In more advanced QFT, one can show that a spin 2 boson (such as the graviton, hypothesized to mediate the gravitational force) also attracts particles that have like spin 2 field charges. For gravity, the spin 2 field charge is mass. So, positive mass particles attract one another.

In general, even number spin bosons mediate an attractive force between like charges. Odd number spin bosons mediate a repulsive force between like charges.

Wholeness Chart 16-1. Boson Spin and Like Charges

	Spin 0	**Spin 1**	**Spin 2**
Like charges	attract	repel	attract
Example	(pseudo) scalar mesons	photons	gravitons
Potential	Yukawa	Coulomb	gravitational
Charge type	Yukawa charge	electric	mass

16.4 Anomalous Magnetic Moment

Anomalous magnetic moment

16.4.1 Sophomore Physics Review

As a refresher, an elementary review of the electron magnetic moment is hereby provided.

Review of basic physics

Consider a circular loop of current I encompassing an area A, which acts like a magnetic dipole, i.e., acts just as if fictitious positive and negative "magnetic charges" were separated by a small distance. If the current loop is placed in an external magnetic field $\mathbf{B}^e$, the torque $\boldsymbol{\tau}$ it experiences (which can be visualized as equal magnitude, opposite direction forces on the two fictitious "magnetic charges") is (where $\mathbf{A}$ has magnitude of area A and direction normal to the plane of the loop aligned with the thumb of the right hand when the fingers curve in the direction of the current)

Torque on classical magnetic moment $\boldsymbol{\mu} = I\mathbf{A}$

$$\boldsymbol{\tau} = I\mathbf{A}\times\mathbf{B}^e = \boldsymbol{\mu}\times\mathbf{B}^e\,, \tag{16-32}$$

where $\boldsymbol{\mu} = I\mathbf{A}$ is called the magnetic moment of the current loop. The energy of the loop/external field, with θ the angle between $\boldsymbol{\mu}$ and $\mathbf{B}^e$, where we define the $\theta = \pi/2$ position as zero potential energy, is

$$E_{loop/field} = \int_{\pi/2}^{\theta}\tau\, d\theta' = \int_{\pi/2}^{\theta}\mu B^e \sin\theta' d\theta' = -\mu B^e\cos\theta = -\boldsymbol{\mu}\bullet\mathbf{B}^e \qquad \left(\boldsymbol{\mu} = \mu\mathbf{e}_\perp\right). \tag{16-33}$$

If the current is composed of a single particle of charge $-e$ (e.g., as one would have in the Bohr theory orbit of an electron in an atom), its speed is v, its time for one orbit is T_{orbit}, and the circular area A has radius r, then the magnetic moment due to the orbital angular momentum of the charge is

$\boldsymbol{\mu}$ in terms of angular momentum $\mathbf{L}$

$$\boldsymbol{\mu} = I\mathbf{A} = \frac{-e}{T_{orbit}}\pi r^2\mathbf{e}_\perp = \frac{-ev}{2\pi r}\pi r^2\mathbf{e}_\perp = -\frac{1}{2}evr\,\mathbf{e}_\perp \xrightarrow{L=mvr} \boldsymbol{\mu} = -\frac{1}{2}\frac{e}{m}\mathbf{L} \quad (\text{for 1 electron}), \tag{16-34}$$

where $\mathbf{e}_\perp$ is a unit vector pointing in the direction of the right hand thumb above, and $\mathbf{L}$ is orbital angular momentum. In an atomic orbit, angular momentum magnitude is

Magnitude of angular mom L of atomic orbit

$$L = \hbar m_l \tag{16-35}$$

with m_l an orbital quantum number. So, with (16-34), we can define the Bohr magneton μ_B via

Above used to define Bohr magneton μ_B

$$\boldsymbol{\mu} = \mu\mathbf{e}_\perp \xrightarrow[\text{in orbit}]{\text{1 electron}} \boldsymbol{\mu} = -\frac{1}{2}\frac{e}{m}\mathbf{L} = -\frac{1}{2}\frac{e\hbar}{m}m_l\,\mathbf{e}_\perp = -\mu_B m_l\,\mathbf{e}_\perp \qquad \mu_B = \frac{1}{2}\frac{e\hbar}{m}. \tag{16-36}$$

As to the electron itself, one can view it classically as a charge that is distributed internally, rather than being point-like, and that rotates, or "spins" around some internal axis. So, in effect, we would have a circulating current loop of sorts similar to that described above for an atom. In quantum theory, that spin of the electron is quantized, and the intrinsic (different from orbital contribution) angular momentum is spin angular momentum $\mathbf{S} = \hbar m_s\mathbf{e}_\perp = \pm\hbar/2\,\mathbf{e}_\perp$ ($m_s = \pm$ ½ is spin quantum number for one electron). So, one might consider

For electron, we know angular momentum (spin), but not internal charge distribution

$$\boldsymbol{\mu} = \mu\mathbf{e}_\perp \xrightarrow[\text{spinning}]{\text{1 electron}} \boldsymbol{\mu} \overset{\text{maybe?}}{=} -\frac{1}{2}\frac{e}{m}\mathbf{S} = \mp\frac{1}{2}\frac{e\hbar}{m}\frac{1}{2}\mathbf{e}_\perp = -\mu_B m_s\mathbf{e}_\perp = \mp\frac{\mu_B}{2}\mathbf{e}_\perp \qquad |\mu| = \frac{\mu_B}{2}, \tag{16-37}$$

where for positive spin (and negatively charged electron) we get the minus sign on the RHS in (16-37). However, the RHS of (16-37) is derived assuming charge is distributed as a neat current loop, as in (16-36), which is naïve. Given the unknown nature of this distribution, researchers introduced a constant g, called the gyromagnetic ratio[1] or the g-factor, which could be determined by experiment. So, the most general form for the magnetic moment $\boldsymbol{\mu}$ of the electron and its magnitude μ would be (where, after the third equal sign, the negative sign holds for positive spin)

The unknown charge distribution contribution labeled as g, gyromagnetic ratio

$$\boldsymbol{\mu} = \mu \mathbf{e}_{\perp} \xrightarrow[\text{spinning}]{\text{1 electron}} \boldsymbol{\mu} = -g\frac{\mu_B}{2} m_s \mathbf{e}_{\perp} = \mp g \frac{\mu_B}{2}\mathbf{e}_{\perp} \qquad |\mu| = g\frac{\mu_B}{2} = g\frac{e\hbar}{4m} \quad \left(g\frac{e}{4m}, \text{ natural units} \right). \tag{16-38}$$

If our naïve analysis (current distributed in a neat loop) were correct, then the gyromagnetic ratio g would be found equal to 1.

[1] This term is used in the literature for two things, the g-factor described herein (which is dimensionless) and the ratio of magnetic dipole moment to angular momentum (which is often denoted by the symbol γ, and which has SI dimensions of radians per second per tesla). In this book, the term gyromagnetic ratio will be used as equivalent to g, as shown above.