Relativistic Point Particle vs Relativistic String Solutions
Light-Cone Gauge & Light-Cone Coordinates: Classical Mechanics

As an aid for Zwiebach (which equation numbers below reference) Robert D Klauber January 25, 2023

	Point Particle		**Open String Field**	
Independent variables	x^I, x_0^-, p^I, p^+	(11.25), pg. 220	$X^I, x_0^-, \mathcal{P}^{\tau I}, p^+$	(12.5) $\mathcal{P}^\tau$ = momentum density
Full description of motion	$x^\mu(\tau) = (x^+, x^-, x^I)$		$X^\mu(\tau,\sigma) = (X^+, X^-, X^I)$	
Eq of motion	$\dot{p}_\mu = \dfrac{dp_\mu}{d\tau} = m^2 \ddot{x}_\mu = 0$	m^2 instead of m gives unitless τ (nat units)	$\ddot{X}^\mu - X''^\mu = 0$	(9.39)
General solution	$x^\mu = x_0^\mu + \dfrac{p^\mu}{m^2}\tau$	Motion of point particle with no external force	$X^\mu = x_0^\mu + \underbrace{\sqrt{2\alpha'}\alpha_0^\mu}_{2\alpha' p^\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\dfrac{1}{n}\alpha_n^\mu e^{-in\tau}\cos n\sigma$	(9.56), p^μ from (9.52) Neumann B.C.s Wave motion of string with no external force
Transverse motion (given)	$x^I = x_0^I + \dfrac{p^I}{m^2}\tau$	(11.15), pg. 218	X^I = (9.56) above with $\mu = I$	(9.69)
Dependent variables	$x^+ = \dfrac{p^+}{m^2}\tau \quad \left(x_0^+ = 0\right)$ $x^- = x_0^- + \dfrac{p^-}{m^2}\tau$	← Light-cone gauge cond. (11.7) & (11.29) (11.14) & (11.30)	$X^+ = 2\alpha' p^+\tau = \sqrt{2\alpha'}\alpha_0^+\tau$ $\left(\mu = +,\ x_0^+ = \alpha_n^+ = 0 \text{ in } (9.56)\right)$ X^- = (9.56) with $\mu = -$	← Light-cone gauge cond. (9.70) (9.72)
Auxiliary to get above	$p^- = \dfrac{1}{2p^+}\left(p^I p^I + m^2\right)$	(11.12) & (11.31)	$\sqrt{2\alpha'}\alpha_n^- = \dfrac{1}{p^+}L_n^\perp \qquad L_n^\perp = \dfrac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_{n-p}^I\alpha_p^I$ $\sqrt{2\alpha'}\alpha_0^- = 2\alpha' p^- = \dfrac{1}{p^+}L_0^\perp \qquad L_0^\perp = 2\alpha' p^+ p^-$ $L_0^\perp = \alpha'\left(p^I p^I + m^2\right) = \alpha' p^I p^I + \sum_{p=1}\alpha_p^{I\dagger}\alpha_p^I$	(9.77) $L_n^\perp =$ transverse Viasoro mode (only used for α_n^-) (9.78) (p^- from point particle) (12.105), m^2 from (9.83)
Momenta density	N/A		$\mathcal{P}^\tau = \mathcal{P}^{\tau\mu} = (\mathcal{P}^{\tau+}, \mathcal{P}^{\tau-}, \mathcal{P}^{\tau I}) \qquad \mathcal{P}^{\tau\mu} = \dfrac{1}{2\pi\alpha'}\dot{X}^\mu$	
Momenta of motion	$p^\mu = (p^+, p^-, p^I)$		$p^\mu = (p^+, p^-, p^I) \qquad p^\mu = \int \mathcal{P}^{\tau\mu} d\sigma$	
Hamiltonian	$H = \dfrac{p^+ p^-}{m^2} = \dfrac{\left(p^I p^I + m^2\right)}{2m^2}$	(11.34)	$H = 2\alpha' p^+ p^- = L_0^\perp$	(12.16)
Valuable relation			$\dot{X}^- \pm X^{-\prime} = \dfrac{1}{\beta\alpha'}\dfrac{1}{2p^+}\left(\dot{X}^I \pm X^{I\prime}\right)^2$ $= \dfrac{1}{p^+}\sum_{n\in\mathbb{Z}} L_n^\perp e^{-in(\tau\pm\sigma)}$	(9.65) $\beta = 2$ for open (9.79)