Lagrangians: A Summary of Different Cases

An aid for Zwiebach (which plain eq nums reference – "K" eq num refs to Klauber, Vol. 1) Robert D Klauber March 6, 2023

[xx] reference page number in the text

	Non-Relativistic				**Relativistic**			
	Free		**Interacting**		**Free**		**Interacting**	
Particle, L=	$T=\frac{1}{2}m\dot{x}^2$ $=\frac{p^2}{2m}$	(4.22) [78] with V=0	$T-V=$ $\frac{1}{2}m\dot{x}^2-V(x)$	(4.22) [78]	$T=-mc^2\sqrt{1-v^2/c^2}$ t system $=-mc^2$ τ system	(5.8) [92] (5.7) [92]	$T-V=-mc^2+\frac{q}{c}A_\mu\frac{dx^\mu}{d\tau}$ τ system	(5.33) [97]
Harmonic oscill	$\frac{1}{2n}\dot{q}_n^2-\frac{n}{2}q_n^2$	(12.68) [247]	Not common		Not common		Not common	
Field, $\mathcal{L}$=					t system below		t system below	
Scalar	$\mathcal{T}-\mathcal{U}=$ $\frac{1}{2}\rho\dot{x}^2-\mathcal{U}$	$\mathcal{U}$= internal pot energy	$\mathcal{T}-\mathcal{U}-\mathcal{V}=$ $\frac{1}{2}\rho\dot{x}^2-\mathcal{U}-\mathcal{V}$	$\mathcal{V}$= external pot energy	$\partial_\alpha\phi^\dagger\,\partial^\alpha\phi-\mu^2\phi^\dagger\phi$	K(3-32) [49]	$\partial_\alpha\phi^\dagger\,\partial^\alpha\phi-\mu^2\phi^\dagger\phi-\mathcal{V}$	
Spinor	Not common		Not common		$\bar{\psi}\left(i\gamma^\alpha\partial_\alpha-m\right)\psi$	K(4-60) [104]	$\bar{\psi}\left(i\gamma^\alpha\partial_\alpha-m\right)\psi+e\bar{\psi}\gamma^\mu\psi A_\mu$	K(7-20) [186]
Photon	N/A		N/A		$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$	$\leftarrow A_\mu=0$ in$\rightarrow$	$-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}+e\bar{\psi}\gamma^\mu\psi A_\mu$	K(11-7) [288]
String					τ,σ parameter space below			
General	$\frac{1}{2}\mu_0(\dot{y})^2$ $-\frac{1}{2}T_0(y')^2$	(4.35) [81]	$\frac{1}{2}\mu_0(\dot{y})^2$ $-\frac{1}{2}T_0(y')^2-\mathcal{V}$		$-\frac{T_0}{c}\sqrt{-\gamma}=$ $-\frac{T_0}{c}\sqrt{(\dot{X}\bullet X')^2-(\dot{X})^2(X')^2}$	(6.44) & (6.46) [112] Min area of world sheet		
Static gauge	N/A		N/A		$-\frac{T_0}{c}\sqrt{-(\dot{X})^2(X')^2}=-T_0\frac{\partial\mathbf{x}}{\partial\sigma}$ $=-T_0\frac{ds}{d\sigma}\sqrt{1-v_\perp^2/c^2}$	(6.66) [118] (6.89) [123]		
↑ & new σ parametrization	N/A		N/A		As above			
$n\cdot X=\alpha' n\cdot p\tau$ $n\bullet\mathcal{P}^\tau=\frac{\beta}{2\pi}n\cdot p$	N/A		N/A		As above			
Light cone gauge	N/A		N/A		As above			
Simple surrogate	N/A		N/A		$\frac{1}{4\pi\alpha'}\left(\dot{X}^I\dot{X}^I-X^{I'}\dot{X}^{I'}\right)$	(12.81) [248]		
World sheet fermions	N/A		N/A		$\mathcal{L}_\psi=\frac{1}{2\pi}\left\{\begin{array}{l}\psi_1^I\left(\partial_\tau+\partial_\sigma\right)\psi_1^I \\ +\psi_2^I\left(\partial_\tau-\partial_\sigma\right)\psi_2^I\end{array}\right\}$	(14.10) [310]		