WHOLENESS CHART 3-X: QED/FIELD THEORY OVERVIEW

Part 2. From Operators and Propagators to Feynman Rules

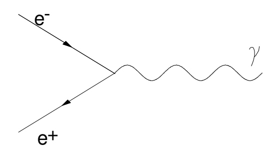
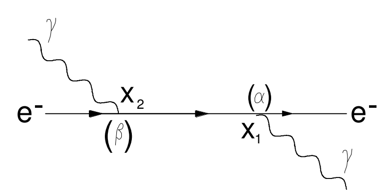
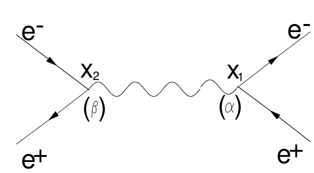
⇩ INTERACTING FIELDS ⇩

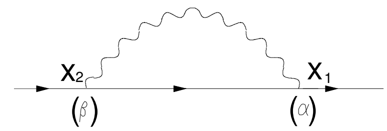
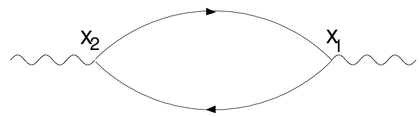
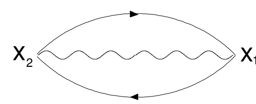
	In theory, the non-linear coupled partial differential interaction fields equations of Part 1 can be solved simultaneously to get interacting fields solutions and hence complete descriptions of all interactions. In practice this has not been possible and the following perturbation scheme has been developed. Note though that the treatment is exact until approximation is made in "Dyson Expansion of S Operator" block below.

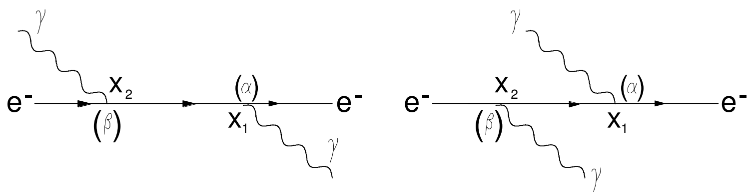
Interaction Picture Approach

Interaction picture $H^I = H_0^I + H_I^I$	Motion of <u>states</u> governed by H_I^I: $\quad i\frac{d}{dt}\left\|\alpha^I(t)\right\rangle = H_I^I(t)\left\|\alpha^I(t)\right\rangle$ Motion of <u>operators</u> governed by $H_0^I = H_0$ $\quad i\frac{dO^I(t)}{dt} = \left[O^I(t), H_0\right]$ ϕ, ψ, A^μ are operators, so depend on H_0 only. Further, for them the above operator equation reduces to the free field equations in the first block of Part 1 of this chart. See <u>Heisenberg Eq of Motion to Field Eq</u>.
Results of Interaction Picture	Can use: 1. free field operator solutions of Part 1 for interaction picture fields 2. free field number operators for interactions. 3. free field observables operators. 4. free field Feynman propagators 5. state equations of motion in H_I^I to determine change in state in time (i.e., interactions)
H_I^I	Spatial integral of $\mathcal{H}_I^{1/2,1} = -\mathcal{L}_I^{1/2,1}$ with operators taken as free field solutions = H_I^I. e.g., for QED $H_I^I = \int d^3\mathbf{x}\mathcal{H}_I^I$ with $\mathcal{H}_I^I = -e\bar{\psi}\gamma^\mu A_\mu \psi$
New Notation	Use $H_I, \mathcal{H}_I$ for $H_I^I, \mathcal{H}_I^I$ with free field solutions used in usual expressions for $H_I^I, \mathcal{H}_I^I$. Drop superscript "I" on states and other operators as well.
S Operator	General scattered state: $\quad \left\|\phi(t=\infty)\right\rangle = S\left\|\phi(t=-\infty)\right\rangle = S\left\|i\right\rangle$ $\left\|i\right\rangle$ = initial state, an eigenstate. $\left\|f\right\rangle$ = a final eigenstate. (Eigenstates are often multiparticle.) General final state (sum of final eigenstates) = $\left\|\phi(t=\infty)\right\rangle$. S is non-zero for time of interaction by adiabatic hypothesis.
S Matrix	$S_{fi} = \left\langle f\right\|S\left\|i\right\rangle = \left\langle f \mid \phi(t=\infty)\right\rangle$ so $\left\|\Phi(t=\infty)\right\rangle = \sum_f \left\|f\right\rangle S_{fi}$ For given $\left\|i\right\rangle$, probability of finding eigenstate $\left\|f\right\rangle$ is $\|S_{fi}\|^2$. Conservation of probability (not particles) is $\sum_f \|S_{fi}\|^2 = 1$

Dyson expansion of S operator exact: 2nd order:	Integrating state equation of motion (I.P. block near top): $\lvert\phi(t=\infty)\rangle = \lvert i\rangle + (-i)\int_{-\infty}^{\infty} dt' H_I(t')\lvert\phi(t')\rangle$ Cannot integrate in closed form, but via iteration $S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!}\int_{-\infty}^{\infty}.....\int_{-\infty}^{\infty} d^4x_1 d^4x_2 ... d^4x_n T\{\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)...\mathcal{H}_I(x_n)\}$ To this point treatment is exact. Perturbation arises from using fewer than an infinite number of terms in the above. $S \cong I \; + \; (-i)\int_{-\infty}^{\infty} d^4x_1 \mathcal{H}_I(x_1) \; + \; \frac{(-i)^2}{2!}\int_{-\infty}^{\infty} d^4x_1 d^4x_2 T\{\mathcal{H}_I(x_1)\mathcal{H}_I(x_2)\}$
Contractions of operators	Definition: $\underbrace{AB} = \langle 0\lvert T\{AB\}\rvert 0\rangle$ = Feynman propagators if A and B are fields. $\underbrace{\phi(x_1)\phi^\dagger(x_2)} = \underbrace{\phi^\dagger(x_2)\phi(x_1)} = i\Delta_F(x_1 - x_2)$ Special cases: $\underbrace{\psi_\alpha(x_1)\bar\psi_\beta(x_2)} = -\underbrace{\bar\psi_\beta(x_2)\psi_\alpha(x_1)} = iS_{F\alpha\beta}(x_1 - x_2)$ $\underbrace{A^\mu(x_1)A^\nu(x_2)} = iD_F^{\mu\nu}(x_1 - x_2)$
Extended Wick's Theorem	$T\left\{:(AB...)_{x_1}.......:(AB...)_{x_n}\right\} = :\left((AB...)_{x_1}.......:(AB...)_{x_n}\right)$ $+:\left((A\underbrace{B...)_{x_1}(A}B...)_{x_2}...\right)$ $+:\left((\underbrace{AB...)_{x_1}(AB}...)_{x_2}...\right)$ $+ ....$ $+:\left(...(A...\underbrace{Z)_{x_{n-1}}(A}...Z)_{x_n}\right)$ $+\left(\underbrace{A\underbrace{B...)_{x_1}(A}B}...)_{x_2}...\right)$ + (all other normal ordered non equal time double contractions) + (all normal ordered non equal time triple contractions) + etc.
$\mathcal{H}_I$	$\mathcal{H}_I = -\mathcal{L}_I^{1/2,1} = -e\sum_l :\{\bar\psi_l A^\mu \gamma_\mu \psi_l\} = -e\sum_l :\{(\bar\psi_l^{+} + \bar\psi_l^{-})(\not{A}^{+} + \not{A}^{-})(\psi_l^{+} + \psi_l^{-})\}$ where $l = 1$ for electrons (and positrons), 2 for muons, and 3 for tauons.
Dyson Expansion	Using above in Dyson expansion of S operator and using extended Wick's theorem to evaluate the time ordered normal ordered integrand yields $S = \sum_{n=0}^{\infty} S^{(n)} = S^{(1)} + S^{(2)} + S^{(3)} + ...$ (higher order terms)
	For simplicity, only $l = 1$ (electrons and positrons) treated below.

	Operator	Matrix Elements
$S^{(0)}$	$= I$ no transition of particles $\lvert f\rangle = \lvert i\rangle$	$S^{(0)}_{fi} = \langle f \lvert S^{(0)} \rvert i\rangle$ typical process: $\lvert e^-,\gamma\rangle \rightarrow \lvert e^-,\gamma\rangle$ no virtual particles, no 4 momentum change
$S^{(1)}$	$= (-i)\int d^4x_1 : \{-e\overline{\psi}_l A^{\mu}\gamma_{\mu}\psi_l\}_{x_1}$ = 8 terms but these processes are <u>not real physical processes.</u>	Typical non-physical process: 
$S^{(2)}$		
$S_A^{(2)}$	$= \frac{-e^2}{2!}\int d^4x_1 d^4x_2 : \left\{(\overline{\psi}\not{A}\psi)_{x_1}(\overline{\psi}\not{A}\psi)_{x_2}\right\}$ No real physical processes.	Two processes like $S^{(1)}$ above going on independently.
$S_B^{(2)}$	$= \frac{-e^2}{2!}\int d^4x_1 d^4x_2 : \left\{(\overline{\psi}\not{A}\underbrace{\psi)_{x_1}(\overline{\psi}}\not{A}\psi)_{x_2}\right\}$ $+ : \left\{(\underbrace{\overline{\psi}\not{A}\psi)_{x_1}(\overline{\psi}\not{A}\psi})_{x_2}\right\}$ $= -e^2\int d^4x_1 d^4x_2 : \left\{(\overline{\psi}\not{A}\underbrace{\psi)_{x_1}(\overline{\psi}}\not{A}\psi)_{x_2}\right\}$ = terms describing Compton scattering of electrons and positrons by photons, electron-positron creation and annihilation, and a number of non-physical processes = all two external lepton, two external photon interaction terms.	$S^{(2)}_{Bfi} = \langle f \lvert S_B^{(2)} \rvert i\rangle$ typical process (Compton scattering): $\lvert i\rangle = \lvert e^-,\gamma\rangle \rightarrow \lvert f\rangle = \lvert e^-,\gamma\rangle$ with virtual electron mediating scatter. 
$S_C^{(2)}$	$= \frac{-e^2}{2!}\int d^4x_1 d^4x_2 : \left\{(\overline{\psi}\not{A}\underbrace{\psi)_{x_1}(\overline{\psi}\not{A}}\psi)_{x_2}\right\}$ = all four external lepton interaction terms	$S^{(2)}_{Cfi} = \langle f \lvert S_C^{(2)} \rvert i\rangle$ typical process (Bhabbha scattering): $\lvert e^-,e^+\rangle \rightarrow \lvert e^-,e^+\rangle$ 

$S_D^{(2)}$	$= -e^2 \int d^4x_1 d^4x_2 : \left\{ (\bar{\psi} \not{A} \psi)_{x_1} (\bar{\psi} \not{A} \psi)_{x_2} \right\}$ = 2 physical processes	$S_{Dfi}^{(2)} = \langle f \| S_D{}^{(2)} \| i \rangle$ electron and positron self energy 
$S_E^{(2)}$	$= \frac{-e^2}{2!} \int d^4x_1 d^4x_2 : \left\{ (\bar{\psi} \not{A} \psi)_{x_1} (\bar{\psi} \not{A} \psi)_{x_2} \right\}$	$S_{Efi}^{(2)} = \langle f \| S_E{}^{(2)} \| i \rangle$ photon self energy 
$S_F^{(2)}$	$= \frac{-e^2}{2!} \int d^4x_1 d^4x_2 : \left\{ (\bar{\psi} \not{A} \psi)_{x_1} (\bar{\psi} \not{A} \psi)_{x_2} \right\}$	$S_{Ffi}^{(2)} = \langle f \| S_F{}^{(2)} \| i \rangle$ vacuum bubble 
$S^{(3)}$, $S^{(4)}$, etc.	Higher order terms. Ignored for now.	

Sample probability determination	Compton scattering, two ways:  $S_{B1}^{(2)}$ $S_{B2}^{(2)}$ See Box 3-1 for derivation of the following $\langle f \| S \| i \rangle_{Comp} = \langle f \| S_{B1}{}^{(2)} + S_{B2}{}^{(2)} \| i \rangle$ $= \left\{ (2\pi)^2 \delta^{(4)}(p'+k'-p-k) \left(\frac{m}{VE_p}\right)^{1/2} \left(\frac{m}{VE_{p'}}\right)^{1/2} \left(\frac{1}{2V\omega_k}\right)^{1/2} \left(\frac{1}{2V\omega_{k'}}\right)^{1/2} \right\} \{M_{B1} + M_{B2}\}$ where $M_{B1} = -e^2 \bar{u}_{s',\alpha}(\mathbf{p}')\varepsilon^{\mu}_{r'}(\mathbf{k}')\gamma^{\alpha\beta}_{\mu} iS_{F\beta\delta}(q=p+k)\varepsilon^{\nu}_{r}(\mathbf{k})\gamma^{\delta\eta}_{\nu}u_{s,\eta}(\mathbf{p})$ $M_{B2} = -e^2 \bar{u}_{s',\alpha}(\mathbf{p}')\varepsilon^{\mu}_{r'}(\mathbf{k})\gamma^{\alpha\beta}_{\mu} iS_{F\beta\delta}(q=p-k)\varepsilon^{\nu}_{r}(\mathbf{k}')\gamma^{\delta\eta}_{\nu}u_{s,\eta}(\mathbf{p})$ Probability of Compton scattering = $\left\| \langle f \| S \| i \rangle_{Comp} \right\|^2$ Assumption: Particles are plane waves in a box where V = volume of box.
Adding amplitudes	When two or more diagrams have the same external particles in and out, add amplitudes for each contributing diagram, then square the absolute value of result to get probability. For probability that any of two or more outcomes (different external particles out) may occur from the same external particles in, square absolute value of individual amplitudes first and then add.
2 ways to calculate probability	1) go through tedious derivation like Box 3-1 for each interaction 2) use short cut of Feynman rules (listed in Appendix ?)
	All three lepton types treated below.
Mixed lepton S operator	Each $\bar{\psi}\not{A}\psi$ term in S expression above for single lepton type replaced by $\sum_l \bar{\psi}_l \not{A}\psi_l$ term. $S = \sum_{n=0}^{\infty} \frac{(ie)^n}{n!} \int_{-\infty}^{\infty} ..... \int_{-\infty}^{\infty} d^4x_1 ... d^4x_n \sum_{l_1=1}^{3} .... \sum_{l_n=1}^{3} T\left\{ :\left(\bar{\psi}_{l_1}\not{A}\psi_{l_1}\right)_{x_1} ... :\left(\bar{\psi}_{l_n}\not{A}\psi_{l_n}\right)_{x_n} \right\}$ = terms like previous blocks for e^-, e^+ + " " " " " muons + " " " " " tauons + terms mixing lepton types.
Typical interaction	$e^- + e^+ \rightarrow \mu^- + \mu^+$ (with photon mediating.)

Mixed lepton summary	1) Draw all relevant Feynman diagrams which conserve $N(e)$, $N(\mu)$, $N(\tau)$ at each vertex. 2) Write Feynman amplitude for each diagram directly (from Feynman rules.)
	Part 3: Scattering and Decay
	To be included in the future.
	Part 4: Renormalizaton
	To be included in the future.