The understanding gained by this approach should provide a reasonable foundation for i) moving on to applications in Part Four and thereafter drawing this book to a close, and ii) studying higher order renormalization in detail when, and if, the need arises in your career.

Many other texts provide in depth development of higher order renormalization. The purpose of this text is to get you grounded in the fundamentals of QFT, and not get bogged down in complications along the way. It is easier grinding through the details after one knows basic theoretical principles, rather than while one is trying to assimilate them.

14.5 Higher Order Renormalization Example: Compton Scattering

Compton scattering to nth order

We begin, parallel to what we did at second order, by considering an example, Compton scattering, now to arbitrarily high order *n*. In addition to the two first order and 18 second order Feynman diagrams of Fig. 14-1 (pg. 340), we have many additional diagrams, the actual number of which depends on the order *n* being considered.

Associated Feynman diagrams

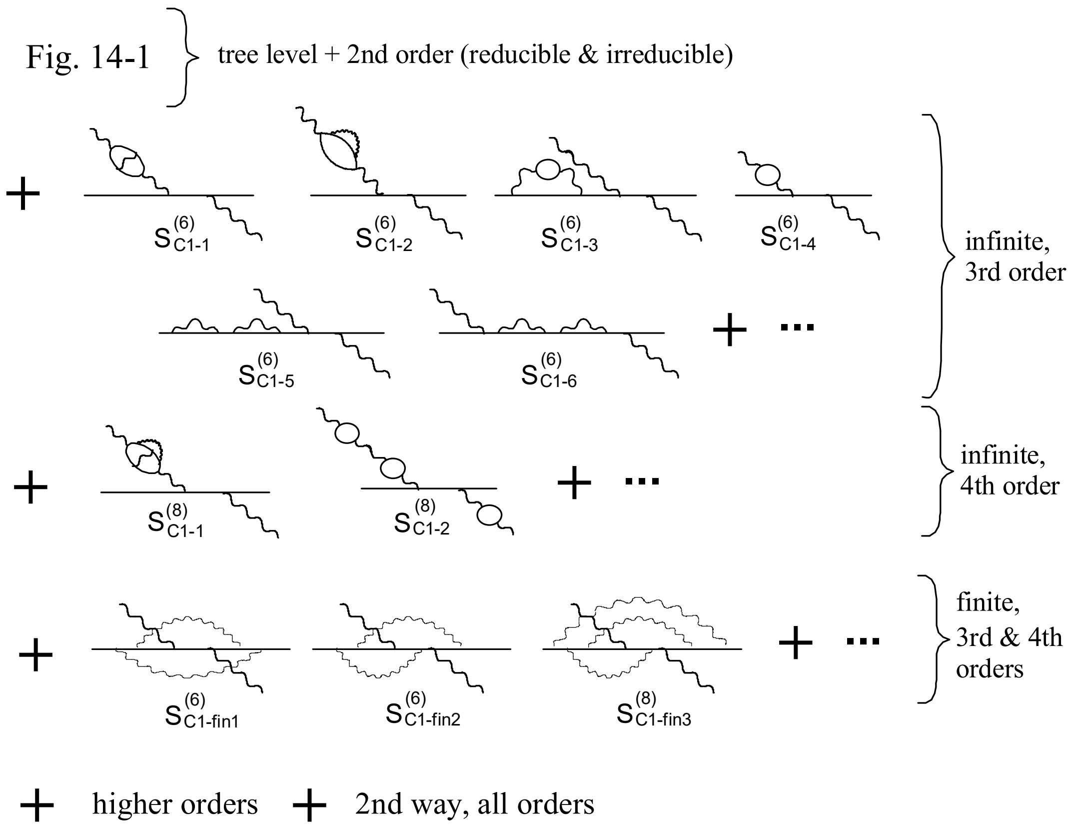


Figure 14-5. Compton Scattering Showing Some Higher Order Feynman Diagrams

The first row of Fig. 14-5, represented by Fig. 14-1, includes all the 1st and 2nd order diagrams we have seen before. The second and third rows show certain (not all) 3rd order diagrams. The fourth row shows certain (not even close to all) 4th order diagrams. Rows two to four contain diagrams whose amplitudes diverge. The fifth row illustrates some (not all) of the finite amplitude diagrams for 3rd and 4th order. These do not contain any of the divergent loop diagrams. The last row indicates we also have to consider the second way for Compton scattering to occur plus orders higher than 4th, if we are including them. We don't include triangle, pentangle, etc. diagrams because, due to Furry's theorem, they all cancel out.

We now introduce another category for distinguishing Feynman diagrams, proper vs improper. This distinction will help us in renormalizing photon, fermion, and vertex loop amplitudes to orders higher than second.

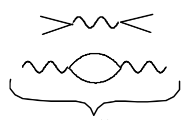
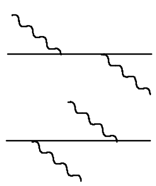
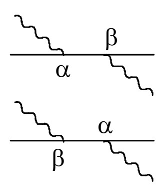
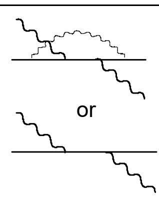
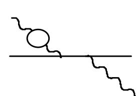
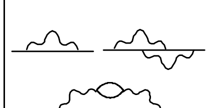
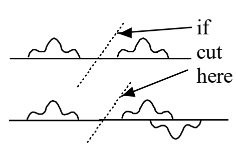
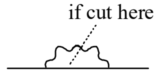
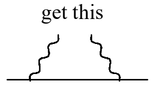
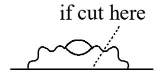
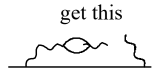
Definitions of proper and improper diagrams

An improper Feynman diagram is a non-tree level diagram having at least one internal line which, if cut, would leave two disconnected diagrams. A proper Feynman diagram is any i) tree-level diagram or ii) a non-tree level (reducible) diagram having no such internal line. In many cases, an improper diagram has side-by-side loops, whereas proper diagrams do not, though that is not a general principle. See Wholeness Charts 14-2 (2nd and 3rd columns, next to last row) and 14-3 below.

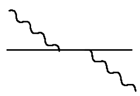
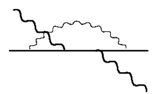
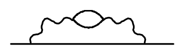
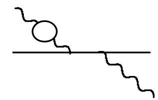
Primitively divergent diagrams

Wholeness Chart 14-2 is provided to organize and summarize the various types of Feynman diagrams we have dealt with and are now dealing with. The last row depicts yet another category called primitively divergent diagrams, which for QED are simply the photon, fermion, and vertex loops. Every QED divergent graph is composed of at least one of these. If you find the definition in the chart confusing, particularly in light of the definition of proper diagrams, ignore that definition and for now, simply think of primitively divergent diagrams as the three single loop diagrams for the photon, fermion, and vertex. Primitively divergent diagrams are used to prove that only reducible diagrams diverge. See footnote on page 342.

Wholeness Chart 14-2. Types of Feynman Diagrams

Graph Type	Examples	Counter Examples	Definition	Use	Chapter
Connected	 connected	 same diagram unconnected	All lines connected to all others	Feynman rules apply only to connected diagrams	8
Topologically Distinct	 topologically distinct	 topologically similar	Changing vertex label does not yield topologically distinct diagram	Eliminates $1/n!$ factor of Dyson-Wicks amplitude expansion in Feynman rules	8 (pg. 235) 9 (pg. 257)
Irreducible (Skeleton)	 irreducible (skeleton graph)	 reducible	Irreducible = no self-energy or vertex mod parts. Reducible = self-energy or vertex mod part(s). If remove all, becomes irreducible.	Irreducible are finite. Reducible are ∞ (before renormalization).	14
Proper	 proper diagrams	 improper diagrams	i) Tree level, or ii) cannot be split into two disconnected graphs by cutting an internal line.	Renormalization for order > 2nd.	14
Primitively Divergent	if cut here  primitive divergence get this  convergent	if cut here  non-primitive divergence get this  still divergent	Divergent graph which is converted to a convergent graph if any internal line is cut (replaced by 2 external lines)	All divergent graphs made of one or more primitive divergences. Used to prove that only reducible diagrams diverge.	Several (In QED, photon loop, fermion loop, and vertex loop)

Wholeness Chart 14-3. Comparing Certain Types of Feynman Diagrams[1]

Irreducible (Skeleton) (no loops, finite)		**Reducible** (loops, infinite)	
Tree Diagram	Non-tree Diagrams	Non-tree Diagrams	Non-tree Diagrams
Proper	Proper	Proper	Improper
Examples			
			

[1] Peskin & Schroeder use somewhat different terminology that we do here. Mandl & Shaw use the terms and definitions we have in this text.