Pair Popping Addendum Vol 1 Sect 10.11 pg 284

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10.11 Appendix D: Free Fields and "Pair Popping" Re-visited

One possible issue some might raise with this chapter needs to be addressed. That is, by using the Interaction Picture (I.P.), where operator fields are free and particle behavior is described by the interaction Hamiltonian, are we somehow obscuring some physics? Recall we derived Feynman diagrams from only the interaction term in the Hamiltonian. If we included the free Hamiltonian in such a derivation, would we possibly find Feynman diagrams producing and destroying particle/antiparticle pairs?

Specifically, $\mathcal{H}_0$ has creation and destruction operators paired together in the same terms. For scalar fields, in the Heisenberg Picture (H.P.), we found

$$\mathcal{H}_0^0 = \pi_0^0\dot{\phi} + \pi_0^{0\dagger}\dot{\phi}^\dagger - \mathcal{L}_0^0 = \left(\dot{\phi}\dot{\phi}^\dagger + \nabla\phi^\dagger\cdot\nabla\phi + \mu^2\phi^\dagger\phi\right) = \left(\pi_0^{0\dagger}\pi_0^0 + \nabla\phi^\dagger\cdot\nabla\phi + \mu^2\phi^\dagger\phi\right) \tag{10-27}$$

$$\text{with} \quad \phi(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{2V\omega_{\mathbf{k}}}}\left(a(\mathbf{k})e^{-ikx} + b^\dagger(\mathbf{k})e^{ikx}\right). \tag{10-28}$$

So, we might end up with a creation ($a^\dagger(\mathbf{k})$ or $b^\dagger(\mathbf{k})$) and a destruction ($a(\mathbf{k})$ or $b(\mathbf{k})$) operator in the same term in H_0^0. In particular, if we had terms containing factors of $a^\dagger(\mathbf{k})b^\dagger(\mathbf{k})$ or $a^\dagger(\mathbf{k})b^\dagger(-\mathbf{k})$, we might expect creation of a particle and an antiparticle at the same event in the vacuum.

To see how states might change, let's use the Schrödinger picture (S.P.), in which operators do not change in time, but states do. In the S.P. for free fields, scalar states are governed by

$$i\frac{d}{dt}\left|\Phi\right\rangle = H_0^0\left|\Phi\right\rangle, \tag{10-29}$$

where $|\Phi\rangle$ is in general a multi-scalar particle state and H_0^0 is the same in the H.P. as the S.P. (See Wholeness Chart 2-4, pg. 28.) Using H_0^0, specifically the RHS of (10-27), along with (10-28) and its associated quantities, we can follow the same steps as we did in Chap. 8 to determine the evolution of the state $|\Phi\rangle$. (See Wholeness Chart 8-4, pgs. 248-251.) This leads us to

$$\left|\Phi(t_f)\right\rangle = S\left|\Phi(t_i)\right\rangle = e^{-i\int_{-\infty}^{\infty}H_0^0 dt'}\left|\Phi(t_i)\right\rangle \qquad S_{fi} = \left\langle\Phi(t_f)\right|S\left|\Phi(t_i)\right\rangle = \left\langle f\right|S\left|i\right\rangle. \tag{10-30}$$

With the Dyson expansion, we find

$$S = \underbrace{I}_{S^{(0)}} \underbrace{-i\int_{-\infty}^{\infty}\mathcal{H}_0^0(x_1)d^4x_1}_{S^{(1)}} \underbrace{-\frac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}T\left\{\mathcal{H}_0^0(x_1)\mathcal{H}_0^0(x_2)\right\}d^4x_1 d^4x_2}_{S^{(2)}} + \ldots\ldots. \tag{10-31}$$

For the $S^{(1)}$ term, the integration over all space was carried out in Chap. 3, Sect. 3.4.1, pgs. 53-54. In that derivation, we saw all terms containing factors $a^\dagger(\mathbf{k})b^\dagger(-\mathbf{k})$, $a^\dagger(\mathbf{k})b^\dagger(\mathbf{k})$, $a(\mathbf{k})b(-\mathbf{k})$, and $a(\mathbf{k})b(\mathbf{k})$ dropped out. That is, no terms remain that create a particle/anti-particle pair at the same event in the vacuum. Ditto for destruction of such a pair.

For the $S^{(2)}$ term, at any given point in time, we can integrate $H_0^0(x_1)$ over $d^3\mathbf{x_1}$ without regard to the integration of $H_0^0(x_2)$ over $d^3\mathbf{x_2}$. For that integration, we would get the same result as for $S^{(1)}$, i.e., no terms with factors creating or destroying a particle/anti-particle pair. The same result would hold for $S^{(n)}$ for any n.

The transition amplitude S_{fi} would, therefore, not contain any terms creating/destroying such pairs. And so, we would have no Feynman diagrams representing such a thing.

Further, by reviewing the above cited section of Chap. 3, one can see that the ½ quanta terms, commonly considered "vacuum fluctuations" come from the $a(\mathbf{k})a^\dagger(\mathbf{k})$ and $b(\mathbf{k})b^\dagger(\mathbf{k})$ terms and the coefficient commutation relations. Even if we chose to use these terms directly, without employing the commutation relations, the $a(\mathbf{k})a^\dagger(\mathbf{k})$ term is not coupled to the $b(\mathbf{k})b^\dagger(\mathbf{k})$term so both terms together would not represent a vertex in a Feynman diagram. In that interpretation, one might think of the $a(\mathbf{k})a^\dagger(\mathbf{k})$ as representing creation of a particle and destruction of the same particle at the same event, i.e., nothing would happen as time evolves. No evanescence. No pair popping.

In summary, for free fields

- Terms in the free Hamiltonian density containing two creation operators that might create a particle/antiparticle pair at an event drop out of the full (not density) Hamiltonian.
- The only terms surviving in the full Hamiltonian have creation and destruction operators paired. These would create and destroy the same particle at the same event, i.e., nothing would effectively happen.

We conclude that the free field components of the Hamiltonian do not lead to particle/antiparticle pairs popping in and out of the vacuum.

End of Sect. 10.11

1 First Additional Consideration

Consider the second order term(10-31) with the Wick's theorem, where we get propagators from x_2 to x_1.

$$
\begin{aligned}
&-\tfrac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T\left\{\mathcal{H}_0^0(x_1)\mathcal{H}_0^0(x_2)\right\} d^4x_1 d^4x_2 \\
&\quad = -\tfrac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T\left\{\left(\dot\phi\dot\phi^\dagger + \nabla\phi^\dagger\cdot\nabla\phi + \mu^2\phi^\dagger\phi\right)_{x_1}\left(\dot\phi\dot\phi^\dagger + \nabla\phi^\dagger\cdot\nabla\phi + \mu^2\phi^\dagger\phi\right)_{x_2}\right\} d^4x_1 d^4x_2
\end{aligned}
\tag{1}
$$

Consider one value of 3-momentum in each of the sums for ϕ in the bi-linear terms of (1).

$$
\begin{aligned}
&\mathbf{k} \text{ at } x_1 \text{ and } \mathbf{k}' \text{ at } x_2 \\
&= -\tfrac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T\left\{\left(-\omega_{\mathbf{k}}^2\phi\phi^\dagger + \mathbf{k}^2\phi\phi^\dagger + \mu^2\phi^\dagger\phi\right)_{x_1}\left(-\omega_{\mathbf{k}'}^2\phi\phi^\dagger + \mathbf{k}'^2\phi\phi^\dagger + \mu^2\phi^\dagger\phi\right)_{x_2}\right\} d^4x_1 d^4x_2
\end{aligned}
\tag{2}
$$

$$
= -\tfrac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T\left\{
\begin{array}{l}
\left(\omega_{\mathbf{k}}^2\phi\phi^\dagger\right)_{x_1}\left(\omega_{\mathbf{k}'}^2\phi\phi^\dagger\right)_{x_2} - \left(\omega_{\mathbf{k}}^2\phi\phi^\dagger\right)_{x_1}\left(\mathbf{k}'^2\phi\phi^\dagger\right)_{x_2} - \left(\omega_{\mathbf{k}}^2\phi\phi^\dagger\right)_{x_1}\left(\mu^2\phi^\dagger\phi\right)_{x_2} \\
-\left(\mathbf{k}^2\phi\phi^\dagger\right)_{x_1}\left(\omega_{\mathbf{k}'}^2\phi\phi^\dagger\right)_{x_2} + \left(\mathbf{k}^2\phi\phi^\dagger\right)_{x_1}\left(\mathbf{k}'^2\phi\phi^\dagger\right)_{x_2} + \left(\mathbf{k}^2\phi\phi^\dagger\right)_{x_1}\left(\mu^2\phi^\dagger\phi\right)_{x_2} \\
-\left(\mu^2\phi^\dagger\phi\right)_{x_1}\left(-\omega_{\mathbf{k}'}^2\phi\phi^\dagger\right)_{x_2} + \left(\mu^2\phi^\dagger\phi\right)_{x_1}\left(\mathbf{k}'^2\phi\phi^\dagger\right)_{x_2} + \left(\mu^2\phi^\dagger\phi\right)_{x_1}\left(\mu^2\phi^\dagger\phi\right)_{x_2}
\end{array}
\right\} d^4x_1 d^4x_2
$$

Apply Wick's theorem, but focus on only one term, the first, as all others will follow the same behavior.

For only the first term above, where ϕ now stands for a fixed value of 3-momentum instead of a sum

$$
\begin{aligned}
&= -\tfrac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} T\left\{\left(\omega_{\mathbf{k}}^2\phi\phi^\dagger\right)_{x_1}\left(\omega_{\mathbf{k}'}^2\phi\phi^\dagger\right)_{x_2}\right\} d^4x_1 d^4x_2 \\
&= -\tfrac{1}{2!}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} N\left\{\left(\omega_{\mathbf{k}}^2\phi\phi^\dagger\right)_{x_1}\left(\omega_{\mathbf{k}'}^2\phi\phi^\dagger\right)_{x_2}\right\} d^4x_1 d^4x_2 - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} N\left\{\left(\omega_{\mathbf{k}}^2\phi\underbrace{\phi^\dagger\right)_{x_1}\left(\omega_{\mathbf{k}'}^2\phi}\phi^\dagger\right)_{x_2}\right\} d^4x_1 d^4x_2 \\
&\quad - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \omega_{\mathbf{k}}^2\omega_{\mathbf{k}'}^2 \left\{\underbrace{\left(\phi\underbrace{\phi^\dagger\right)_{x_1}\left(\phi}\phi^\dagger\right)_{x_2}}\right\} d^4x_1 d^4x_2
\end{aligned}
\tag{3}
$$

In (3) we get propagators from one event to another, and at the x_2 event, it seems we could have, in the vacuum, a particle and an antiparticle created, followed by their mutual destruction at x_1. In other words, it seems like pair-popping.

From the procedures of Chap. 8 in *SFQFT Vol. 1*, we know we get a Dirac delta function at each event forcing the incoming 4-momentum to equal the outgoing 4-momentum. Consider the presumed creation event at x_2, where we label the two particles as 1 and 2. Particle 1 has 4-momentum k_a, and particle 2 has 4-momentum k_b. At that event, we get a factor in the amplitude of

$$
\delta\left(k_a + k_b\right). \tag{4}
$$

This tells us the only possible way we get a non-zero amplitude is if

$$
k_b = -k_a \qquad \rightarrow \qquad \omega_a = -\omega_b \quad \text{and} \quad \mathbf{k}_a = -\mathbf{k}_b . \tag{5}
$$

In other words, one of the particles in the propagator must have negative energy. This is OK, as we know, for virtual particles, but it means the sum total of energy for the vacuum bubble is zero. The pair-popping scenario, on the other hand, claims the bubble has net positive energy (over short time scales) due to the uncertainty principle. (For non-infinite volume and time integration, see Appendix E of Chap. 10, for 3-particle vacuum bubbles. The same logic applies here.)

Further, even if 2-particle bubbles could manifest, as suggested by (3), the effective coupling constant would be $\omega_{\mathbf{k}}^2$ (the ϕ each have a square root of $\omega_{\mathbf{k}}$ factor in their denominator). In other words, bubbles with greater energy per particle would be more likely, the opposite of what the pair-popping/uncertainty principle story tells.

Bottom line: Using Wick's theorem with the free Hamiltonian does not lead to pair-popping because 1) net energy must be zero, and 2) the probability of a pair-pop increases with energy, rather than decreases.

2 Second Additional Consideration

If one insists on considering the free terms as representing vertices, then that means one is including them in the Dyson-Wicks expansion (which no one does). Consider that we do that. What happens?

From the kinetic term (a free Hamiltonian term), we would get Dyson-Wicks terms like (6) normal ordered, and also like (3), (7) and (8) with propagators (contractions). (3) looks like pair-popping, but we would also get terms like (7) and (8), as well as a number more. Those terms are not found in traditional QFT.

$$m\bar{\psi}_{x1}\psi_{x1} \qquad e\bar{\psi}_{x2}A^{\mu}_{x2}\gamma_{\mu}\psi_{x2} \qquad e\bar{\psi}_{x2}A^{\mu}_{x2}\gamma_{\mu}\psi_{x2} \tag{6}$$

$$m\bar{\psi}_{x1}\psi_{x1} \qquad e\bar{\psi}_{x2}A^{\mu}_{x2}\gamma_{\mu}\psi_{x2} \qquad e\bar{\psi}_{x2}A^{\mu}_{x2}\gamma_{\mu}\psi_{x2} \tag{7}$$

$$m\bar{\psi}_{x1}\psi_{x1} \qquad e\bar{\psi}_{x2}A^{\mu}_{x2}\gamma_{\mu}\psi_{x2} \qquad e\bar{\psi}_{x2}A^{\mu}_{x2}\gamma_{\mu}\psi_{x2} \tag{8}$$

Feynman diagrams for (7) and (8) look like

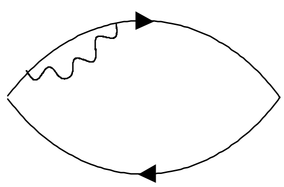

Figure 1.

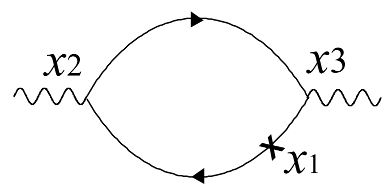

Figure 2.

So, when we calculate the photon loop correction, we would not have just the coupling constants *e* in the sub-amplitude, but also a factor of *m* from x_2. And for the $\bar{\psi}_{x1}\gamma^{\mu}\partial_{\mu}\psi_{x1}$ kinetic terms in place of the mass term in (8), we would get another, different term with the energy and 3-momentum (from the derivative) as factors.

In other words, the contributions from the free Hamiltonian would give us a different amplitude value for the photon loop calculation, as well as for every amplitude QFT now gives us where we don't include free Hamiltonian terms. Since the traditional theory without those terms gives us answers that match experiment to 10 decimal places, we would get the wrong predictions by including (7) and (8) in the theory.

Additionally, for Fig. 1, we would mass factors in the amplitude calculation. The higher the mass, the greater the probability of a vacuum bubble. So, higher energy (mass) means greater likelihood of a bubble, whereas the Heisenberg principle implies quite the opposite.

Bottom line: Since including free Hamiltonian terms in the Wicks expansion gives us the wrong answers for interactions we know of, any predictions for vacuum 2-particle bubbles from terms like (3) must be wrong. Further, for vacuum bubbles, the behavior would be totally incompatible, quantitatively, with the Heisenberg uncertainty principle.