Energy in a Superstring

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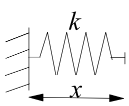
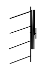
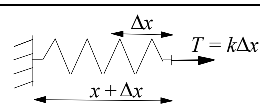
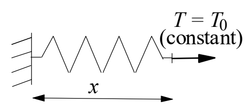
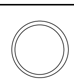
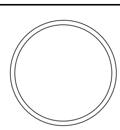
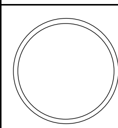
1 Tension Energy: Classical Strings vs Superstrings

1.1 Comparison

The tension force as modeled in a superstring is a bit weird by classical standards, as it does not depend (as a classical elastic string would) on how much the string has been stretched. The tension in the superstring is the same constant value for any amount of stretching. It is even more weird in that the initial length of the superstring is considered to be zero. This means the potential energy in the string is directly proportional to the length of the string (unlike a classical elastic string). This is summarized below.

	Classical String	**Superstring**	**Comment**
Open			
Unstretched	 Elastic string modeled as spring		Superstring considered to have zero length unstretched, unlike classical elastic string.
Stretched			Classical force proportional to stretch Δx. Superstring force constant for any stretch, and stretch is x (unlike classical string Δx).
Potential Energy	$E = V = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}T\Delta x$	$E = V = T_0 x$	Classical tension (potential) energy proportional to $(\Delta x)^2$; superstring proportional to x.
Tension Force	$T = \frac{dV}{dx} = k\Delta x$ (varies with Δx)	$T = \frac{dV}{dx} = T_0$ (constant)	Classical tension grows with stretch Δx; superstring is constant with stretch x.
Closed			
Unstretched	 Radius R	○ Zero radius	Superstring considered to have zero circumference unstretched, unlike classical rubber band.
Stretched	 $T = k\Delta C$ $\Delta C = 2\pi \Delta R$	 $T = T_0$ (constant)	Classical force proportional to stretch ΔC. Superstring force constant for any stretch, and stretch is C (unlike rubber band ΔC).
Potential Energy	$E = V = \frac{1}{2}k(\Delta C)^2 = \frac{1}{2}T\Delta C$	$E = V = T_0 C$	Classical tension (potential) energy proportional to $(\Delta C)^2$; Superstring energy proportional to C.
Tension Force	$T = \frac{dV}{dC} = k\Delta C$ (varies with ΔC)	$T = \frac{dV}{dC} = T_0$ (constant)	Classical tension grows with stretch ΔC; superstring is constant with stretch C.

1.2 Superstring Loops and Tension Energy

We define the quantity α' and express string energy in terms of it and the radius of a superstring circular loop.

$$\text{Tension } E_{1loop} = 2\pi TR \qquad \frac{1}{\alpha'} \equiv 2\pi T \quad \Rightarrow \quad \text{Tension } E_{1loop} = \frac{R}{\alpha'} \tag{1}$$

α' reflects the amount of tension in the string. It is proportional to the inverse of the tension. For multiple wrappings (loops) of the string

$$\text{Tension } E = \frac{mR}{\alpha'} \qquad m = \text{ number of wrappings ("wrapping, or winding, number")} . \tag{2}$$

Note the units.

$$T\,(\text{tension force})\text{ units }(\text{from Newton's law})\;\frac{ml}{s^2}\;\xrightarrow[\text{units}]{\text{natural}}\;\frac{1}{l^2}\text{ or } M^2 \tag{3}$$

So, from (1), α' has units of M^{-2} (or l^2, as it is sometimes taken as cm^2).

2 Loops and Vibration Energy (= Kinetic Energy)

Quantized energy is

$$\text{Kinetic } E = \hbar\omega = \hbar\frac{2\pi c}{\lambda}\,(\text{massless}) \qquad \xrightarrow[\text{units}]{\text{natural}}\;\frac{1}{l}\text{ or } M^1 . \tag{4}$$

The lowest (first or fundamental) mode of vibration has wavelength $\lambda = \lambda_1$ equal to $C = 2\pi R$ in (4), giving, in natural units,

$$\text{Kinetic } E_{\substack{lowest\,mode\\ 1\,wrapping}} = \frac{1}{R} . \tag{5}$$

For more wrappings with the same wavelength (one circumference), kinetic energy would not change.

$$\text{Kinetic } E_{\substack{lowest\,mode\\ m\;wrappings.\\ same\;\lambda_1}} = \frac{1}{R} . \tag{6}$$

But, we also have higher vibration modes to consider, where the shorter wavelength means higher kinetic energy.

$$\lambda = \lambda_n = \frac{\lambda_1}{n} \qquad \text{Kinetic } E_{\substack{nth\,mode\\ m\;wrappings.\\ same\;\lambda}} = \frac{n}{R} \qquad n = \text{mode number} \tag{7}$$

So, greater wrapping number increases potential (tension) energy via (2), and higher modes increase kinetic (vibration) energy via (7). They both contribute to the total energy of the string, which is

$$E_{\substack{m\;wrappings,\\ nth\;mode}} = \text{Tension } E + \text{Vibration } E = \frac{mR}{\alpha'} + \frac{n}{R} \qquad \text{for given fundamental mode } \lambda_1 . \tag{8}$$

If a massless superstring has energy E with its wrapping around dimensions in a higher dimensional compactified space, then to us in the non-compactified 4D spacetime, it looks to have (rest) mass

$$m = \frac{E}{c^2} \tag{9}$$

3 Ramifications

Note that one term in (8) has R in the numerator, and one has R in the denominator. Consider one case where (we will drop subscripting on E)

$$E = \frac{mR}{\alpha'} + \frac{n}{R} == \frac{10R}{1} + \frac{1}{R} \qquad \frac{m}{\alpha'} = 10 \qquad n = 1 . \tag{10}$$

For small R = .01 this has large kinetic energy of 100 units, but small potential energy of 0.1 units with a total energy of 100.1 units.

Now, consider the case where R = 10. This has small kinetic energy 0.1 and large kinetic energy of 100, for the same total of 100.1 units.

One superstring has large radius; the other, small, but the string energy is the same, and via (9), they both manifest in our world as having the same mass, i.e., being the same particle, at least effectively.

Suppose one of these cases can be solved perturbatively, but the other not. We can get the same prediction for mass either way, so we take the easy way, the perturbative way, and the answer is good for both cases. A very large radius string wrapped in Calabi-Yau space has identical eigenvalues (like energy) as a very small radius string.

We can thus solve an intractable problem by solving a tractable equivalent problem. The answers to the easy problem are answers to the hard one. As one no doubt knows, this is a great boon.

This is called a duality. There are dual systems, which are different, yet effectively the same. In this case, it is a T-duality, where T stands for "target space", whose meaning is a topic for another day. I do seem to recall some saying the T stands for "topological", as compactified space is characterized by its topology, which relates to how strings can be wrapped around openings within it.

Caveat: This treatment is simplified. There is a bit more to calculating wound string energy beyond the scope of this discussion.