# Number Operators in QFT vs String Theory

Note: We ignore the ordering factor (ZPE in QFT, *a* in string theory)

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| | **<u>Quantum Field Theory</u>** (Eq & pg nums, Klauber, Vol. 1) | | **<u>String Theory</u>** (Eq & pg nums from Zwiebach) | |
|---|---|---|---|---|
| Commutator | $\left[a_{\mathbf{k}}, a_{\mathbf{k}}^{\dagger}\right] = \delta_{\mathbf{k}\mathbf{k}'}$ | (3.41) [51] | $\left[a_m^I, a_n^J\right] = \delta_{mn}\eta^{IJ}$ | (12.64) [245] |
| Creation & Destruction Operators | $n_{\mathbf{k}}$ is the number of particles of 3-momentum $\mathbf{k}$<br>$a_{\mathbf{k}}^{\dagger}\vert 0\rangle = \vert n_{\mathbf{k}} = 1\rangle$<br>$a_{\mathbf{k}}^{\dagger}\vert n_{\mathbf{k}}\rangle = \sqrt{n_{\mathbf{k}}+1}\,\vert n_{\mathbf{k}}+1\rangle$<br>$a_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle = \sqrt{n_{\mathbf{k}}}\,\vert n_{\mathbf{k}}-1\rangle$ | (3-81) [59] | $\hat{n}_n^I$ is number of strings in mode $n$ in $I$th direction<br>$a_n^{I\dagger}\vert 0\rangle = \vert \hat{n}_n^I = 1\rangle$<br>$a_n^{I\dagger}\vert \hat{n}_n^I\rangle = \sqrt{\hat{n}_n^I+1}\,\vert \hat{n}_n^I+1\rangle$<br>$a_n^{I}\vert \hat{n}_n^I\rangle = \sqrt{\hat{n}_n^I}\,\vert \hat{n}_n^I-1\rangle$ | (12.65) [246], more or less, but for alpha $\alpha_n^{I\dagger} = \sqrt{n}\,a_n^{I\dagger}$ |
| Number Operators | $N_{\mathbf{k}} = a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}$<br>$N_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle = a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle = n_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle$<br>$N = \sum_{\mathbf{k}'} N_{\mathbf{k}'} = \sum_{\mathbf{k}'} a_{\mathbf{k}'}^{\dagger}a_{\mathbf{k}'}$<br>$N\vert n_{\mathbf{k}}\rangle = \sum_{\mathbf{k}'} a_{\mathbf{k}'}^{\dagger}a_{\mathbf{k}'}\vert n_{\mathbf{k}}\rangle = n_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle$ | (3-56) [54] | $N_n^I = a_n^{I\dagger}a_n^I$<br>$N_n^I\vert \hat{n}_n^I\rangle = a_n^{I\dagger}a_n^I\vert \hat{n}_n^I\rangle = \hat{n}_n^I\vert \hat{n}_n^I\rangle$<br>$N^I = \sum_{I,n} N_n^I = \sum_{I,n} a_n^{I\dagger}a_n^I$<br>$N^I\vert \hat{n}_n^I\rangle = \sum_{I,n} a_n^{I\dagger}a_n^I\vert \hat{n}_n^I\rangle = \hat{n}_n^I\vert \hat{n}_n^I\rangle$ | Not shown. |
| Eigenvalue | $n_{\mathbf{k}}$ is the number of particles of 3-momentum $\mathbf{k}$<br>If only one or no particles, $n_{\mathbf{k}} = 1$ or 0 | | $\hat{n}_n^I$ is number of strings in mode $n$ in $I$th direction<br>If only one or no strings, $\hat{n}_n^I = 1$ or 0 | |
| Other number operator | $\tilde{N}_{\mathbf{k}} = \mathbf{k}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}$<br>$\tilde{N}_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle = \mathbf{k}a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle = \mathbf{k}n_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle$<br>$\tilde{N} = \sum_{\mathbf{k}'} \tilde{N}_{\mathbf{k}'} = \sum_{\mathbf{k}'} \mathbf{k}' a_{\mathbf{k}'}^{\dagger}a_{\mathbf{k}'}$<br>$\tilde{N}\vert n_{\mathbf{k}}\rangle = \sum_{\mathbf{k}'} \mathbf{k}' a_{\mathbf{k}'}^{\dagger}a_{\mathbf{k}'}\vert n_{\mathbf{k}}\rangle = \mathbf{k}n_{\mathbf{k}}\vert n_{\mathbf{k}}\rangle$ | Not shown | $N_n^{\perp I} = n\,a_n^{I\dagger}a_n^I$<br>$N_n^{\perp I}\vert \hat{n}_n^I\rangle = n\,a_n^{I\dagger}a_n^I\vert \hat{n}_n^I\rangle = n\,\hat{n}_n^I\vert \hat{n}_n^I\rangle$<br>$N^{\perp} = \sum_{I,n'} n' N_{n'}^I = \sum_{I,n'} n' a_{n'}^{I\dagger}a_{n'}^I$<br>$N^{\perp}\vert \hat{n}_n^I\rangle = \sum_{I,n'} n' a_{n'}^{I\dagger}a_{n'}^I\vert \hat{n}_n^I\rangle = n\,\hat{n}_n^I\vert \hat{n}_n^I\rangle$ | (12.164) [264] |
| Eigenvalue | $\mathbf{k}n_{\mathbf{k}}$ is the number of particles $n_{\mathbf{k}}$ times the 3-momentum $\mathbf{k}$<br>If only one or no particles, $\mathbf{k}n_{\mathbf{k}} = \mathbf{k}$ or 0 | | $n\,\hat{n}_n^I$ is number of strings in mode $n$ in $I$th direction $\hat{n}_n^I$ times the mode number $n$<br>If only one or no particles, $n\,\hat{n}_n^I = n$ or 0 | |
| Conclusion | For single particle state, eigenvalue of $\tilde{N}_{\mathbf{k}}$ is $\mathbf{k}$ | Not shown | For a single string in a single mode $n$, the eigenvalue of $N^{\perp}$ is $n$ | above (12.169) [264] |
| | QFT states can be single particle or multiparticle | | String states studied in Zwiebach are single strings | |
| | As an aside, $\tilde{N}_{\mathbf{k}}$ is actually the 3-momentum operator P | (3-101) [64] | For a single string with modes in more than one direction $I$, see Zwiebach, bottom of page 264 | [264] |