as a necessary condition for any divergent proper diagram with at least one vertex. We re-write (16-26) as (16-27) below, in terms of a function of only external fermions and bosons, which we label K_0.

which can be expressed in terms of a function of f_e, f_b, along with n

$$K_{non-min} = \underbrace{4 - \tfrac{3}{2} f_e - 2b_e}_{K_0(f_e, b_e)} + n = \underbrace{K - b_e}_{K_0(f_e, b_e)} + n \geq 0 \qquad \left(K = K_{min}\right). \tag{16-27}$$

The part of (16-27) after the second equal sign is incidental, presented only for comparison purposes, and not needed in what follows.

From (16-27) the superficial divergence increases with increasing n (number of vertices), whereas in the comparable relation (16-8) of the minimal theory, there is no such n dependence. Hence, higher-order diagrams in the non-minimal theory mean higher K values and, as the order (number of vertices) goes to infinity, we will get K approaching infinity. Although K values only indicate superficial divergence, the actual divergences can generally only be somewhat smaller than K, so the actual divergences go to infinity, as well. Such an infinite number of different divergences cannot be absorbed into a finite number of redefined physical parameters. Conclusion: for non-minimal interaction terms (such as, for example, (16-23)), the non-minimal theory is non-renormalizable.

This means we need infinite parameters to renormalize

We can re-express (16-27), in terms of the coupling constant from our earlier example, as

Re-expressing $K_{non-min}$ in terms of the coupling constant dimension

$$K_{non-min} = K_0\left(f_e, b_e\right) - nD_{g_1} \geq 0\,, \tag{16-28}$$

where D_{g_1} is the dimension of the coupling constant g_1, which is –1 in this case.

This result can be generalized to other non-minimal terms having coupling constants with other dimensions, and to Lagrangians with more than one type of non-minimal term. Where n_i is the order of the graph in the particular coupling g_i, we have

Most general form of QED $K_{non-min}$, for any number of non-minimal terms

$$K_{non-min} = K_0\left(f_e, b_e\right) - \sum_i n_i D_{g_i} \geq 0\,. \tag{16-29}$$

(16-29) makes sense, as we know a more negative dimension coupling constant implies more factors of four-momentum in the numerator, and thus, greater divergence in integrals containing four-momenta.