Wholeness Chart
Summary of QED Green Functions and Generating Functional
Canonical Quantization vs Path Integral Formulations

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	Canonical Approach	**Path Integral Approach**	**Comments**
S operator	$S=\sum_{n=0}^{\infty}\frac{i^n}{n!}\int T\left\{\mathcal{L}_I(\underline{x}_1)..\mathcal{L}_I(\underline{x}_n)\right\}d^4\underline{x}_1..d^4\underline{x}_n$ $=T\left\{e^{i\int\mathcal{L}_I(\underline{x})d^4\underline{x}}\right\}$	N/A	Fields $A^\mu,\psi,\bar{\psi}$ are operators in canonical approach; functions, not operators, in P.I.
Transition amplitude	$S_{fi}=\langle f\|S\|i\rangle=\delta_{if}$ $+\prod_{\mathbf{p}''}\sqrt{\frac{m}{VE_{\mathbf{p}''}}}\prod_{\mathbf{k}''}\sqrt{\frac{1}{2V\omega_{\mathbf{k}''}}}(2\pi)^4\delta(p_f-p_i)\mathcal{M}$	$S_{fi}=\lim_{x_i,x_f\to-\infty,+\infty}U(i,f;T)$ $S_{fi}=C\int e^{i\overbrace{\int\mathcal{L}d^4x}^{\text{label }X}}\mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi$	See Klauber, Chap 18, pg. 508, (18-64). for P.I. and extrapolate to 3 fields over all such fields and all x^μ
Green functions	Using them: Green function will be all propagators. For each external particle, change associated propagator to external line (e.g., $S_F(p)\to u(\mathbf{p})$) to get amplitude.	← Ditto →	All Green function momenta inward. Need certain $p\to -p$ to get amplitude
	$G^{\mu..}(x_1,.y_1,.\,z_1,.)=\dfrac{\langle 0\|T\left\{SA^\mu(x_1)...\psi(y_1)....\bar{\psi}(z_1)...\right\}\|0\rangle}{\langle 0\|S\|0\rangle}$ Ignore denominator above for connected Feynman diagrams	$G^{\mu..}(x_1,.y_1,.\,z_1,.)=\dfrac{\int\left\{e^{iX}A^\mu(x_1)..\psi(y_1)..\bar{\psi}(z_1)\right\}\mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi}{\int e^{iX}\mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi}$ $X=\int\mathcal{L}d^4x=\int\left(\mathcal{L}_0+\mathcal{L}_I\right)d^4x$	When evaluated these both give the same result. Proof below. Analogous results hold for other field theories (e.g., P.I. for weak, strong).
Generating functional Z	Z used to find Green functions	← Ditto	
	$Z[J_k,\sigma,\bar{\sigma}]=\dfrac{\langle 0\|S'\|0\rangle}{\langle 0\|S\|0\rangle}$ where ↓ $S'=\sum_{n=0}^{\infty}\frac{i^n}{n!}\int T\left\{\mathcal{L}_I'(\underline{x}_1)..\mathcal{L}_I'(\underline{x}_n)\right\}d^4\underline{x}_1..d^4\underline{x}_n$ $=T\left\{e^{i\int\mathcal{L}_I'(\underline{x})d^4\underline{x}}\right\}$ $\mathcal{L}_I'=\mathcal{L}_I+\mathcal{L}_S$ where $\mathcal{L}_S=J_\kappa A^\kappa+\bar{\sigma}\psi+\bar{\psi}\sigma$	$Z[J_k,\sigma,\bar{\sigma}]=\dfrac{\int e^{iX'}\mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi}{\int e^{iX}\mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi}$ where $X'=\int d^4x\left(\mathcal{L}_0+\mathcal{L}_I+\mathcal{L}_S\right)$	$\mathcal{L}_S$ called "source Lagrangian". $J_\kappa,\bar{\sigma},\sigma$ sources are fictitious and classical fields, not operators. Analogous results for other field theories.
Green function from generating functional	$G^{\mu..}=(-1)^{\bar{n}}\left(\frac{1}{i}\right)^n\left.\dfrac{\delta^n Z[J_k,\sigma,\bar{\sigma}]}{\delta J_\mu(x_1)..\delta\bar{\sigma}(y_1)..\delta\sigma(z_1)}\right\|_0$	← Ditto	$\|_0$ means $J=\bar{\sigma}=\sigma=0$ after taking derivatives
	Defs: n = tot num source fields = total num derivatives = <u>points</u> of Green funct $\bar{n}$ = tot num of σ source fields = tot num derivs wrspt σ <u>order</u> = number of vertices in graph (Green funct has all orders in it due to $\mathcal{L}_I$)		
	Plug and chug Z (for canonical approach) in above to prove get G^μ of canonical	Plug and chug Z (for P.I. approach) in above to prove get G^μ of P.I.	Yet to show two approaches equiv
Significance	Knowing $G^{\mu..}$ for a case with particular fields A^μ, ψ, $\bar{\psi}$, one knows the amplitude for that case. Knowing Z, one knows all possible $G^{\mu..}$. Z contains the whole theory.		

Proving equivalence of the two approaches			
Free field case	Take $\mathcal{L}_I = 0$ in canonical Z above $Z_0[J_k,\sigma,\bar{\sigma}] = e^{-i[J_\mu D_F^{\mu\nu} J_\nu]} e^{-i[\bar{\sigma} S_F \sigma]}$ where $[AKB] = \iint A(x)K(x,y)B(y)\,d^4x\,d^4y$ Klauber proof: See "Green function and …" link on *SFQFT* text home web page M&S proof: Sect. 12.5.1, pgs. 267-270.	Take $\mathcal{L}_I = 0$ in P.I. Z above. Result ↓ ← Ditto (same result as at left) M&S proof: Sects. 13.2.3 & 13.2.4, pgs. 289-294	Subscript "0" in Z_0 means free field. M&S refers to Mandl and Shaw
Interacting fields case	$Z[J_k,\sigma,\bar{\sigma}] = \dfrac{\sum_{n=0}^{\infty}\frac{(ie)^n}{n!}\left[\int d^4x\, I_\delta(x)\right]^n Z_0[J_k,\sigma,\bar{\sigma}]}{\langle 0 \vert S \vert 0\rangle}$ $I_\delta(x) = \left(-\frac{1}{i}\frac{\delta}{\delta\sigma(x)}\right)\gamma_\mu\left(\frac{1}{i}\frac{\delta}{\delta\bar{\sigma}(x)}\right)\left(\frac{1}{i}\frac{\delta}{\delta J_\mu(x)}\right)$ M&S Sect. 12.5.2, 270-271, eq (12.122).	← Ditto (same form as at left but with P.I. Z_0) Sect. 13.2.2, pgs 287-289, eq. (13.74) with (13.77)	Proof: Insert Z_0 (Z form for each approach with $\mathcal{L}_I = 0$) in this relation, then plug and chug.
Proof of equivalence	Since Z_0 is shown the same in both approaches for the free field case above, then in above row, Z must be the same in both approaches.		
Alternative symbolism for Green function			
	$\langle A^\mu(x_1)..\psi(y_1)..\bar{\psi}(z_1)..\rangle = G^{\mu .}(x_1,.y_1,.\,z_1,.)$	← Ditto	
Notation	Free fields case: $\langle A^\mu(x_1)..\psi(y_1)..\bar{\psi}(z_1)..\rangle_0$	← Ditto	
Key P.I. Relations			
		$[AKB] = \iint A(x)K(x,y)B(y)\,d^4x\,d^4y$	M&S (13.6) pg. 276
		$K_{ij} = \iint u_i(x)K(x,y)u_j(x)\,d^4x d^4y$ where $A(x) = \sum_{i-1}^{\infty}\alpha_i u_i(x)$	M&S (13.8) pg. 277 and (13.3) pg. 276
		$\int e^{-\frac{1}{2}[\phi K \phi]}\mathcal{D}\phi = \frac{1}{\sqrt{Det\,K}}$ $Det\,K = \lim_{n\to\infty} K_n$ $\int e^{-[\theta M \tilde{\theta}]}\mathcal{D}\theta\mathcal{D}\tilde{\theta} = Det\,M$ $\int e^{-\prod_{i,j=1}^{n}\theta_i N_{ij}\tilde{\theta}_j}\left(\prod_{i=1}^{n}\mathcal{D}\theta_i\mathcal{D}\tilde{\theta}_i\right) = Det\,N_n$	M&S (13.12) pg. 277 M&S (13.61) pg. 285 M&S (13.53) pg. 283
		$A[J,\phi] = -\frac{1}{2}\iint \phi(x)K(x,y)\phi(y)\,d^4x\,d^4y + \int J(x)\phi(x)\,d^4x$	M&S (13.24) pg. 279
		$\int e^{A[J,\phi]}\mathcal{D}\phi = e^{\frac{1}{2}[JK^{-1}J]}$	M&S (13.33) pg. 280