

A Symmetry for Null Zero Point Energy, Null Higgs Condensate Energy, and Resolution of the Gauge Hierarchy Problem without SUSY

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Abstract

Alternative forms for solutions of the quantum field equations and their implications for physical theory, are considered. Incorporation of these alternative solutions, herein deemed “supplemental solutions”, into the development of quantum field theory leads to simple resolutions of more than one extant problem in high energy physics. The symmetry between the traditional and supplemental solutions results in a direct and natural zero-point energy value of zero, and, as well, a possible mechanism for cancelling the Higgs condensate energy, thereby providing a potential resolution of the large cosmological constant problem. Further, this symmetry may also resolve the Higgs gauge hierarchy problem. Resolutions of seeming theoretic impediments to supplemental fields, in particular, non-positive definite Fock space metric and vacuum decay, are presented. As the traditional/supplemental symmetry is already inherent in quantum field theory, it requires no extension to the fundamental mathematics of the theory.

1 Introduction

“... given that everything turned out to be very simple [experimentally], yet extremely puzzling — puzzling in its simplicity ... We have to ... find the new principles that will explain the simplicity.”

Neil Turok

“... a good part of science is distinguishing between useful crazy ideas and those that are just plain nutty”

Princeton University Press book advertisement

1.1 Unresolved Issues in Physics

As summarized by Peebles and Ratra¹, Padmanabhan², and others, there are presently three overriding cosmologic issues involving observed phenomena for which no generally accepted theoretical solutions exist: 1) dark matter (non-baryonic, unseen “normal” matter), 2) dark energy (with effective negative pressure and positive mass-energy), and 3) a vanishing sum of zero-point energies (ZPE). These may be related, or unrelated.

Additionally, with the increasing lack of experimental support for supersymmetry (SUSY), the Higgs gauge hierarchy problem has grown more intractable.

The ZPE could give rise, in a manner suggested by Zeldovich³, and further refined by Martin⁴, to a cosmological constant, and thus, there are two aspects of what is known as the cosmological constant (c.c.) problem. First, why is vacuum energy not enormous (“the large c.c. problem”), and second, why is it almost null, but not completely null (“the small c.c. problem”).

This article focuses on a possible resolution of the large c.c. and gauge hierarchy problems via incorporation into quantum field theory of an inherent, but heretofore apparently unappreciated, symmetry. Though possibly seeming, at first, to be similar to other approaches to one or both of these issues presented, or summarized, in Nobbenhuis⁵, Sakharov⁶, Linde^{7,8}, Chen^{9,10}, Henry-Couannier^{11,12}, Faraoni¹³, Quiros¹⁴, D’Agostini et al¹⁵, Habara et al¹⁶, Kaplan and Sundrum¹⁷, Moffat^{18,19}, Elze²⁰, ’t Hooft and Nobbenhuis²¹, and Andrianov et al²², the approach shown herein is, at its foundation, fundamentally different.

1.2 Negative Frequencies and Supplemental Solutions

The issue of negative-frequency solutions to the relativistic counterparts of the Schrödinger equation has a long and variegated history. Such solutions constitute a second way to solve the quantum field equations beyond those of positive frequency. The question of interpretation of negative-frequency solutions, one of the most famous in the history of science, was answered via field (second) quantization. The solutions to the field equations could then be shown to be operators that create and destroy (positive energy) states, rather than being states themselves.

By including the negative-frequency solutions, the set of solutions to the quantum field equations was doubled in size, and thus so were the number of particles. In this article, it is noted that alternative solution forms to the field equations, interpreted within a framework of physical law, double the number of particle states once again; i.e. they introduce new particles. The typically unused set of solution forms is designated herein as the set of “*supplemental* solutions”, and the new particles as “*supplemental* particles”.

2 Alternative Solutions to the Field Equations

For simplicity, we focus on the scalar field equation (in natural units)

$$(\partial_\mu \partial^\mu + \mu^2)\phi = 0 \quad (1)$$

with traditional eigensolutions

$$e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}} = e^{-ikx}, \quad (2)$$

$$e^{i\omega t - i\mathbf{k}\cdot\mathbf{x}} = e^{ikx}. \quad (3)$$

Note, however, that by taking $\omega \rightarrow -\omega$ (consider the symbol ω as always a positive number) in (2) and (3), one obtains alternative solution forms for (1)²³ of

$$e^{i\omega t + i\mathbf{k}\cdot\mathbf{x}} = e^{ikx} \quad (4)$$

$$e^{-i\omega t - i\mathbf{k}\cdot\mathbf{x}} = e^{-ikx}. \quad (5)$$

New notation (see underscoring above) herein introduced, wherein (4) and (5) are the aforementioned supplemental solutions to (1). Upon introduction to supplemental solutions, some at first consider them to be already contained in the traditional solutions to the field equations. This is true in a certain sense, as discussed in the appendix. Nevertheless, one finds they give rise to distinctly *different* physical properties for particles. This is shown explicitly in the appendix for relativistic quantum mechanics (RQM) and derived later in this article for quantum field theory (QFT).

That is, and regardless of the arguments in the appendix, if one simply starts with (4) and (5), and proceeds (as we do in this article) through the same steps used to develop traditional quantum

field theory (QFT), one finds states created and destroyed by supplemental field operators that have distinctly different eigenvalues (quantum numbers) from traditional particle states, and thus comprise an independent species of particle.

Note that the Dirac and Proca equations are solved by supplemental solutions as well. As shown in Klauber²⁴, the Dirac equation actually has eight eigenspinor solutions, a set of four for $e^{+(iEt-i\mathbf{p}\cdot\mathbf{x})}$ and another set of four for $e^{-(iEt-i\mathbf{p}\cdot\mathbf{x})}$, as one would expect in solving a 4X4 matrix eigenvalue problem. Four (two from the first set and two from the second set) have spacetime dependence like that of (2) and (3) and are employed in traditional QFT. Four (the other two from the first set and the other two from the second set) have spacetime dependence like that of (4) and (5) and are not employed in traditional QFT.

In relativistic quantum mechanics (RQM), the precursor to QFT²⁵, (3) presented a problem as it represented a negative-energy state. As noted, QFT resolved this, but examination of (4) and (5) in the context of RQM leads to a similar issue of negative energy, as well as an additional one. Momentum direction in (4) and (5), if they represent physical states, is in the opposite direction of wave velocity [see the appendix], and hence (4) and (5) are unlikely candidates for physical particle states. Possibly for this reason, solutions of the form of (4) and (5) were not included in the development of RQM.

In parallel with the historical development of (2) and (3) in QFT, however, we can apply second quantization to (4) and (5), determine the resultant field operator solutions, and see if those solutions might provide anything of value in helping to match experiment with theory.

3 Supplemental Solutions and QFT

3.1 Symmetry of the Free Lagrangian

If (4) and (5) solve the same free field equations as (2) and (3), then it follows that the free scalar Lagrangian is symmetric under the transformation $\omega \rightarrow -\omega$.

3.2 Klein-Gordon Supplemental Solutions

Quantum field theory formalism for the supplemental solutions can be developed in parallel with the standard approach, using, for scalar fields, the following definitions.

$$\underline{\phi} = \underline{\phi}^+ + \underline{\phi}^- = \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k} \right)^{1/2} \{ \underline{a}(\mathbf{k})e^{i\mathbf{k}\mathbf{x}} + \underline{b}^\dagger(\mathbf{k})e^{-i\mathbf{k}\mathbf{x}} \} \quad (6)$$

$$\underline{\phi}^\dagger = \underline{\phi}^{\dagger+} + \underline{\phi}^{\dagger-} = \sum_{\mathbf{k}} \left(\frac{1}{2V\omega_k} \right)^{1/2} \{ \underline{b}(\mathbf{k})e^{i\mathbf{k}\mathbf{x}} + \underline{a}^\dagger(\mathbf{k})e^{-i\mathbf{k}\mathbf{x}} \} \quad (7)$$

$$\underline{\mathcal{L}}_0^0 = \partial_\mu \underline{\phi}^\dagger \partial^\mu \underline{\phi} - m^2 \underline{\phi}^\dagger \underline{\phi}, \quad (8)$$

where (8) is an extra component added to the Lagrangian density representing the scalar supplemental solutions. The superscript “0” refers to “free” Lagrangian; the subscript “0”, to scalar fields.

3.3 Quantizing Supplemental Solutions

Applying second quantization to the supplemental solutions where

$$\left[\underline{\phi}_r(\mathbf{x}, t), \underline{\pi}_s(\mathbf{y}, t) \right] = \left[\underline{\phi}_r(\mathbf{x}, t), \dot{\underline{\phi}}_s^\dagger(\mathbf{y}, t) \right] = i\delta_{rs}\delta(\mathbf{x} - \mathbf{y}), \quad (9)$$

with (6) and (7), yields the coefficient commutation relations²⁴ (see ref. 25. pp. 52-53)

$$\left[\underline{a}(\mathbf{k}), \underline{a}^\dagger(\mathbf{k}') \right] = \left[\underline{b}(\mathbf{k}), \underline{b}^\dagger(\mathbf{k}') \right] = -\delta_{\mathbf{k}\mathbf{k}'}. \quad (10)$$

Note the above relations differ from their traditional solution counterparts by the minus sign on the RHS. This resulted from the time derivative in (9), since the supplemental solutions have opposite signs from the traditional solutions for the time (energy) term in the exponent.

3.4 The Scalar Supplemental Hamiltonian

Using (6) and (7) in the relevant term in the Hamiltonian density

$$\mathcal{H}_0^0 = \sum_r \pi_r \dot{\underline{\phi}}_r^\dagger - \underline{\mathcal{L}}_0^0 = \dot{\underline{\phi}}\dot{\underline{\phi}}^\dagger + \dot{\underline{\phi}}^\dagger\dot{\underline{\phi}} - \underline{\mathcal{L}}_0^0 = \dot{\underline{\phi}}\dot{\underline{\phi}}^\dagger + \nabla\underline{\phi}^\dagger\nabla\underline{\phi} + m^2\underline{\phi}^\dagger\underline{\phi} \quad (11)$$

and integrating over all space, in parallel with that of ref. 25, pp. 51-53, for the traditional fields, yields

$$\underline{H} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left\{ \underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k}) - \frac{1}{2} + \underline{b}^\dagger(\mathbf{k})\underline{b}(\mathbf{k}) - \frac{1}{2} \right\}, \quad (12)$$

where for notational streamlining we here and from henceforth drop the sub and superscript notation. Note that, due to (10), the $\frac{1}{2}$ terms in (12) have the opposite sign from similar terms in the traditional Hamiltonian (13),

$$H = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left\{ a^\dagger(\mathbf{k})a(\mathbf{k}) + \frac{1}{2} + b^\dagger(\mathbf{k})b(\mathbf{k}) + \frac{1}{2} \right\}. \quad (13)$$

3.5 Dirac and Proca Supplemental Solutions

As should be expected, the above analysis has its analogues for spin- $\frac{1}{2}$ and spin-1 fields. The Proca equation is so closely related to the Klein-Gordon equation that all results of the preceding sections can be readily extrapolated to spin-1 fields.

Since we do not have a classical spinor-field theory, one cannot obtain quantum spinor-field theory by quantizing it. The typical approach for spinors is to assume coefficient anti-commutation relations, in contrast to the coefficient commutation relations used for bosons. Doing so for supplemental fields, we obtain

$$\left[\underline{c}_r(\mathbf{p}), \underline{c}_s^\dagger(\mathbf{p}') \right]_+ = \left[\underline{d}_r(\mathbf{p}), \underline{d}_s^\dagger(\mathbf{p}') \right]_+ = -\delta_{rs}\delta_{\mathbf{p}\mathbf{p}'}. \quad (14)$$

Using (14) in the free Hamiltonian density for Dirac supplemental particles results in a Hamiltonian analogous to (12), and having $\frac{1}{2}$ terms of opposite sign from those in the traditional theory.

3.6 Supplemental Operators, Propagators, and Observables

Supplemental field number operators, Hamiltonian, creation/destruction operators, other observables, and propagators are derived in ref. 24 following steps parallel to those in the development of traditional QFT. These are (where for simplicity we ignore the $\frac{1}{2}$ quanta terms in \underline{H})

$$\begin{aligned} \underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k})|\underline{n}_{\mathbf{k}}\rangle &= \underline{N}_a(\mathbf{k})|\underline{n}_{\mathbf{k}}\rangle = n_{\mathbf{k}}|\underline{n}_{\mathbf{k}}\rangle = -|n_{\mathbf{k}}||\underline{n}_{\mathbf{k}}\rangle & n_{\mathbf{k}} \leq 0 \\ \underline{b}^\dagger(\mathbf{k})\underline{b}(\mathbf{k})|\underline{\bar{n}}_{\mathbf{k}}\rangle &= \underline{N}_b(\mathbf{k})|\underline{\bar{n}}_{\mathbf{k}}\rangle = \bar{n}_{\mathbf{k}}|\underline{\bar{n}}_{\mathbf{k}}\rangle = -|\bar{n}_{\mathbf{k}}||\underline{\bar{n}}_{\mathbf{k}}\rangle & \bar{n}_{\mathbf{k}} \leq 0 \end{aligned} \quad (15)$$

$$\begin{aligned} \underline{H}|\underline{n}_{\mathbf{k}}\rangle &= \sum_{\mathbf{k}'} \omega_{\mathbf{k}'} \{ \underline{a}^\dagger(\mathbf{k}')\underline{a}(\mathbf{k}') + \underline{b}^\dagger(\mathbf{k}')\underline{b}(\mathbf{k}') \} |\underline{n}_{\mathbf{k}}\rangle = \sum_{\mathbf{k}'} \omega_{\mathbf{k}'} \{ \underline{N}_a(\mathbf{k}') + \underline{N}_b(\mathbf{k}') \} |\underline{n}_{\mathbf{k}}\rangle \\ &= \underline{n}_{\mathbf{k}} \omega_{\mathbf{k}} |\underline{n}_{\mathbf{k}}\rangle = -|n_{\mathbf{k}}| \omega_{\mathbf{k}} |\underline{n}_{\mathbf{k}}\rangle \end{aligned} \quad (16)$$

$$\begin{aligned} \text{lowering operator increases} & & \underline{a}(\mathbf{k})|\underline{n}_{\mathbf{k}}\rangle &= \sqrt{n_{\mathbf{k}}}|n_{\mathbf{k}}+1\rangle \\ \underline{n}_{\mathbf{k}} \text{ by 1 (one less particle)} & & & \end{aligned} \quad (17)$$

$$\begin{aligned} \text{raising operator decreases} & & \underline{a}^\dagger(\mathbf{k})|\underline{n}_{\mathbf{k}}+1\rangle &= \sqrt{\bar{n}_{\mathbf{k}}}|n_{\mathbf{k}}\rangle \\ \underline{n}_{\mathbf{k}} \text{ by 1 (one more particle)} & & & \end{aligned}$$

$$\begin{aligned} \text{3-momentum} & & \mathbf{P}^{oper}|\underline{n}_{\mathbf{k}}=-1\rangle &= \mathbf{k}|n_{\mathbf{k}}=-1\rangle \\ \text{of a single supplemental particle} & & & \end{aligned} \quad (18)$$

$$\begin{aligned} \text{probability current density (} \propto \text{ velocity)} & & \mathbf{j}^{oper}|\underline{n}_{\mathbf{k}}=-1\rangle &= -\frac{\mathbf{k}}{V\omega_{\mathbf{k}}}|n_{\mathbf{k}}=-1\rangle \\ \text{of a single supplemental particle} & & & \end{aligned} \quad (19)$$

$$\begin{aligned} \text{pressure} & & T_{11}|\underline{n}_{\mathbf{k}_1}\rangle &= \frac{1}{V} n_{\mathbf{k}_1} \frac{(k_1)^2}{\omega_{\mathbf{k}_1}} |n_{\mathbf{k}_1}\rangle = -\frac{1}{V} |n_{\mathbf{k}_1}| \frac{(k_1)^2}{\omega_{\mathbf{k}_1}} |n_{\mathbf{k}_1}\rangle \end{aligned} \quad (20)$$

$$\begin{aligned} \text{Feynman propagator} & & \underline{\Delta}_F(k) &= -\Delta_F(k) \end{aligned} \quad (21)$$

Number operators (15) yield number eigenvalues of opposite sign (negative) from their traditional counterparts. The energy (16) of a supplemental particle state is negative. Supplemental propagators (21) have the same form, but opposite sign from traditional propagators. And the total three-momentum (18) for a supplemental particle state is in the opposite direction of its velocity (19). Such characteristics, particularly the lattermost, are not those of real particles in our universe, though they can be (and often are) so for the virtual particles of traditional QFT.

For example, the virtual exchange between two oppositely charged macroscopic bodies traveling the same line of action must entail three-momentum in the opposite direction of travel of the virtual particles in order for the bodies to attract. And virtual loop diagrams are integrated over both positive and negative energies for the individual virtual particles therein. Further, traditional scalar (timelike polarization) virtual photons have negative energies.²⁶ Still further, traditional fermion zero-point energies are negative.

3.7 Nature of Supplemental Particles

Hence, we consider herein that if supplemental particles are indeed realized in the spectrum of states, they are necessarily constrained to be virtual, cannot be real, and are never directly observed. No symmetry breaking mechanism (at least none much above contemporary energy levels) is envisioned between the traditional and supplemental fields.

And to be clear, the supplemental particles, though having negative-energy states, are not a reincarnation of Dirac's sea of negative energy, but quite a different thing entirely. Neither are they related to the Wheeler-Feynman absorber theory, which is compared to the present theory in ref. 24.

Note that if supplemental particles were to interact with traditional particles, then for every traditional Feynman diagram with a virtual particle represented by a traditional propagator, we would

have another diagram, the same in every regard, except that a supplemental virtual particle would replace the traditional one. From (21), this would result in only a sign change in the Feynman sub-amplitude for the interaction. When we add the two diagrams, which we need to do if the incoming and outgoing particles are the same, they cancel, leaving zero for the transition amplitude.

Thus, we would have zero probability for any interaction to take place, and nothing would happen in the world. Hence, we posit that for standard model gauge interactions, the traditional and supplemental particles are uncoupled. Quantum gravitational interactions are considerably more complex, however, and interactions of some sort between traditional and supplemental particles may occur.

4 Potential Resolutions of Certain Extant Problems

4.1 Cancellation of Zero-Point Energy Fluctuations

Weinberg²⁷ and Klauber²⁵ (pg. 279), among others, show summing (or integration) of the zero-point energies of a boson field of mass m up to a wave number cutoff $k_c \gg m$ on the order of the Planck scale yields a vacuum energy density exceeding the observed value by a factor of more than 120 orders of magnitude. Martin⁴, citing Zeldovich's³ original work, notes that such an approach, though widely disseminated, is not valid, as it violates Lorentz invariance. That is, the cutoff wave number momentum is observer dependent, and just as the cutoff method fails for that reason in renormalization, it fails here, as well.

Using dimensional regularization (which is Lorentz covariant), instead of the cutoff method, to evaluate the integration over zero-point wave numbers, Martin finds a vacuum energy density differing from the observed value by an order of “only” 55 orders of magnitude. He also shows, via the same approach, a ZPE pressure of equal magnitude (in natural units) of, but opposite sign from, the ZPE energy density, i.e., an equation of state $w = -1$, which, of course, is that of a cosmological constant.

The Zeldovich/Martin approach nevertheless leaves us with the same qualitative problem – a mind bending discrepancy in magnitude between theory and observation.

However, if we consider the total (free, scalar) Hamiltonian as the sum of (12) and (13),

$$H_{\text{tot}} = H + \underline{H} = \sum_{\mathbf{k}} \omega_{\mathbf{k}} \left\{ a^\dagger(\mathbf{k})a(\mathbf{k}) + b^\dagger(\mathbf{k})b(\mathbf{k}) + \underline{a}^\dagger(\mathbf{k})\underline{a}(\mathbf{k}) + \underline{b}^\dagger(\mathbf{k})\underline{b}(\mathbf{k}) \right\}, \quad (22)$$

then the $\frac{1}{2}$ terms all drop out and the expectation energy of the vacuum from the free Hamiltonian is naturally zero. In similar fashion, the concomitant expectation value of pressure is likewise zero. In this scenario, the vacuum stress energy tensor is zero, i.e., the associated cosmological constant is zero.

4.2 Cancellation of the Higgs Condensate Energy

In electroweak symmetry breaking²⁸, the potential \mathcal{V} , expressed in terms of the Higgs field doublet Φ , is

$$\mathcal{V} = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \quad (23)$$

For the unitary gauge, the minimum value of the potential (23), where the Higgs field takes the VEV v , we find a Higgs condensate energy density with VEV

$$\mathcal{V}_{\min}^{\text{expect}} = \bar{\mathcal{V}}_{\min} = -\frac{\lambda v^4}{4} \quad \text{where } v = \sqrt{\frac{-\mu^2}{\lambda}} \quad \mu^2 < 0, \lambda > 0. \quad (24)$$

As noted in ref. 28, the relation (23) does not really make sense numerically, because Φ is an operator that creates and destroys fields. Φ does not take on different values, so a numerical quantity, such as \mathcal{V} is often presumed to be in (23), cannot be functionally dependent on it. What (23) implies is an expectation value for the potential

$$\begin{aligned} \bar{\mathcal{V}} = \langle n_{\mathbf{k}} \Phi_{\mathbf{k}}, n_{\mathbf{k}'} \Phi_{\mathbf{k}'}, \dots | \mathcal{V} | n_{\mathbf{k}} \Phi_{\mathbf{k}}, n_{\mathbf{k}'} \Phi_{\mathbf{k}'}, \dots \rangle &= \mu^2 \langle n_{\mathbf{k}} \Phi_{\mathbf{k}}, n_{\mathbf{k}'} \Phi_{\mathbf{k}'}, \dots | \Phi^\dagger \Phi | n_{\mathbf{k}} \Phi_{\mathbf{k}}, n_{\mathbf{k}'} \Phi_{\mathbf{k}'}, \dots \rangle \\ &+ \lambda \langle n_{\mathbf{k}} \Phi_{\mathbf{k}}, n_{\mathbf{k}'} \Phi_{\mathbf{k}'}, \dots | (\Phi^\dagger \Phi)^2 | n_{\mathbf{k}} \Phi_{\mathbf{k}}, n_{\mathbf{k}'} \Phi_{\mathbf{k}'}, \dots \rangle^2 \end{aligned} \quad (25)$$

Given the operator nature of Φ , (25) reduces to a dependence on the number densities

$$\bar{\mathcal{V}} = \mu^2 \frac{1}{V} (n_{\mathbf{k}} + n_{\mathbf{k}'} + \dots) + \lambda \frac{1}{V^2} (n_{\mathbf{k}}^2 + n_{\mathbf{k}'}^2 + \dots). \quad (26)$$

Thus, the famous Mexican hat diagram corresponds to the expectation value of the potential on the vertical axis, and the number density of Higgs particle states on the horizontal axes.

Hence, the relations (24) are actually deduced in a framework where number densities are positive, i.e., $n_{\mathbf{k}} > 0$. However, a parallel analysis, with the same algebraic form (23) for supplemental particles, would yield

$$\bar{\mathcal{V}} = \mu^2 \frac{1}{V} (n_{\mathbf{k}} + n_{\mathbf{k}'} + \dots) + \lambda \frac{1}{V^2} (n_{\mathbf{k}}^2 + n_{\mathbf{k}'}^2 + \dots) = -\mu^2 \frac{1}{V} (|n_{\mathbf{k}}| + |n_{\mathbf{k}'}| + \dots) + \lambda \frac{1}{V^2} (n_{\mathbf{k}}^2 + n_{\mathbf{k}'}^2 + \dots). \quad (27)$$

Finding the minimum of (27), where the supplemental field $\underline{\Phi}$ takes the VEV value v , implies, symbolically and parallel to (23), that effectively, the sign has changed in the μ^2 term, i.e.,

$$\underline{\mathcal{V}} = -\mu^2 \underline{\Phi}^\dagger \underline{\Phi} + \lambda (\underline{\Phi}^\dagger \underline{\Phi})^2. \quad (28)$$

And again, in parallel fashion, finding the stationary (maximum, in this case [see Section 5.3]) value of (28) yields

$$\bar{\mathcal{V}}_{\max} = -\frac{\lambda v^4}{4} = \frac{|\lambda| v^4}{4} \quad \text{where } v = \sqrt{\frac{\mu^2}{\lambda}} \quad \mu^2 < 0, \lambda < 0. \quad (29)$$

Adding the traditional Higgs energy VEV (24) to the supplemental Higgs value (29) yields zero, which matches observational constraints.

4.3 Gauge Hierarchy and Supplemental Solutions

As noted in (21), supplemental field propagators have opposite sign from, but equal magnitude of, their sibling traditional field propagators. Thus, if a single propagator in a loop of a problematic transition sub-amplitude with traditional fields were replaced by its sibling supplemental propagator, the resulting transition sub-amplitude would have opposite sign from, but equal magnitude of, the original sub-amplitude. And adding them would yield zero contribution to the total amplitude.

To this end, consider the traditional Higgs-lepton Lagrangian at the false vacuum,

$$\mathcal{L}^{LH} = -g_l \left(\bar{\Psi}_l^L \psi_l^R \Phi + \Phi^\dagger \bar{\psi}_l^R \Psi_l^L \right) - g_{\nu_l} \left(\bar{\Psi}_{\nu_l}^L \psi_{\nu_l}^R \tilde{\Phi} + \tilde{\Phi}^\dagger \bar{\psi}_{\nu_l}^R \Psi_{\nu_l}^L \right) \quad (30)$$

and make the replacement

$$\Psi_l^L \rightarrow \Psi_l^L + \underline{\Psi}_l^L \quad \psi_l^R \rightarrow \psi_l^R + \underline{\psi}_l^R, \quad (31)$$

to get (where for simplicity we ignore the neutrino terms in ν_l)

$$\begin{aligned} \mathcal{L}_l^{LH} &= -g_l \left(\left(\bar{\Psi}_l^L + \bar{\underline{\Psi}}_l^L \right) \left(\psi_l^R + \underline{\psi}_l^R \right) \Phi + \Phi^\dagger \left(\bar{\psi}_l^R + \bar{\underline{\psi}}_l^R \right) \left(\Psi_l^L + \underline{\Psi}_l^L \right) \right) \\ &= -g_l \bar{\Psi}_l^L \psi_l^R \Phi - g_l \bar{\underline{\Psi}}_l^L \underline{\psi}_l^R \Phi - g_l \bar{\Psi}_l^L \psi_l^R \Phi - g_l \bar{\underline{\Psi}}_l^L \underline{\psi}_l^R \Phi + h.c. \end{aligned} \quad (32)$$

Focusing on terms in (32) where $l = 1$ (i.e., only the electron family), consider the Higgs mass radiative correction interaction of Figure 1, where in the RH sub-diagram, we introduce a pentagon symbol to represent a supplemental particle trajectory and underlining for supplemental particles themselves.

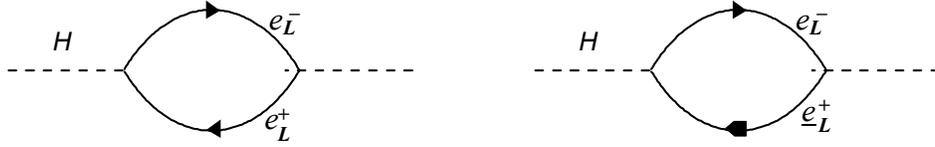


Figure 1. Traditional (LHS) vs Supplemental (RHS) Higgs Mass Radiative Corrections

The LH vertex in the LHS sub-diagram of Figure 1 arises from a traditional radiative correction term, the first term in second row of (32). The RH vertex in the same sub-diagram arises from the h.c. term in (32) corresponding to the first term.

Now consider the RHS sub-diagram, which has a supplemental positron propagator in place of the traditional positron propagator of the LHS sub-diagram. The LH vertex of the RHS sub-diagram arises from the second term in the second row of (32); the RH vertex, from the associated h.c. term.

The two sub-diagrams in Figure 1 have equal magnitude, but due to the sign reversal for supplemental propagators (21), they have Feynman amplitudes of opposite sign. They cancel.

Extending the analysis to other leptons and quarks, and to other relevant sub-diagrams, one finds the same cancellation. From this perspective, there are no radiative corrections to the Higgs mass.

Note: it is presumed that traditional and supplemental particles do not couple to the vector bosons of the standard model (SM). If they did, then the same type of cancellation would occur, and we would not see the running SM coupling constants observed in experiment. In fact, as noted earlier, transition sub-amplitudes would all cancel with one another, probability for anything to happen would be zero, and we would have no phenomenological universe of interacting particles at all.

5 Resolution of Seeming Inconsistencies

5.1 Non-Positive Definite Metric

As shown in ref. 24, supplemental eigenstate norms are not positive definite^{29,30,31}. Specifically, for suitable normalization, where numbers refer to $\underline{a}(\mathbf{k})$ -type particles [having negative particle numbers] of the same \mathbf{k} ,

$$\langle 0||0\rangle=1 \quad \langle -1||-1\rangle=-1 \quad \langle -2||-2\rangle=1 \quad \langle -3||-3\rangle=-1 \quad \dots\dots\dots \quad (33)$$

So, the non-positive definite metric in Fock space for such particles is

$$\underline{g}_{mn}^{Fock} = \begin{bmatrix} 1 & 0 & 0 & 0 & .. \\ 0 & -1 & 0 & 0 & .. \\ 0 & 0 & 1 & 0 & .. \\ 0 & 0 & 0 & -1 & .. \\ .. & .. & .. & .. & \dots \end{bmatrix}, \quad (34)$$

This result differs from the identity matrix of the traditional-particle Fock space metric. Pauli^{32,33} investigated fields with commutation relations such as (10), which generate metrics such as (34), and concluded it was impossible for them to be realized in the spectrum of physical states. In pre-QFT quantum theory, this would imply negative probabilities for some states, and positive ones for others.

However, if supplemental particles are only virtual and never real, then their metric in Fock space should be irrelevant, as that metric is a relationship between real states. Nevertheless, we can resolve this issue, even if such particles were real, as follows.

In QFT, one focuses on expectation values of operators/observables. In the traditional theory, for an eigenstate $|n_k\rangle$ having an eigenvalue a_k of an operator \mathcal{O} ,

$$\langle \mathcal{O} \rangle \equiv \langle n_k | \mathcal{O} | n_k \rangle = a_k \langle n_k || n_k \rangle = a_k . \quad (35)$$

Directly extrapolating (35) to supplemental particles, one gets

$$\langle \mathcal{O} \rangle = \langle \underline{n}_k | \mathcal{O} | \underline{n}_k \rangle = a_k \langle \underline{n}_k || \underline{n}_k \rangle = a_k (-1)^{\underline{n}_k} , \quad (36)$$

a not very satisfying, consistent, or usable result, as our expectation value is positive for every other multiparticle eigenstate and negative for the ones in between.

However, (35) was defined in a manner that would give us a numerical value corresponding to what we would measure for traditional particles. It is a simple and straightforward matter to define a mathematical procedure slightly different from (36) that will give us what we would measure for supplemental particle states, i.e., in the present case, the eigenvalue (with correct sign). Thus, we define expectation values for supplemental eigenstates as

$$\langle \mathcal{O} \rangle \equiv (-1)^{\underline{n}_k} \langle \underline{n}_k | \mathcal{O} | \underline{n}_k \rangle = (-1)^{\underline{n}_k} a_k \langle \underline{n}_k || \underline{n}_k \rangle = a_k (-1)^{2\underline{n}_k} = a_k . \quad (37)$$

Hence, any physical quantity we might wish to measure related to supplemental particles can be determined theoretically via (37), and the impasse that stymied Pauli dissolves. This definition would also apply to multi-particle states (with a factor of $(-1)^{\underline{n}_k}$ for each \mathbf{k} and similar factors for supplemental \underline{b}_k -type antiparticles) and transition amplitudes.

(35) and (37) can be generalized using the coefficient commutation relations (10) along with their traditional counterparts for traditional fields, where a bar through the middle of a quantity indicates it can be for either a traditional or supplemental field.

$$\langle \mathcal{O} \rangle \equiv \left[\bar{a}_k, a_k^\dagger \right]^{\underline{n}_k} \langle \bar{n}_k | \mathcal{O} | \bar{n}_k \rangle = \bar{a}_k \quad (38)$$

5.2 Vacuum Decay

Linde^{7,8} conjectured “some mechanism, associated probably with some kind of symmetry of the elementary particle theory, which would automatically lead to a vanishing of the cosmological constant in a wide class of theories”. He considered the possibility of two quasi-independent universes, occupying the same physical space and having the same particle equations of motion, but with opposite signs for their Lagrangians, for which he coined the term “antipodal symmetry”. The two universes would not interact via standard model forces, but would be linked gravitationally. Due to its negative Lagrangian, the vacuum energy of the antipodal (shadow) universe would be of opposite sign from ours, so the total vacuum energy would net to zero. Linde’s approach and the one shown herein have some similarities, but are quite different at their roots.

However, in personal correspondence with the present author, Linde noted his dissatisfaction with his idea as it seemed inevitably to imply vacuum instability and decay. Interaction between the two types of particles, even if only gravitationally, would lead to instability of the vacuum. In essence, a positive-energy traditional particle and a negative-energy supplemental particle could arise spontaneously from the vacuum, as they would comprise a net total energy of zero.

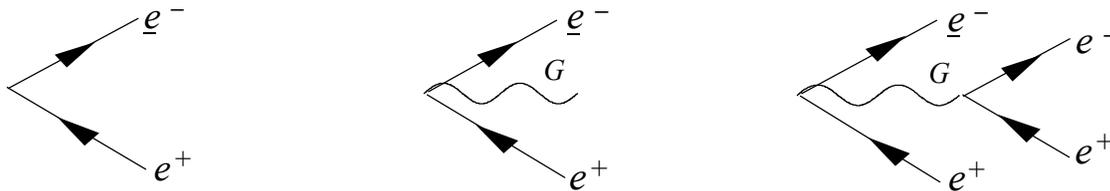


Figure 2. Conjectured Modes of Vacuum Decay Prohibited in One Form of Theory

As supplemental particles and traditional particles are presumed to be prohibited from interacting via standard model forces, the interactions displayed in Figure 2 are solely gravitational. The G symbol represents a traditional graviton, but in possible embodiments of supplemental field theory, it could also represent a supplemental graviton.

Further, with reference to Section 4.3, the G in the figure could be replaced by an H , i.e., the Higgs field could also interact with leptons and quarks in the manner shown in Figure 2.

Via the form of the theory suggested herein whereby real (on-shell) supplemental particles are prohibited, no interaction leaving a final state with real supplemental particles can occur. So, none of the interactions of Figure 2 would be allowable, even though they maintain energy and momentum balance. Thus, the vacuum could not decay in a manner like those of the diagrams in Figure 2, and it appears that for this interpretation of supplemental particles, Linde’s concern is resolved.

5.3 Supplemental Energy Bounded from Above

Resistance to negative-energy particles can also arise with regard to the fundamental physical principle that configurations of particles spontaneously seek the lowest attainable energy state. Thus, negative-energy particles would have no lowest state and be inherently unstable.

5.3.1 For Supplemental Particles Constrained to be Virtual Only

If supplemental particles can never be real, but only virtual, this concern is not relevant. There would be no real supplemental particles subject to such constraint. Virtual particles are not subject to it.

5.3.2 If Supplemental Particles Could be Real

If supplemental particles were real, it is wholly reasonable to presume they would seek the highest (least-negative) energy state. That is, with regard to energy levels, they would behave as traditional particles under a mirror reflection of the energy axis.

6 Additional Implications

6.1.1 Arrow of Time

Some, such as Barbour et al³⁴, and Carroll and Chen³⁵, have suggested the possible existence of a mirror universe to ours moving, since the Big Bang, in the opposite direction of time.

The absence of observed real supplemental particles may be conjectured as being related to the fact that reversing the arrow of time in the supplemental solutions produces the traditional solutions. That may mean, with regard to perception, that real supplemental particles travel backward in time. Thus, one might speculate that the universe needed no initial energy from which to begin, with equal amounts of traditional and supplemental particles emerging from nothing. From there, the real negative-energy supplemental particles traveled backward in time, and in the process created their own universe, which would appear to any beings in that universe to possess positive energy. The traditional particles, traveling forward in time, created our universe and appear to us as having positive energy. Each universe would have only one kind of real particle, but virtual particles of both types. And this would result in null (or near null, perhaps for other reasons) vacuum energy in both.

6.1.2 Inflation, GUTS, and More

Inflation, GUT, and possibly other vacuum condensate energies could cancel when account is taken of the associated supplemental fields in similar manner to that shown herein for the Higgs condensate.

6.1.3 Dark Energy

A slight asymmetry, from some presently unknown cause, between traditional and supplemental fields could give rise, via ZPE or symmetry breaking mechanisms, to a small cosmological constant, similar to what is observed.

7 Summary, Caveats, and Questions

7.1 Summary

A previously unrecognized, but fundamental, symmetry in elementary particle theory exists in which supplemental (alternative) solutions to the QFT free-field equations are obtained from the traditional solutions by taking $\omega \rightarrow -\omega$. The implementation of this symmetry in the theory results in a total Hamiltonian yielding null ZPE and possible resolutions of the Higgs hierarchy and condensate energy problems. Agreement with observations should be maintained if supplemental states occur only as virtual, and not as real, particles, and given the properties of the supplemental states, this appears reasonable.

The supplemental/traditional solution symmetry can be maintained over all energy scales, unlike other attempts to null out vacuum energy, such as supersymmetry, which only succeed down to particular, non-contemporary, energy levels.

Resolution of certain significant issues in physics and cosmology is direct and simple. Nothing more, no new theory, is needed, other than the inclusion in QFT of alternative solution forms to the field equations.

7.2 Caveats

There may be ways to falsify supplemental field theory with extant data. The author invites attempts at such falsification from those more qualified in this regard than he.

It may seem somewhat *ad hoc* to simply allocate terms selectively to the Lagrangian that give a desired result, even though that is what physics has done from day one. In the present case, we have added Higgs-supplemental fields interaction and Higgs supplemental field potential terms. At present there is no hint of a more encompassing theory from which these terms may spring.

While Section 4.2 presents a way in which the Higgs condensate energy density can be nulled out, it appears to contain an implicit assumption that supplemental Higgs particles are real. (The horizontal axes in the traditional and supplemental Mexican hat diagrams represent expectation values for real particle density.) But, a fundamental assumption in the version of the theory championed herein, used to potentially resolve the Higgs hierarchy problem, and employed to avoid vacuum decay, was that supplemental particles could only be virtual, and not real. Perhaps, the Higgs condensate issue, and the stable true vacuum location determination, can still be applied in a sense for which the supplemental Higgs is purely virtual. Possibly the virtual interactions between the traditional Higgs and the supplemental Higgs result in an effective supplemental potential that tracks the traditional potential.

The author has not found any way, in the pure mathematics of the theory, to prohibit supplemental particles from manifesting as real particles. The presumption for constraining them to be virtual is based on the empirical reality that negative-energy particles with momentum in the opposite direction of velocity are simply not observed, and in fact, do not even make sense in our universe as we know it.

7.3 Question

The fundamental question is whether or not nature in her physical manifestation parallels the nature of her mathematics. Supplemental field mathematical expressions solve the field equations, but do supplemental particles actually exist? The intractable nature of several outstanding problems in physics may lend support to them in some form. But as the University of Chicago cosmologist Rocky Kolb has said

“... for every complex phenomenon there is a simple, elegant, compelling explanation that is wrong.”

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Appendix: Are Supplemental Solutions Independent from the Traditional Solutions?

Since the traditional independent solutions of the Klein-Gordon equation

$$\phi = \sum_{\mathbf{k}=-\infty}^{+\infty} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}} a(\mathbf{k}) e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} + \sum_{\mathbf{k}=-\infty}^{+\infty} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}} b^\dagger(\mathbf{k}) e^{+i(\omega t - \mathbf{k} \cdot \mathbf{x})} \quad (39)$$

are summed over all \mathbf{k} , one might reason, that for each $+\omega$ in the summation with a $-k_x$, there is a $+\omega$ with a $+k_x$, and similarly for $-\omega$, leading to the solution forms (4) and (5). We respond to this argument below.

Consideration 1

There are subtle differences between sign in an algebraic relation and sign of an algebraic quantity within that relation. Consider the difference between sign in an algebraic expression and sign of an algebraic quantity in that expression. For example,

$$x - y = 0 \quad (40)$$

has, for every positive value of y , a negative value of the same magnitude. But that does not mean that the relation

$$x + y = 0 \quad (41)$$

is contained within (40). Parallel logic applies to supplemental vs traditional solutions.

Consideration 2

Consider traditional RQM where the independent solutions to the Klein-Gordon equation are shown in the summations of (39). Supplemental (or alternative) solutions, represented by underbars are, from (4) and (5), shown in the summations of

$$\underline{\phi} = \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}} \underline{a}(\mathbf{k}) e^{i(\omega t + \mathbf{k} \cdot \mathbf{x})} + \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}} \underline{b}^\dagger(\mathbf{k}) e^{-i(\omega t + \mathbf{k} \cdot \mathbf{x})}. \quad (42)$$

Mathematically, the terms in (42) are not independent of those in (39), since

$$a(\mathbf{k}) e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})} = \underline{b}^\dagger(-\mathbf{k}) e^{-i(\omega t + (-\mathbf{k}) \cdot \mathbf{x})} \quad \text{for } a(\mathbf{k}) = \underline{b}^\dagger(-\mathbf{k}). \quad (43)$$

Physically, however, consider the phase velocity of a traditional solution

$$e^{-i(\omega t - k_1 x_1)} \xrightarrow{\text{constant phase}} \omega t - k_1 x_1 = \text{constant} \rightarrow v_{\text{phase}} = \frac{dx_1}{dt} = \frac{\omega}{k_1}, \quad (44)$$

and phase is in the direction of wave number k_1 . For k_1 in the positive direction, the wave travels in the positive direction.

Then, consider the phase velocity of a supplemental (alternative) solution

$$e^{i(\omega t + k_1 x_1)} \xrightarrow{\text{constant phase}} \omega t + k_1 x_1 = \text{constant} \rightarrow v_{\text{phase}} = \frac{dx_1}{dt} = -\frac{\omega}{k_1}, \quad (45)$$

and phase is in the opposite direction of wave number k_1 . For k_1 in the positive direction, the wave travels in the negative direction.

One could, of course, define wave number as being in the opposite direction of phase velocity and have a consistent theory. Wave number direction is arbitrarily defined.

However, if k_1 is proportional to momentum,

$$k_1 = \frac{p_1}{\hbar} , \quad (46)$$

then traditional and supplemental solutions with momentum in the same direction have phase velocity in opposite directions. (This is also true for states in QFT, as shown in the main body of this article.)

If we developed our theory, using only (42) and not (39), and also revised our assumption in (46) to

$$k_1 = -\frac{p_1}{\hbar} , \quad (47)$$

we would find (45) as

$$v_{phase} = \frac{dx_1}{dt} = -\frac{\omega}{k_1} = \frac{\hbar\omega}{p_1} , \quad (48)$$

and phase velocity and momentum would be in the same direction, as we find in the world.

One could then consider the difference between the two theories is in assumptions (46) and (47). As long as we use either (46) with (39) or (47) with (42), we get the same results. The choice between the two is essentially only a choice of gauge.

However, one could instead postulate a different species of particle governed by (47) instead of (46), but with solutions of form (39). Or equivalently, one could get the same results by postulating that particle is governed by (46) with solutions of form (42).

In this article we do the latter, as it is simpler to develop, more suited to QFT, and seems more natural (since we do not change fundamental physical relationships between quantities, but merely re-cast the solution forms to the field equations.)

In essence, mathematically, the supplemental solutions can be considered contained within the traditional solutions, but, physically, they give rise to particle states not found in the traditional theory, and in this context, can be considered independent.

Consideration 3

As demonstrated in this article, by simply assuming supplemental solutions having the form shown herein, and proceeding step-by-step through the development of QFT with these solutions included, one gets distinctly different results, which cannot be subsumed into the traditional theory. Specifically, states created and destroyed by supplemental field operators have distinctly different eigenvalues (quantum numbers) from traditional particle states, and states with different quantum numbers are independent states.

Consideration 4

Supplemental particles may be thought of as simply a new species of particle, much like SUSY particles are a different species. SUSY and standard model field equation solutions are not linearly independent with regard to spacetime coordinates, but the associated operators are independent.

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