$$= \frac{-e^2}{2V\omega_{\mathbf{k}}} \delta^{(4)}(k-k') \int d^4\bar{p} \underbrace{S_{F\eta\alpha}(\bar{p})\gamma^{\mu}_{\alpha\beta} S_{F\beta\delta}(\bar{p}+k)\gamma^{\nu}_{\delta\eta}}_{\text{trace in spinor space of matrix, } M^{\mu\nu}_{\eta\eta}} \varepsilon_{\mu,r'}(\mathbf{k})\varepsilon_{\nu,r}(\mathbf{k}) \qquad (8\text{-}85)$$

Our final result for the photon loop transition amplitude, with Tr indicating the trace in spinor space, is

$$\mathcal{M}_{\underset{loop}{\gamma}} = \frac{-e^2}{(2\pi)^4} \left\{ \text{Tr} \int d^4\bar{p} S_F(\bar{p}) \gamma^{\mu} S_F(\bar{p}+k) \gamma^{\nu} \right\} \varepsilon_{\mu,r'}(\mathbf{k})\varepsilon_{\nu,r}(\mathbf{k})$$

$$S_{\underset{loop}{\gamma}} = \left(\prod_{\mathbf{k}}^{\substack{\text{all ext} \\ \text{bosons}}} \sqrt{\frac{1}{2V\omega_{\mathbf{k}}}} \right) (2\pi)^4 \delta^{(4)}(k-k') M_{\gamma\, loop} \,. \qquad (8\text{-}86)$$