Quantum Field Theory Vector Spaces

Operators	Fields	States
Physical quantities like energy, charge, momentum, spin	Solutions to field equations that create and destroy particles	Physical particles, one or more (though vacuum state has none)
$H, Q, \mathbf{P}, \Sigma_3$	$\phi,\ \psi,\ A^\mu$	|particles> like $\|e^+\rangle$ or $\|e^+ e^-\rangle$

Operators ← A vector space and operators → Vectors

Operators ← Another vector space and operators → Vectors

Operators ← Yet another vector space and operators → Vectors

Example $$\Sigma_3 \psi_{\substack{spinup \\ \mathbf{p}=0}} = \frac{1}{2}\begin{bmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}\left(c_r(\mathbf{p}) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ipx} + d_r^\dagger(\mathbf{p}) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ipx} \right) = \frac{1}{2}\left(c_r(\mathbf{p}) \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-ipx} + d_r^\dagger(\mathbf{p}) \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} e^{ipx} \right) = \frac{1}{2} \psi_{\substack{spinup \\ \mathbf{p}=0}} .$$

Example $$\text{QFT}\,\Sigma_3 \left| \underset{\mathbf{p}_1=0}{e^-_{up}(r=1)} \right\rangle = \left(\int_V \psi^\dagger \Sigma_3 \psi \, d^3x \right) \left| \underset{\mathbf{p}_1=0}{e^-_{up}(r=1)} \right\rangle$$

$$= \sum_{r,\mathbf{p}} \frac{m}{E_\mathbf{p}} \left(u_r^\dagger(\mathbf{p}) \Sigma_3 u_r(\mathbf{p}) N_r(\mathbf{p}) + v_r^\dagger(\mathbf{p}) \Sigma_3 v_r(\mathbf{p}) \bar{N}_r(\mathbf{p}) \right) \left| \underset{\mathbf{p}_1=0}{e^-_{up}(r=1)} \right\rangle = \frac{1}{2} \left| \underset{\mathbf{p}_1=0}{e^-_{up}(r=1)} \right\rangle$$