf{p}}\right|t+i\mathbf{p}\cdot\mathbf{x}} \quad \text{for} \quad B_{-\mathbf{p}} = A_{\mathbf{p}} \ . \end{aligned} \tag{72}$$

From similar relations with other solutions forms, we can conclude that each supplemental type solution [the second row of (69) and the second row of (70)], is equivalent mathematically to a corresponding traditional solution [the first rows of (69) and (70)] with 3-momentum direction reversed.

2.2.3 Physical Non-equivalence of Traditional and Supplemental Solutions

However, following the same procedure carried out with scalars in Ref. 2 (setting exponents in (69) to constants and taking the time derivative), one finds the phase velocity of a supplemental spinor wave is in the opposite direction of its 3-momentum. Continuing with that procedure, one also finds supplemental spinors to have negative energy.

So though the supplemental spinor solutions are not mathematically independent from the traditional ones with regard to contributions to the general solution, if they are to represent physical entities, then they are distinctly different, and in fact, independent.

2.3 General Case

The following are the most general case, normalized solutions to the Dirac equation for any **p**, which one can justify by substituting them into (61) [i.e., (62)] or (56). In each case,

$$E = \pm\sqrt{m^2 + \mathbf{p}^2} \qquad \left|E\right| = \sqrt{m^2 + \mathbf{p}^2} \ . \tag{73}$$

As an aside, we note that relativistic theories are already replete with negative energy solutions, i.e., those with a minus sign in (73).

We follow notation of Ref. 2, in which supplemental solutions have underbars.

$$\begin{aligned} &\psi^{(1)}_{\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} 1 \\ 0 \\ \dfrac{p^3}{|E|+m} \\ \dfrac{p^1+ip^2}{|E|+m}\end{pmatrix}}_{u_1} e^{-i|E|t+i\mathbf{p}\cdot\mathbf{x}} \qquad \psi^{(2)}_{\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} 0 \\ 1 \\ \dfrac{p^1-ip^2}{|E|+m} \\ \dfrac{-p^3}{|E|+m}\end{pmatrix}}_{u_2} e^{-i\left|E^{(1)}_{\mathbf{p}}\right|t+i\mathbf{p}\cdot\mathbf{x}} \\ &\underline{\psi}^{(3)}_{-\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} \dfrac{-p^3}{|E|+m} \\ \dfrac{-\left(p^1+ip^2\right)}{|E|+m} \\ 1 \\ 0\end{pmatrix}}_{u_3} e^{+i|E|t+i\mathbf{p}\cdot\mathbf{x}} \qquad \underline{\psi}^{(4)}_{-\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} \dfrac{-\left(p^1-ip^2\right)}{|E|+m} \\ \dfrac{p^3}{|E|+m} \\ 0 \\ 1\end{pmatrix}}_{u_4} e^{+i|E|t+i\mathbf{p}\cdot\mathbf{x}} \end{aligned} \tag{74}$$

$$\psi'^{(1)}_{\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} \frac{p^1 - ip^2}{|E|+m} \\ \frac{-p^3}{|E|+m} \\ 0 \\ 1 \end{pmatrix}}_{v_1} e^{+i|E|t - i\mathbf{p}\cdot\mathbf{x}} \qquad \psi'^{(2)}_{\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} \frac{p^3}{|E|+m} \\ \frac{p^1 + ip^2}{|E|+m} \\ 1 \\ 0 \end{pmatrix}}_{v_2} e^{+i|E|t - i\mathbf{p}\cdot\mathbf{x}}$$

$$\underline{\psi}'^{(3)}_{\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} 0 \\ 1 \\ \frac{-p^1 + ip^2}{|E|+m} \\ \frac{p^3}{|E|+m} \end{pmatrix}}_{v_3} e^{-i|E|t - i\mathbf{p}\cdot\mathbf{x}} \qquad \underline{\psi}'^{(4)}_{\mathbf{p}} = \underbrace{\sqrt{\frac{|E|+m}{2m}}\begin{pmatrix} 1 \\ 0 \\ \frac{-p^3}{|E|+m} \\ \frac{-\left(p^1 + ip^2\right)}{|E|+m} \end{pmatrix}}_{v_4} e^{-i|E|t - i\mathbf{p}\cdot\mathbf{x}} \tag{75}$$

Note that by taking $\mathbf{p} \rightarrow -\mathbf{p}$ in each of the supplemental solutions, we get one of the traditional solutions. That is, mathematically,

$$\underline{\psi}^{(3)}_{-\mathbf{p}} = \psi'^{(2)}_{\mathbf{p}} \qquad \underline{\psi}^{(4)}_{-\mathbf{p}} = \psi'^{(1)}_{\mathbf{p}} \qquad \underline{\psi}'^{(3)}_{-\mathbf{p}} = \psi^{(2)}_{\mathbf{p}} \qquad \underline{\psi}'^{(4)}_{-\mathbf{p}} = \psi^{(1)}_{\mathbf{p}} \ . \tag{76}$$

However, once again in each case, the supplemental solution has phase velocity in the opposite direction of its 3-momentum. Additionally, and importantly, the energy eigenvalues for the supplemental solutions are negative.

3 Wheeler-Feynman Absorber Theory and Supplemental Fields Theory

One might at first consider the present approach akin to the Wheeler-Feynman (WF) absorber theory, where supplemental solutions here correspond to retarded solutions for WF. There are major differences.

For one, WF absorber theory considered both advanced and retarded waves to occur in the same interaction between particles (as does Cramer's transactional interpretation[10][11] of QM.) Traditional and supplemental solutions, on the other hand, when considered as virtual particles mediating interactions, either occur as one or the other, not as acting in tandem.

Further, supplemental fields, as considered herein, are not coupled to the traditional ones, except possibly via the Higgs and gravitons. WF fields, in contrast, interact directly with traditional particles. Additionally, the present author understands that the WF absorber theory was limited in not providing higher order effects leading to the Lamb shift. The supplemental fields approach does not conflict with the Lamb shift analysis.

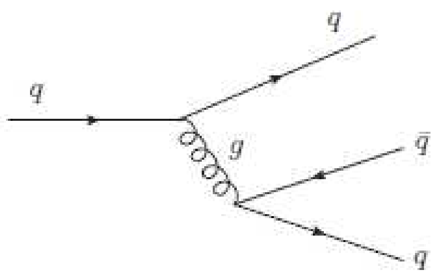
4 Higgs Interactions at LHC

The primary interactions for production of Higgs particles and subsequent Higgs decays are shown in the charts below[12]. Impact on supplemental fields theory is outlined in the last three columns of each chart.

Section 4.4 shows the scattering branching fractions for various masses of the Higgs, though, of course, it is now known to be 125 GeV.

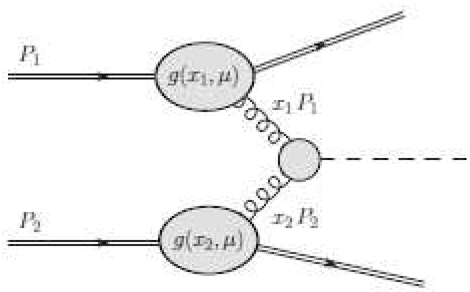
4.1 Production of Higgs

4.1.1 Background: Mechanism for Gluon Production



All quarks on shell. Gluon off shell.

Pair production by a virtual gluon inside a proton.

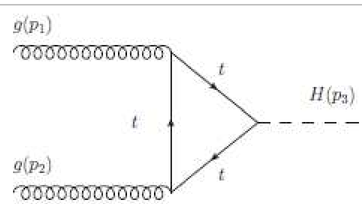
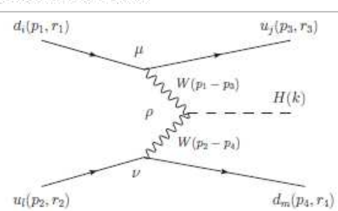


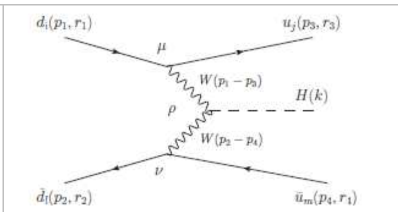
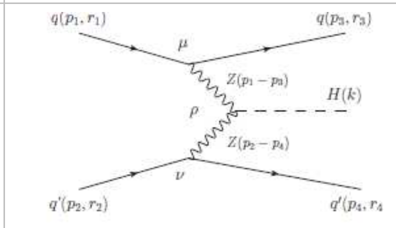
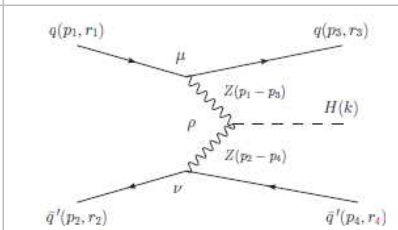
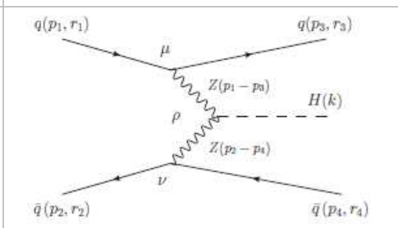
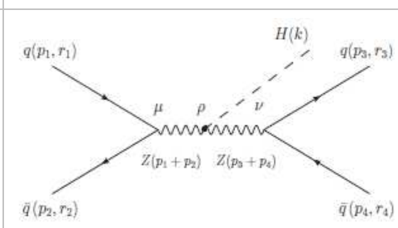
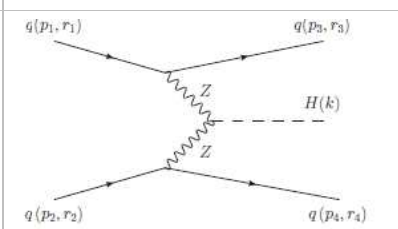
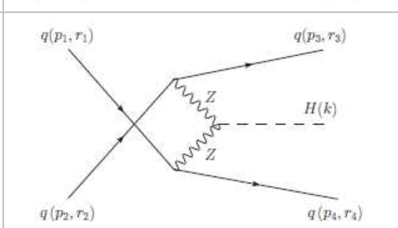
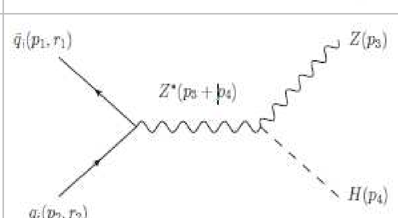
Two off shell gluons produce an H.

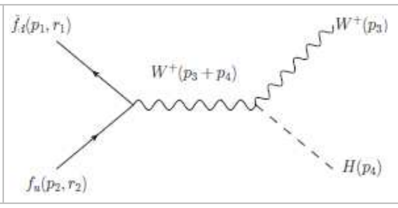
Partonic model of the gluon-gluon scattering process.

4.2 Summary of Primary Sources for Higgs in Proton-Proton Collisions

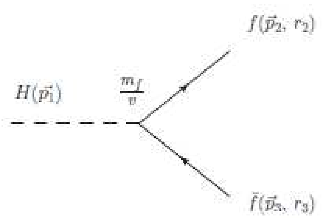
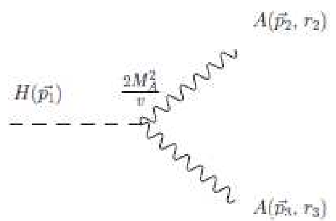
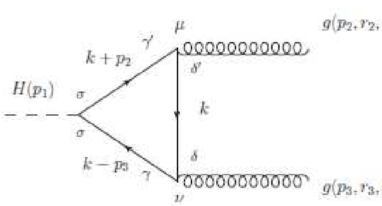
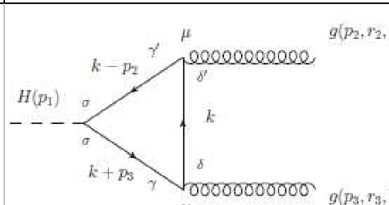
Need no extra diagrams contributing from supplemental particles to agree with data.

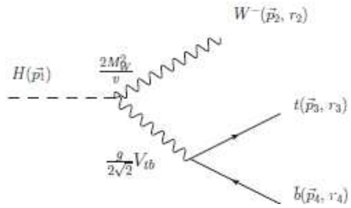
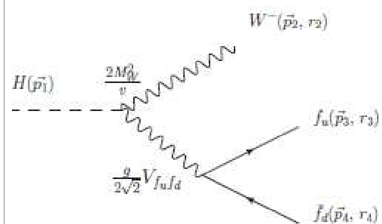
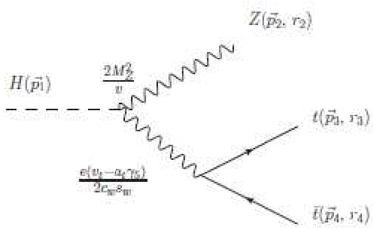
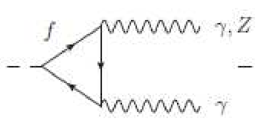
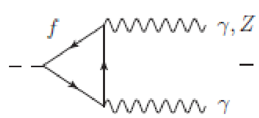
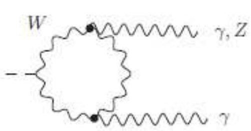
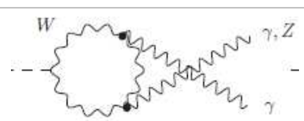
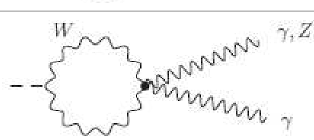
Interaction		**Possible Supplemental Theory Impact**	**Supplemental Theory**	**Theory OK?**
Gluon-Gluon fusion		No $t \rightarrow \underline{t}$ diagram if g not coupled to $\underline{t}$ No $H \rightarrow \underline{H}$ diagram if t not coupled to $\underline{H}$	g not coupled to $\underline{t}$ t not coupled to $\underline{H}$	Y
Weak boson fusion		No $W \rightarrow \underline{W}$ diagram if d,u not coupled to $\underline{W}$ No $H \rightarrow \underline{H}$ diagram if W not coupled to $\underline{H}$	d,u not coupled to $\underline{W}$ W not coupled to $\underline{H}$	Y

2^{nd} contrib		No $W \rightarrow \underline{W}$ diagram if d,u not coupled to $\underline{W}$ No $H \rightarrow \underline{H}$ diagram if W not coupled to $\underline{H}$	d,u not coupled to $\underline{W}$ W not coupled to $\underline{H}$	Y
3^{rd} contrib		No $Z \rightarrow \underline{Z}$ diagram if q not coupled to $\underline{Z}$ No $H \rightarrow \underline{H}$ diagram if Z not coupled to $\underline{H}$	q not coupled to $\underline{Z}$ Z not coupled to $\underline{H}$	Y
4^{th} contrib		No $Z \rightarrow \underline{Z}$ diagram if q not coupled to $\underline{Z}$ No $H \rightarrow \underline{H}$ diagram if Z not coupled to $\underline{H}$	q not coupled to $\underline{Z}$ Z not coupled to $\underline{H}$	Y
5^{th} contrib		No $Z \rightarrow \underline{Z}$ diagram if q not coupled to $\underline{Z}$ No $H \rightarrow \underline{H}$ diagram if Z not coupled to $\underline{H}$	q not coupled to $\underline{Z}$ Z not coupled to $\underline{H}$	Y
6^{th} contrib		No $Z \rightarrow \underline{Z}$ diagram if q not coupled to $\underline{Z}$ No $H \rightarrow \underline{H}$ diagram if Z not coupled to $\underline{H}$	q not coupled to $\underline{Z}$ Z not coupled to $\underline{H}$	Y
7^{th} contrib		No $Z \rightarrow \underline{Z}$ diagram if q not coupled to $\underline{Z}$ No $H \rightarrow \underline{H}$ diagram if Z not coupled to $\underline{H}$	q not coupled to $\underline{Z}$ Z not coupled to $\underline{H}$	Y
8^{th} contrib		No $Z \rightarrow \underline{Z}$ diagram if q not coupled to $\underline{Z}$ No $H \rightarrow \underline{H}$ diagram if Z not coupled to $\underline{H}$	q not coupled to $\underline{Z}$ Z not coupled to $\underline{H}$	Y
Higgs-strahlung		No $Z \rightarrow \underline{Z}$ diagram if q not coupled to $\underline{Z}$ No $H \rightarrow \underline{H}$ diagram if Z not coupled to $\underline{H}$	q not coupled to $\underline{Z}$ Z not coupled to $\underline{H}$	Y

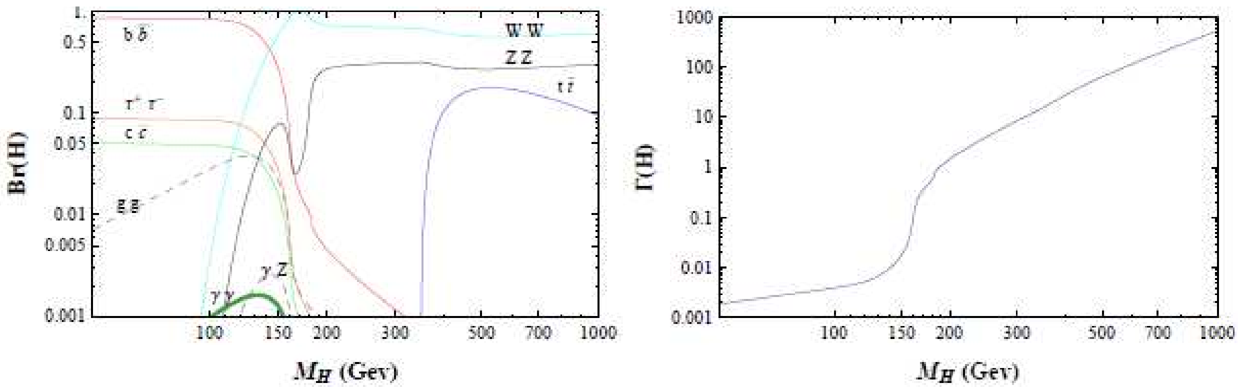
2^{nd} contrib		No $W \rightarrow \underline{W}$ diagram if f not coupled to $\underline{W}$ No $H \rightarrow \underline{H}$ diagram if W not coupled to $\underline{H}$	f not coupled to $\underline{W}$ W not coupled to $\underline{H}$	Y

4.3 Decay Modes of Higgs

Final Particles	**Interactions**	**Possible Supplemental Theory Impact**	**Supplemental Theory**	**Theory OK?**
		2 Body Decays		
Fermions		No $f \rightarrow \underline{f}$ diagram if $\underline{f}$ not coupled to H No $f \rightarrow \underline{f}$ diagram if $\underline{f}$ not allowed as final, real particle	$\underline{f}$ is coupled to H $\underline{f}$ not allowed as final, real particle	Y
Weak bosons ($A = W$, Z)		No $W,Z \rightarrow \underline{W},\underline{Z}$ diagram if H not coupled to $\underline{W},\underline{Z}$ No $W,Z \rightarrow \underline{W},\underline{Z}$ diagram if $\underline{W},\underline{Z}$ not allowed as final, real particles No $W,Z \rightarrow \underline{W},\underline{Z}$ diagram if $\underline{W},\underline{Z}$ not give rise final, real particles	H may, or may not, be coupled to $\underline{W},\underline{Z}$ $\underline{W},\underline{Z}$ not allowed as final, real particles $\underline{W},\underline{Z}$ not coupled to traditional (final, real) particles	Y
Gluons 1st contrib		No $g \rightarrow \underline{g}$ diagram if $\underline{g}$ not coupled to f No $g \rightarrow \underline{g}$ diagram if $\underline{g}$ cannot yield final, real q No $f \rightarrow \underline{f}$ diagram if $\underline{f}$ not coupled to H No $f \rightarrow \underline{f}$ diagram if $\underline{f}$ not coupled to g No $f,g \rightarrow \underline{f},\underline{g}$ if $\underline{g}$ cannot yield final, real q	$\underline{g}$ not coupled to f $\underline{g}$ not coupled to q $\underline{f}$ is coupled to H $\underline{f}$ not coupled to g $\underline{g}$ not coupled to q	Y
2^{nd} contrib		Ditto of above.	Ditto of above.	Y

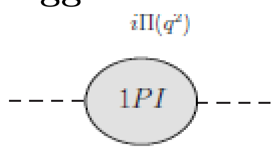
3 Body Decays				
W, 2 quarks 1st contrib		No $W \to \underline{W}$ diagram if H not coupled to $\underline{W}$ No $W \to \underline{W}$ diagram if $\underline{W}$ not allowed as final, real particles No $W \to \underline{W}$ diagram if $\underline{W}$ does not give rise final, real particles	H may, or may not, be coupled to $\underline{W}$ $\underline{W}$ not allowed as final, real particle $\underline{W}$ not coupled to traditional (final, real) particles	Y
2nd contrib		Ditto of above.	Ditto of above.	Y
Z, 2 quarks 1st contrib		Ditto of above using Z in place of W.	Ditto of above using Z in place of W.	Y
2 Photons or 1 Photon and 1 Z (actually 2 body as above section)				
$\gamma\gamma$ and γZ 1st contrib		No $f \to \underline{f}$ if $\underline{f}$ not coupled to H No $f \to \underline{f}$ if $\underline{f}$ not coupled to γ, Z	$\underline{f}$ is coupled to H $\underline{f}$ not coupled to γ, Z	Y
2nd contrib		Ditto of above.	Ditto of above.	Y
3rd contrib		No $W \to \underline{W}$ diagram if H not coupled to $\underline{W}$ No $W \to \underline{W}$ diagram if $\underline{W}$ not coupled to γ, Z	H may, or may not, be coupled to $\underline{W}$ $\underline{W}$ not coupled to γ, Z	Y
4th contrib		Ditto of above.	Ditto of above.	Y
5th contrib		Ditto of above.	Ditto of above.	Y

4.4 Scattering Branching Fractions

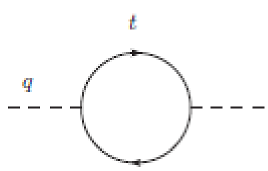
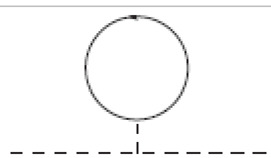
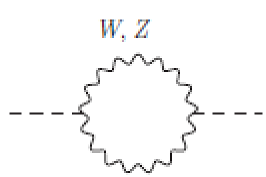
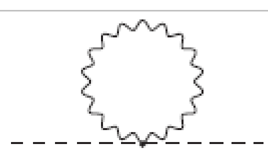
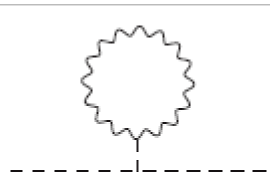


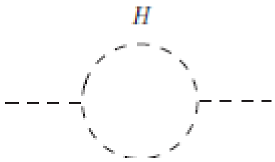
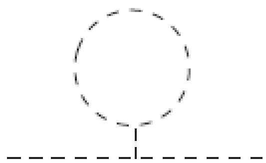
Higgs branching fractions (left) and Higgs decay rate (right) as functions of M_H including 3-body decays

5 Higgs Mass Radiative Corrections Diagrams



equals sum of subdiagrams below

Interaction	**Possible Supplemental Theory Impact**	**Cancels with Diagram of Supplemental Theory?**	**Theory OK?**	**Question**
	$t \to \underline{t}$ in half of loop	Cancels	Y	
	$t \to \underline{t}$	Cancels	Y	
	$W, Z \to \underline{W}, \underline{Z}$	Cancels only if H coupled to $\underline{W}, \underline{Z}$	Y, in one version	Is this diagram meaningful if it weren't cancelled?
	Ditto of above.	Ditto of above.	Y, in one version	Ditto of above.
	Ditto of above for $W, Z \to \underline{W}, \underline{Z}$, but also $H \to \underline{H}$ in tadpole leg	Ditto of above for $W, Z \to \underline{W}, \underline{Z}$, but also cancels if H coupled to $\underline{H}$	Y, in one version	Ditto of above.

	$H \to \underline{H}$	Cancels only if H coupled to $\underline{H}$	Y, in one version	Is this diagram meaningful if it weren't cancelled?
	Ditto of above.	Ditto of above.	Y, in one version	Ditto of above.
	Ditto of above.	Ditto of above.	Y, in one version	Ditto of above.

Conclusion: All diagrams cancel if H is coupled to $\underline{H}$, $\underline{W}$, and $\underline{Z}$. This would not seem to change H production channels or H decay channels, as outlined in Sections 4.2 and 4.3.

Question: Are the diagrams that need the above couplings in order to cancel traditional diagrams in the radiative corrections that lead to the hierarchy problem? Needs an expert to answer.

[1] R. D. Klauber, Mechanism for Zero-Point Energy (2003) http://arXiv.org/abs/astro-ph/0309679

[2] R. D. Klauber, A Symmetry for Resolution of the Gauge Hierarchy Problem without SUSY, Null Higgs Condensate Energy, and Null Zero Point Energy http://arxiv.org/abs/1802.03277 .

[3] R. D. Klauber, *Student Friendly Quantum Field Theory*, Sandtrove, 2013. Chapter 3, cited herein as an aid for deriving scalar supplemental relations, can be found online at http://www.quantumfieldtheory.info/website_Chap03.pdf .

[4] G. Kallén, *Quantum Electrodynamics*, Springer-Verlag (NY), 1972. pp. 17-46.

[5] J.M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons*, Springer-Verlag (NY), 1976. pp. 103-110.

[6] S. N. Gupta, *Quantum Electrondynamics*, Gordon and Breach (NY), 1977. pp. 30-37.

[7] W. Pauli, "Relativistic Field Theories of Elementary Particles", *Rev. Mod. Phys.*, **13**, 203-232 (July 1941); "On Dirac's New Method of Field Quantization", *Rev. Mod. Phys.*, **13**(3), 175-207 (July 1943).

[8] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton Univ Press, 1993), pp. 262-263, 394-396.

[9] R. D. Klauber, Supplemental Propagator Derivation (July 29, 2015), www.quantumfieldtheory.info/suppl_propagator_deriv.pdf

[10] Cramer, John G. "The Transactional Interpretation of Quantum Mechanics". *Reviews of Modern Physics*. **58** (3): 647–688. (July 1986).

[11] Cramer, John G.. *The Quantum Handshake: Entanglement, Nonlocality and Transactions. Springer Science+Business Media.* (2016)

[12] V. Ilisie, "S.M. Higgs Decay and Production Channels", Master's Thesis, University of Valencia (September 2011). http://ific.uv.es/lhcpheno/PhDthesis/master_vilisie.pdf