Electroweak Symmetry Breaking Overview

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The following material is a schematic overview of the Higgs mechanism and its role in electroweak symmetry breaking. Unfortunately, I have not had time to provide a number of details, but since I am leaving on vacation shortly for a month, I felt it best to post this on my book website, as is, before I go. Though not yet complete, it may nevertheless help some in their study of the subject. At some point in the future, there will be more material added.

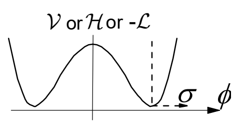
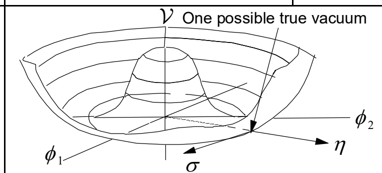
It would therefore be best to treat this overview as a supplement to the text you are using to study electroweak theory. I believe Mandl and Shaw's *Quantum Field Theory* is the best (read "easiest, most efficient") book for a new learner. The notation used herein parallels that of Mandl and Shaw.

Note that some of the wholeness charts presented are my hand-written notes from when I was studying this material many years ago. When I have time, I will type these out. But, I hope that as they are, they are legible and helpful.

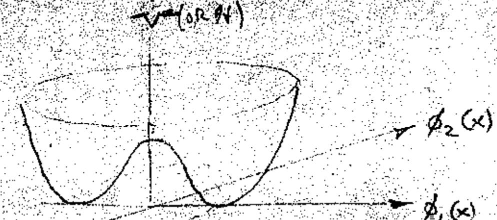
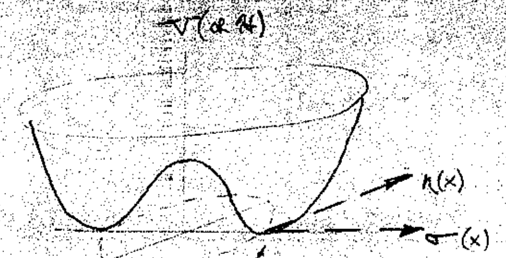
The wholeness chart summarizing three different models of symmetry breaking comes first. Then, the treatment of each model has a wholeness chart for that model first, then typed out backup material for that chart following.

Take caution that no one (including myself) has checked this material carefully for errata. So if something doesn't seem right to you, there is a good chance it is not and needs correcting. If you find such things, or have helpful suggestions to offer, please let me know via the email address in the "Feedback" link on the book website.

Electroweak Symmetry Breaking

	Single Real Scalar	**Complex Scalar Singlet (Isoscalar)**		**Complex Scalar Field Doublet (Isospinor)**	
		Goldstone Model	**Higgs Model**	**(Not treated)**	**Weinberg/Salam**
Higgs Field	$\phi(x) = \text{real}$	$\phi(x) = \phi_1 + i\phi_2$	as at left		$\underset{\sim}{\phi}(x) = \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{bmatrix}$
Symmetry Transform Type	Global reflection $\mathcal{L}(\phi) = \mathcal{L}(-\phi)$	Global U(1) (i.e., no interactions)	Local U(1) (i.e., interactions)	Global SU(2) X U(1)	Local SU(2) X U(1)
$\mathcal{V}(\phi)$ [or $\mathcal{H}(\phi)$] (same symmetry as $\mathcal{L}$)			as at left		Similar in 5D (4 ϕ_i,1 $\mathcal{V}$) to Goldstone. Have true vac at min $\mathcal{V}$. Choose $\sigma_i = \phi_{i0}$
Symmetry Transformation	$\phi(x) \to \phi(-x)$ (discrete)	$\delta\phi = i\alpha\phi$ (infinitesimal) (or $\phi' = e^{i\alpha}\phi$ finite)	$\delta\phi = i\alpha(x)\phi$ $\delta A^\mu = -\frac{1}{e}\partial^\mu\alpha(x)$ $[\& \not\partial \to \not D \text{ in } \mathcal{L}_0]$		$\delta\Phi = \frac{i}{2}\omega_i(x)\tau_i\Phi + ig'(x)Yf\Phi$ $\delta B^\mu = -\frac{1}{g'}\partial^\mu f$ $\delta W_i^\mu = -\frac{1}{g'}\partial^\mu W_i^\mu - \varepsilon_{ijk}\omega_j W_k^\mu$ $[\& \not\partial \to \not D \text{ in } \mathcal{L}_0]$
Degs of Freedom, False Vacuum	1 (ϕ)	2 (ϕ_1, ϕ_2)	4 (ϕ_1, ϕ_2, 2 massless A^μ polarization states) $m_1 = m_2 \quad m_1^2 < 0$		12 (ϕ_1, ϕ_2, ϕ_3, ϕ_4, 2 massless B^μ, 2 each for 3 massless W_i^μ)
New Bosons = Lin Combins of Old					$B^\mu, W_i^\mu \to W_\pm^\mu, Z^\mu, A^\mu$ Still 12 DOFs, still massless
Degs of Freedom, False Vacuum	1 (σ)	2 (σ, η) $m_\eta = 0 \quad m_\sigma^2 > 0$	5 (σ,η, 3 from massive A^μ) 1 non physical field $m_\eta = 0 \quad m_\sigma^2 > 0$		15 (1 each from σ_1,σ_2,σ_3,σ_4, 9 from massive $W_\pm^\mu, Z^\mu$, 2 from massless A^μ) 3 non physical fields
Unitary Gauge			Eliminate one DOF by choosing $\eta = 0$		Get what is observe physically by choosing σ_2, σ_3, $\sigma_4 = 0$. Eliminate 3 DOFs.
Result of Unitary Gauge			1 massive Higgs σ 1 massive A^μ 4 DOFs		1 massive Higgs σ 3 massive $W_\pm^\mu, Z^\mu$ 1 massless A^μ 12 DOFs
Renormalization					Easier to prove by choosing diff gauge where σ_2, σ_3, $\sigma_4 \neq 0$. This results in unphysical (ghost) fields we don't observe. Ghost fields are linear combins of the 12 quantities above

GOLDSTONE MODEL

	AT FALSE VACUUM		AT TRUE VACUUM	
PICTORIALLY	V (OR $\mathcal{H}$)  Note: Univ could be [illegible] at $t=0$, but as expands & cools evolves to [illegible] shape.		V (or $\mathcal{H}$)  COMPLEX REP ϕ CHOSEN AT $\phi_{10} = \frac{v}{\sqrt{2}}$, $\phi_{20}=0$ $\frac{1}{\sqrt{2}}$ FOR CONVENIENCE. Real rep $\phi_{10} = v$	
	COMPLEX REP	REAL REP	COMPLEX REP	REAL REP
HIGGS FIELD, $\phi(x)$ (2 D.O.F.)	$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$	$\underset{\sim}{\phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$	$\Phi = \phi - \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\phi_1 - v + i\phi_2)$ $= \frac{1}{\sqrt{2}}(\sigma(x) + i\eta(x))$	$\underset{\sim}{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 - v\\ \phi_2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sigma\\ \eta\end{pmatrix}$
$\mathcal{L}$	$\partial^\mu\phi^*\partial_\mu\phi - \mu^2\phi^2 - \lambda\phi^4$	$\frac{1}{2}(\partial^\mu\phi_1)^2 + \frac{1}{2}(\partial^\mu\phi_2)^2 - \frac{\mu^2}{2}(\phi_1^2+\phi_2^2)$ $- \frac{\lambda}{4}(\phi_1^2+\phi_2^2)^2$	CAN PLUG & CHUG FROM FAR LEFT, BUT REAL REP IS MORE ILLUMINATING	$\frac{1}{2}(\partial^\mu\sigma)^2 - \frac{1}{2}(2\lambda v^2)\sigma^2 + \frac{1}{2}(\partial^\mu\eta)^2$ } ACTS LIKE $\mathcal{L}_0$ $-\lambda v\sigma[\sigma^2+\eta^2]$ $-\frac{1}{4}\lambda[\sigma^2+\eta^2]^2$ } ACTS LIKE $\mathcal{L}_I$
$\mathcal{H}$	$\underbrace{\partial^0\phi^*\partial_0\phi + D\phi^* D\phi}_{=0 \text{ FOR MIN } \mathcal{H}} + V(\phi)$	$= V$ FOR MIN $\mathcal{H}$		$= V$ FOR MIN $\mathcal{H}$
V	$\mu^2\phi^2 + \lambda\phi^4$ $\therefore$ SYMS OF V ARE SYMS OF $\mathcal{H}$ & $\mathcal{L}$	$\mu^2\phi_1^2 + \mu^2\phi_2^2 + \lambda(\phi_1^2+\phi_2^2)^2$ $\therefore$ AS AT LEFT		V = NON GRADIENT TERMS ABOVE V ($\mathcal{H}$ & $\mathcal{L}$) NOT SYM FN OF σ & η (BROKEN SYM)
CLASSICAL INTERP	CONTINUOUS CLASS FIELDS UNSTABLE.	AT LEFT		CONTIN CLASS FIELD STABLE (OR NOT UNSTABLE IN η DIREC) SMALL OSCILLS DEPEND ON COEF σ^2
Q.M. INTERP.	∞ (?) PARTICLE STATE (ϕ positive state) VEV $\langle 0\|\phi\|0\rangle = 0$			SMALL CHANGES IN σ, η $\Rightarrow$ PARTICLES CREATED ALSO VEV $\langle 0\|\Phi\|0\rangle \neq 0$
HIGGS MASSES	FOR SMALL ϕ $\frac{\delta\mathcal{L}}{\delta\phi} = \frac{\delta V}{\delta\phi^2} = -\mu^2$ (coef of ϕ^2) $\mu^2 < 0$ (IF $\mu^2 > 0$ [illegible] SHAPE)	FOR SMALL ϕ_1, ϕ_2 $\frac{\delta\mathcal{L}}{\delta\phi_1^2} = \frac{\delta V}{\delta\phi_1^2} = -\mu^2$ (coef of ϕ_1^2 & ϕ_2^2) $\mu^2 < 0$ $m_1 = m_2$		FOR SMALL σ, η $\frac{\delta\mathcal{L}}{\delta\sigma^2}$ = coef of σ^2 $\frac{\delta\mathcal{L}}{\delta\eta^2}$ = coef of η^2 $m_\sigma^2 > 0$ $m_\eta^2 = 0$
SLOPE OF V	SLOPING DOWNWARD $\Rightarrow$ IMAGINARY MASS TERM		UPWARD (σ DIREC) $\Rightarrow$ REAL POS MASS LEVEL (η DIREC) $\Rightarrow$ ZERO MASS	
DEGENERACY	FOR V (& $\mathcal{L}$ & $\mathcal{H}$) STATIONARY, A UNIQUE STATE $\phi = 0$. $\therefore$ NO DEGENERACY	AS AT LEFT EXCEPT $\underset{\sim}{\phi} = \begin{pmatrix}0\\0\end{pmatrix}$		FOR V (& $\mathcal{H}$) MIN ($\mathcal{L}$ MAX), MANY (∞) POSSIBLE STATES Φ. $\therefore$ DEGENERACY OF VACUUM (ALTHOUGH ONCE UNIVERSE SETTLES IN A VAC IT STAYS THERE.)
SYM TRANSF ($\delta\mathcal{L} = \delta V = 0$)	$U(1)\phi = e^{i\alpha}\phi$ ← (FINITE) → ROTATION IN COMPLEX SPACE AT $\phi = 0$ $U(1)\phi = 0$ (AT $\phi \neq 0$ $U(1)\phi \neq 0$) } $\delta\mathcal{L} = 0$ $\delta\phi = i\alpha\phi$ ← (SMALL) →	$U(1)\underset{\sim}{\phi} = [R]\underset{\sim}{\phi} = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix}$ ROTATION IN REAL PLANE AT $\underset{\sim}{\phi} = \begin{pmatrix}0\\0\end{pmatrix}$ $U(1)\underset{\sim}{\phi} = 0$ (AT $\underset{\sim}{\phi} \neq \begin{pmatrix}0\\0\end{pmatrix}$ $U(1)\underset{\sim}{\phi} \neq 0$) } $\delta\mathcal{L} = 0$ $\delta\underset{\sim}{\phi} = \begin{pmatrix}\alpha\phi_2\\ -\alpha\phi_1\end{pmatrix} = 0$ IF $\phi_1 = \phi_2 = 0$		SAME TRANSF AT $\underset{\sim}{\Phi} = \begin{pmatrix}0\\0\end{pmatrix}$ (OR $\underset{\sim}{\phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}v\\0\end{pmatrix}$) IS INITIALLY IN η DIREC $\delta\underset{\sim}{\Phi} = \begin{pmatrix}0\\ \alpha\frac{v}{\sqrt{2}}\end{pmatrix} \neq 0$ } BUT $\delta\mathcal{L} = 0$
CONCLUSIONS FOR $\alpha\phi = 0$ & $\delta\mathcal{L} = 0$	AT $\phi = 0$ $\alpha\phi = \delta\phi = 0$ (MORE GENERALLY $T_a\underset{\sim}{\phi} = 0$) $\Rightarrow$ ① NO DEGENERACY OF ϕ ② V HAS SLOPE & PARTICLE HAS MASS (imag) ③ SYM UNBROKEN [V SYM IN ϕ_1, ϕ_2] ④ GENERATOR ANNILS VAC $\alpha\phi = 0$ ⑤ VEV OF Field $= 0$ ← Typo =		AT $\underset{\sim}{\phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}v\\0\end{pmatrix}$ $\alpha\phi = \delta\phi \neq 0 \Rightarrow \delta\underset{\sim}{\Phi} \neq 0 \Rightarrow$ ① DEGENERACY OF $\underset{\sim}{\phi}$ ② V HAS NO SLOPE IN DIREC OF TRANSF & NO MASS FOR FIELD IN THAT DIREC ③ SYM BROKEN [V NOT SYM IN σ, η] ④ GENERATOR α DOES NOT ANNIH VACUUM, $\alpha\phi \neq 0$ ⑤ VEV OF Field $\neq 0$	
GOLDSTONE THM	FOR GEN'L CASE $\alpha \to \epsilon_a T_a$ } FOR ANY GLOBAL SYM OF $\mathcal{L}$ ($\delta\mathcal{L} = 0$ FOR $\epsilon_a T_a\phi$) WHICH IS NOT REALIZED IN THE SPECTRUM OF PHYS STATES (TRUE VAC) (i.e. broken) THERE MUST BE i) A MASSLESS SCALAR PARTICLE ii) DEGENERACY OF PHYS STATES (VAC) iii) CORRESPONDING GENERATOR DOES NOT ANNIH VAC ($T_a\Phi \neq 0$ FOR TRUE (REALIZED) VACUUM, i.e. $\delta\Phi \neq 0$.) ← i.e. vac has non zero eigenvalue / same as saying vac has VEV			

Goldstone Model True Vacuum Calculation

Robert D. Klauber July 3, 2017

The Goldstone model is based on

1) a complex scalar Higgs field ϕ, and
2) no interactions (i.e., a Lagrangian density with global $U(1)$ symmetry).

$$\begin{aligned}\mathcal{L} &= \partial^\mu \phi^\dagger \partial_\mu \phi \quad - \mu^2 \phi^2 \quad - \lambda \phi^4 \qquad \text{Complex } \phi = \tfrac{1}{\sqrt{2}}\left(\phi_1 + i\phi_2\right) \quad \phi_1, \phi_2 \text{ real} \\ &= \tfrac{1}{2}\left(\partial^\mu \phi_1\right)^2 + \tfrac{1}{2}\left(\partial^\mu \phi_2\right)^2 \underbrace{- \tfrac{\mu^2}{2}\left(\phi_1^2 + \phi_2^2\right) - \tfrac{\lambda}{4}\left(\phi_1^2 + \phi_2^2\right)^2}_{-\mathcal{V}}\end{aligned} \tag{1}$$

where we are using the symbolism $\phi^2 = \phi^\dagger \phi$.

The minimum of $\mathcal{V}$ for $\phi_2 = 0$ is located at a value for ϕ_1 denoted by the symbol v. This allows us to find an expression relating μ and v, as follows.

$$\left.\frac{\partial \mathcal{V}}{\partial \phi_1}\right|_{\substack{\phi_1 = v \\ \phi_2 = 0}} = 0 = \left(-\mu^2 \phi_1 - \tfrac{\lambda}{4} 2\left(\phi_1^2 + \phi_2^2\right) 2\phi_1\right)_{\substack{\phi_1 = v \\ \phi_2 = 0}} \quad \rightarrow \quad \mu^2 = -\lambda \phi_1^2\Big|_{\phi_1 = v} = -\lambda v^2 < 0 \ , \tag{2}$$

Thus, the "mass" μ of ϕ (for λ positive) is imaginary.

So, we define new fields

$$\sigma = \phi_1 - v \qquad \eta = \phi_2 \qquad \rightarrow \qquad \phi_1 = \sigma + v \qquad \phi_2 = \eta \tag{3}$$

and substitute the RHS (3) into the potential $\mathcal{V}$ in (1) to get

$$\begin{aligned}\mathcal{V} &= \tfrac{\mu^2}{2}\left(\phi_1^2 + \phi_2^2\right) + \tfrac{\lambda}{4}\left(\phi_1^2 + \phi_2^2\right)^2 = \tfrac{\mu^2}{2}\left(\sigma + v\right)^2 + \tfrac{\mu^2}{2}\eta^2 + \tfrac{\lambda}{4}\left(\left(\sigma + v\right)^2 + \eta^2\right)^2 \\ &= \tfrac{\mu^2}{2}\left(\sigma^2 + \eta^2\right) + \mu^2 \sigma v + \tfrac{\mu^2}{2} v^2 + \tfrac{\lambda}{4}\left(\sigma^2 + 2\sigma v + v^2 + \eta^2\right)^2 \ .\end{aligned} \tag{4}$$

With the RHS of (2), this becomes

$$\mathcal{V} = -\tfrac{\lambda v^2}{2}\left(\sigma^2 + \eta^2\right) - \lambda v^3 \sigma - \tfrac{\lambda v^4}{2} + \tfrac{\lambda}{4}\left(\sigma^2 + 2\sigma v + v^2 + \eta^2\right)^2 \tag{5}$$

Completing the square at the end of (5), we find

$$\begin{aligned}\mathcal{V} = \lambda\Bigg(-\overset{\boxed{1}}{\frac{v^2\sigma^2}{2}} - \overset{\boxed{2}}{\frac{v^2\eta^2}{2}} \overset{\boxed{3}}{- v^3\sigma} - \frac{v^4}{2} \quad &+ \tfrac{1}{4}\sigma^4 + \tfrac{1}{4}\sigma^2 2\sigma v + \overset{\boxed{1a}}{\tfrac{1}{4}\sigma^2 v^2} + \tfrac{1}{4}\sigma^2\eta^2 \\ &+ \tfrac{1}{4}\sigma^2 2\sigma v + \tfrac{1}{4}4\sigma^2 v^2 + \overset{\boxed{3a}}{\tfrac{1}{4}2\sigma v v^2} + \tfrac{1}{4}2\sigma v\eta^2 \\ &+ \overset{\boxed{1a}}{\tfrac{1}{4}\sigma^2 v^2} + \overset{\boxed{3b}}{\tfrac{1}{4}2\sigma v v^2} + \tfrac{1}{4}v^4 + \overset{\boxed{2a}}{\tfrac{1}{4}\eta^2 v^2} \\ &+ \tfrac{1}{4}\sigma^2\eta^2 + \tfrac{1}{4}2\sigma v\eta^2 + \overset{\boxed{2b}}{\tfrac{1}{4}v^2\eta^2} + \tfrac{1}{4}\eta^4 \Bigg).\end{aligned} \tag{6}$$

The terms with the same numbers (e.g., 1,2,3) in boxes above them cancel. So, we are left with (7) (where the terms with letters over them have the same form and can be combined)

$$
\begin{aligned}
\mathcal{V} = {} & \overset{\boxed{A}}{-\tfrac{\lambda}{2}v^4} + \tfrac{\lambda}{4}\sigma^4 + \overset{\boxed{B}}{\tfrac{\lambda}{2}\sigma^3 v} + \overset{\boxed{C}}{\tfrac{\lambda}{4}\sigma^2\eta^2} + \overset{\boxed{B}}{\tfrac{\lambda}{2}\sigma^3 v} + \lambda\sigma^2 v^2 + \overset{\boxed{D}}{\tfrac{\lambda}{2}\sigma v\eta^2} \\
& + \overset{\boxed{A}}{\tfrac{\lambda}{4}v^4} + \overset{\boxed{C}}{\tfrac{\lambda}{4}\sigma^2\eta^2} + \overset{\boxed{D}}{\tfrac{\lambda}{2}\sigma v\eta^2} + \tfrac{\lambda}{4}\eta^4 \\
= {} & \lambda\left(\overset{\boxed{A}}{-\frac{v^4}{4}} + \frac{\sigma^4}{4} + \overset{\boxed{B}}{\sigma^3 v} + \overset{\boxed{C}}{\frac{\sigma^2\eta^2}{2}} + \overset{\boxed{D}}{\sigma v\eta^2} + \sigma^2 v^2 + \frac{\eta^4}{4} \right).
\end{aligned}
\tag{7}
$$

Rearranging (7), we have

$$
\begin{aligned}
\mathcal{V} = {} & \lambda\left(\overset{\boxed{A}}{-\frac{v^4}{4}} + \sigma^2 v^2 + \tfrac{1}{4}\left(\overset{\boxed{B}}{\sigma^3 v} + \overset{\boxed{D}}{\sigma v\eta^2} \right) + \tfrac{1}{4}\left(\sigma^4 + \overset{\boxed{C}}{2\sigma^2\eta^2} + \eta^4 \right) \right) \\
= {} & -\tfrac{1}{4}\lambda v^4 + \tfrac{1}{2}\left(2\lambda v^2\right)\sigma^2 + (\lambda v)\sigma\left(\sigma^2 + \eta^2\right) + \tfrac{\lambda}{4}\left(\sigma^2 + \eta^2\right)^2.
\end{aligned}
\tag{8}
$$

The first term in the last row of (8) is a constant, that represents a contribution to the energy density of the vacuum at the true vacuum, i.e., after symmetry breaking, by the scalar field. Note, from (9), that v in the $-\frac{1}{4}\lambda v^4$ term is the vacuum expectation value (VEV) of the field ϕ_1 at the true vacuum. (Typically in QFT, fields have zero VEVs, i.e., one cannot measure the field directly. In effect, the field destroys the vacuum, since it contains only construction and destruction operators acting alone and no number operators [bilinear combinations of operators].)

$$
\text{VEV of } \phi_1 = \langle 0_{true} | \phi_1 | 0_{true} \rangle = \langle 0_{true} | \sigma + v | 0_{true} \rangle = \langle 0_{true} | v | 0_{true} \rangle = v \tag{9}
$$

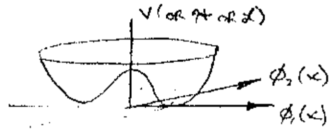
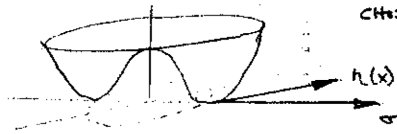
Thus, the vacuum energy density remnant left in the vacuum after symmetry breaking is proportional to the fourth power of this VEV, and linearly dependent on the constant λ.

However, much like what is done in other parts of QFT, we ignore constant vacuum energy, no matter how large (though this practice has yet to be fully justified by anyone). The other terms in the bottom row of (8) have been arranged so constants are on the left and fields on the right.

All of these terms will end up in the Lagrangian density, with opposite signs. There, the 2nd term in the bottom row of (8) acts like a mass term in the free Lagrangian for the σ field, where $2\lambda v^2$ is mass squared. The 3rd and 4th terms are not bilinear (as free terms in the Lagrangian are), but tri and quadrilinear and thus, represent interactions between fields (including the interaction of the σ with itself via σ^3 and σ^4 terms).

For bilinear terms of a certain field in $\mathcal{L}$, the field equation is linear in that field (via the Euler-Lagrange equation). For cubic or quartic terms in $\mathcal{L}$, the field equation is nonlinear.

HIGGS MODEL

	AT FALSE VACUUM		AT TRUE VACUUM	
PICTORIALLY				CHOSEN AT $\phi = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix}$ FOR CONVENIENCE
	COMPLEX REP	REAL REP	COMPLEX REP	REAL REP
HIGG'S FIELD	$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$	$\underset{\sim}{\phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$	$\Phi = \phi - \frac{v}{\sqrt{2}} = \frac{1}{\sqrt{2}}(\phi_1 - v + i\phi_2)$ $= \frac{1}{\sqrt{2}}(\sigma(x) + i\eta(x))$	$\underset{\sim}{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 - v \\ \phi_2\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\sigma \\ \eta\end{pmatrix}$
U(1) LOCAL SYM TRANSF	$\delta\phi = i\alpha(x)\phi$ ($\phi' = e^{i\alpha(x)}\phi$ FINITE) $\delta A_\mu = \partial_\mu \alpha(x)$ $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$	2D ROTATION IN ϕ_1, ϕ_2 SPACE		
$\mathcal{L}$	$[D^\mu\phi]^*[D_\mu\phi] - \mu^2\phi^2 - \cancel{\lambda}\phi^4$ $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (SYM FN OF ϕ & A_μ) NOTE: ϕ, ϕ^3 ARE NOT SYM. ϕ^5, ϕ^6.. MAKE $\mathcal{L}$ NOT RENORMALIZABLE. ∴ $\mathcal{L}$ HERE IS MOST GEN'L	$(D^\mu\phi_1)(D_\mu\phi_1) + (D^\mu\phi_2)(D_\mu\phi_2)$ $-\frac{\mu^2\phi_1^2}{2} - \frac{\mu^2\phi_2^2}{2} - \frac{\cancel{\lambda}}{4}(\phi_1^2+\phi_2^2)^2$ $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ (SYM FN OF ϕ_1, ϕ_2, A_μ)	REAL REP MORE ILLUMINATING UNSYM IN σ, η, A_μ (BROKEN SYM)	USING $D_\mu = \partial_\mu + iqA_\mu$ AND TRANSF TO σ, η COORDS, $\mathcal{L} =$ $\frac{1}{2}(\partial^\mu\sigma)(\partial_\mu\sigma) - \frac{1}{2}(2\lambda v^2)\sigma^2$ $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(qv)^2 A_\mu A^\mu$ $+\frac{1}{2}[\partial^\mu\eta][\partial_\mu\eta]$ $+ qvA^\mu\partial_\mu\eta$ $+ \underbrace{(\text{cubic \& quartic terms})}_{\text{A4's LIKE INTERACTIONS}}$
DIFFICULTIES WITH FORM OF $\mathcal{L}$				PRODUCT TERM $A^\mu\partial_\mu\eta$ SHOWS A^μ & η NOT INDEP NORMAL COORDS & CANNOT CONCLUDE THAT 2ND & 3RD LINES IN $\mathcal{L}$ ABOVE DESCRIBE MASSIVE VEC BOSON A_μ & MASSLESS SCALAR BOSON η.
FIXING UP W/ UNITARY GAUGE			9/03 like having fields real →	NOTE, $\mathcal{L}_{false}$ HAS 4 DEGS OF FREEDOM. $\mathcal{L}_{TRUE}$ HAS 5 ($\underset{(\sigma)}{1} + \underset{(\eta)}{1} + \underset{(A_\mu)}{3} = 5$). CHANGE OF VARS CANNOT ALTER D.O.F.'S. ∴ MUST BE A NON PHYS FIELD IN $\mathcal{L}_{TRUE}$. UNITARY GAUGE ⇒ TAKE $\eta = 0$ ∴ RESTORES 4 D.O.F.
$\mathcal{L}$ UNDER UNITARY GAUGE				$\left.\begin{array}{l}\frac{1}{2}[\partial^\mu\sigma][\partial_\mu\sigma] - \frac{1}{2}(2\lambda v^2)\sigma^2 \\ -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(qv)^2 A_\mu A^\mu\end{array}\right\} \mathcal{L}_0$ $\left.\begin{array}{l}-\lambda v\sigma^3 - \frac{1}{4}\lambda\sigma^4 \\ +\frac{1}{2}q^2 A_\mu A^\mu[2v\sigma + \sigma^2]\end{array}\right\} \mathcal{L}_I$
RESULTS ON QUANTIZATION	AT FALSE VAC: 1 COMPLEX SCALAR FIELD ϕ 1 MASSLESS REAL " A_μ			FOR SMALL σ & A_μ $m_\sigma = \sqrt{2\lambda v^2}$ $m_{A_\mu} = \lvert ev \rvert$ AT TRUE VAC: 1 REAL MASSIVE SCALAR FIELD, σ (HIGGS BOSON) 1 " " VECTOR "
NOTE:				GLOBAL SYM → MASSLESS GOLDSTONE BOSON η & A_μ LOCAL " → MASSIVE A_μ & NO η 1 MASSLESS DOF (η IN GLOBAL CASE) SWITCHED (TO A_μ IN LOCAL CASE) MAKING A_μ MASSIVE

Higgs Mechanism True Vacuum Calculation

Robert D. Klauber July 3, 2017

The difference between the Goldstone and Higgs models is simply that the latter includes interactions, as shown in Wholeness Chart 1 below.

	Goldstone Model	Higgs Model
Higgs fields	complex scalar ϕ	same as at left
Interactions?	no (global $U(1)$ sym of $\mathcal{L}$)	yes (local $U(1)$ sym of $\mathcal{L}$)

Wholeness Chart 1. Difference Between Goldstone and Higgs Models

$$\begin{aligned}\mathcal{L} &= \left|D_\mu\phi\right|^2 \quad -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad -\mu^2\phi^2 \quad -\lambda\phi^4 \qquad \phi=\tfrac{1}{\sqrt{2}}\left(\phi_1+i\,\phi_2\right)\ \ \phi_1,\phi_2 \text{ real}\\ &=\left(D^\mu\phi\right)^\dagger\left(D^\mu\phi\right)-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\underbrace{-\tfrac{\mu^2}{2}\left(\phi_1^2+\phi_2^2\right)-\tfrac{\lambda}{4}\left(\phi_1^2+\phi_2^2\right)^2}_{-\mathcal{V}}\end{aligned} \tag{1}$$

where we are using the symbolism $\phi^2=\phi^\dagger\phi$.

As with Goldstone model, minimum of $\mathcal{V}$ for $\phi_2 = 0$ is located at a positive value for ϕ_1 denoted by the symbol v. This allows us to find an expression relating μ and v, as follows.

$$\left.\frac{\partial\mathcal{V}}{\partial\phi_1}\right|_{\substack{\phi_1=v\\ \phi_2=0}}=0=\left(-\mu^2\phi_1-\tfrac{\lambda}{4}2\left(\phi_1^2+\phi_2^2\right)2\phi_1\right)_{\substack{\phi_1=v\\ \phi_2=0}} \quad\rightarrow\quad \mu^2=-\lambda\phi_1^2\Big|_{\phi_1=v}=-\lambda v^2<0\ , \tag{2}$$

Thus, the "mass" μ of ϕ is imaginary.

<u>$\mathcal{V}$ of $\mathcal{L}$ expressed via new fields σ and η</u>

So, just as we did in the Goldstone model, we define new fields

$$\sigma=\phi_1-v \qquad \eta=\phi_2 \qquad\rightarrow\qquad \phi_1=\sigma+v \qquad \phi_2=\eta \tag{3}$$

and with (3) into the potential $\mathcal{V}$ in (1), we get the same $\mathcal{V}$ we had in the Goldstone case,

$$\mathcal{V}=-\tfrac{1}{4}\lambda v^4 \ +\tfrac{1}{2}\left(2\lambda v^2\right)\sigma^2 \ +\left(\lambda v\right)\sigma\left(\sigma^2+\eta^2\right) \ +\tfrac{\lambda}{4}\left(\sigma^2+\eta^2\right)^2 . \tag{4}$$

<u>Kinetic terms in $\mathcal{L}$ expressed via new fields σ and η</u>

With the gauge covariant derivative definition,

$$D^\mu=\partial^\mu+iqA^\mu\ , \tag{5}$$

we can calculate the derivative terms in $\mathcal{L}$ of (1) in terms of ϕ_1 and ϕ_2,

$$\begin{aligned}\left|D_\mu\phi\right|^2 &=\left(\partial^\mu\phi+iqA^\mu\phi\right)^\dagger\left(\partial_\mu\phi+iqA_\mu\phi\right)=\left(\partial^\mu\phi^\dagger-iqA^\mu\phi^\dagger\right)\left(\partial_\mu\phi+iqA_\mu\phi\right)\\ &=\left(\partial^\mu\phi^\dagger\right)\partial_\mu\phi+iqA_\mu\left(\partial^\mu\phi^\dagger\right)\phi-iqA^\mu\phi^\dagger\partial_\mu\phi+q^2A^\mu A_\mu\phi^\dagger\phi\\ &=\frac{1}{2}\left[\begin{aligned}&\left(\partial^\mu\left(\phi_1-i\phi_2\right)\right)\partial_\mu\left(\phi_1+i\phi_2\right)+iqA_\mu\left(\partial^\mu\left(\phi_1-i\phi_2\right)\right)\left(\phi_1+i\phi_2\right)\\ &\qquad -iqA^\mu\left(\phi_1-i\phi_2\right)\partial_\mu\left(\phi_1+i\phi_2\right)+q^2A^\mu A_\mu\left(\phi_1-i\phi_2\right)\left(\phi_1+i\phi_2\right)\end{aligned}\right]\end{aligned} \tag{6}$$

$$=\frac{1}{2}\left[\begin{array}{l}\left(\partial^\mu\phi_1\right)^2+\left(\partial^\mu\phi_2\right)^2+\overset{\boxed{1}}{iqA_\mu\left(\partial^\mu\phi_1\right)\phi_1}-\overset{\boxed{A}}{qA_\mu\left(\partial^\mu\phi_1\right)\phi_2}+\overset{\boxed{B}}{qA_\mu\left(\partial^\mu\phi_2\right)\phi_1}+\overset{\boxed{2}}{iqA_\mu\left(\partial^\mu\phi_2\right)\phi_2}\\ \qquad -\overset{\boxed{1}}{iqA_\mu\phi_1\partial^\mu\phi_1}+\overset{\boxed{B}}{qA_\mu\phi_1\partial^\mu\phi_2}-\overset{\boxed{A}}{qA_\mu\phi_2\partial^\mu\phi_1}-\overset{\boxed{2}}{iqA_\mu\phi_2\partial^\mu\phi_2}+q^2A^\mu A_\mu\phi_1^2+q^2A^\mu A_\mu\phi_2^2\end{array}\right]. \tag{7}$$

The terms with numbers over them cancel. The terms with letters over them are equal if ϕ_1 and ϕ_2 commute, and they do because they are different fields (and also because they are real, so for any construction/destruction operator $a(\mathbf{k})$ in them, $a(\mathbf{k})=a^\dagger(\mathbf{k})$, so $\left[a(\mathbf{k}),a^\dagger(\mathbf{k})\right]=\left[a(\mathbf{k}),a(\mathbf{k})\right]=0$. This leaves us with

$$\left|D_\mu\phi\right|^2=\tfrac{1}{2}\left[\left(\partial^\mu\phi_1\right)^2+\left(\partial^\mu\phi_2\right)^2-2qA_\mu\left(\partial^\mu\phi_1\right)\phi_2+2qA_\mu\left(\partial^\mu\phi_2\right)\phi_1+q^2A^\mu A_\mu\phi_1^2+q^2A^\mu A_\mu\phi_2^2\right]. \tag{8}$$

With the RHS of (3), and realizing v is a constant. (8) becomes

$$\begin{aligned}\tfrac{1}{2}\left|D^\mu\phi\right|^2&=\tfrac{1}{2}\left(\partial^\mu\sigma\right)\left(\partial^\mu\sigma\right)+\tfrac{1}{2}\left(\partial^\mu\eta\right)\left(\partial_\mu\eta\right)-qA_\mu\left(\partial^\mu\sigma\right)\eta+qA_\mu\left(\partial^\mu\eta\right)(\sigma+v)\\&\qquad+\tfrac{q^2}{2}A^\mu A_\mu(\sigma+v)^2+\tfrac{q^2}{2}A^\mu A_\mu\eta^2\\&=\tfrac{1}{2}\left(\partial^\mu\sigma\right)\left(\partial^\mu\sigma\right)+\tfrac{1}{2}\left(\partial^\mu\eta\right)\left(\partial_\mu\eta\right)-qA_\mu\left(\partial^\mu\sigma\right)\eta+qA_\mu\left(\partial^\mu\eta\right)\sigma+qvA_\mu\left(\partial^\mu\eta\right)\\&\qquad+\tfrac{q^2}{2}A^\mu A_\mu\sigma^2+q^2vA^\mu A_\mu\sigma+\tfrac{q^2}{2}v^2A^\mu A_\mu+\tfrac{q^2}{2}A^\mu A_\mu\eta^2.\end{aligned} \tag{9}$$

Re-arranging so the bilinear (quadratic in fields) terms come first, we have

$$\begin{aligned}\tfrac{1}{2}\left|D^\mu\phi\right|^2&=\tfrac{1}{2}\left(\partial^\mu\sigma\right)\left(\partial^\mu\sigma\right)+\tfrac{1}{2}\left(\partial^\mu\eta\right)\left(\partial_\mu\eta\right)+qvA_\mu\left(\partial^\mu\eta\right)\\&\quad-qA_\mu\left(\partial^\mu\sigma\right)\eta+qA_\mu\left(\partial^\mu\eta\right)\sigma+\tfrac{q^2}{2}A^\mu A_\mu\sigma^2+q^2vA^\mu A_\mu\sigma+\tfrac{q^2}{2}v^2A^\mu A_\mu+\tfrac{q^2}{2}A^\mu A_\mu\eta^2.\end{aligned} \tag{10}$$

<u>$\mathcal{L}$ expressed via new fields σ and η</u>

With (4) and (10), the last row of (1) becomes

$$\begin{aligned}\mathcal{L}=&\tfrac{1}{2}\left(\partial^\mu\sigma\right)\left(\partial^\mu\sigma\right)+\tfrac{1}{2}\left(\partial^\mu\eta\right)\left(\partial_\mu\eta\right)+qvA_\mu\left(\partial^\mu\eta\right)-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+qvA_\mu\left(\partial^\mu\eta\right)\\&-qA_\mu\left(\partial^\mu\sigma\right)\eta+qA_\mu\left(\partial^\mu\eta\right)\sigma+\tfrac{q^2}{2}A^\mu A_\mu\sigma^2+q^2vA^\mu A_\mu\sigma+\tfrac{q^2}{2}v^2A^\mu A_\mu+\tfrac{q^2}{2}A^\mu A_\mu\eta^2\\&\underbrace{+\tfrac{1}{4}\lambda v^4-\tfrac{1}{2}\left(2\lambda v^2\right)\sigma^2-(\lambda v)\sigma\left(\sigma^2+\eta^2\right)-\tfrac{\lambda}{4}\left(\sigma^2+\eta^2\right)^2}_{-\mathcal{V}}.\end{aligned} \tag{11}$$

Dropping the constant term $\frac{1}{4}\lambda v^4$, as is the usual (but yet to be fully justified by anyone) practice in QFT, and re-arranging, (11) becomes

$$\begin{aligned}\mathcal{L}=&\underbrace{\tfrac{1}{2}\left(\partial^\mu\sigma\right)\left(\partial^\mu\sigma\right)-\tfrac{1}{2}\left(2\lambda v^2\right)\sigma^2-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{q^2v^2}{2}A^\mu A_\mu+\tfrac{1}{2}\left(\partial^\mu\eta\right)\left(\partial_\mu\eta\right)}_{=\mathcal{L}_0\ \text{(bilinear terms, with no mixed products, represent free fields)}}\ \underbrace{+\,qvA_\mu\left(\partial^\mu\eta\right)}_{\text{interaction of }A_\mu\text{ and }\eta}\\&\underbrace{-qA_\mu\left(\partial^\mu\sigma\right)\eta+qA_\mu\left(\partial^\mu\eta\right)\sigma-(\lambda v)\sigma\left(\sigma^2+\eta^2\right)-\tfrac{\lambda}{4}\left(\sigma^2+\eta^2\right)^2+\tfrac{q^2}{2}A^\mu A_\mu\sigma^2+q^2vA^\mu A_\mu\sigma+\tfrac{q^2}{2}A^\mu A_\mu\eta^2}_{\text{cubic and quartic terms in the fields, represent interactions}}.\end{aligned} \tag{12}$$

<u>The Unitary Gauge</u>

Employing the unitary gauge $\eta = 0$, we end up with

$$\begin{aligned}\mathcal{L}=&\tfrac{1}{2}\left(\partial^\mu\sigma\right)\left(\partial^\mu\sigma\right)-\tfrac{1}{2}\left(2\lambda v^2\right)\sigma^2-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}+\tfrac{(qv)^2}{2}A^\mu A_\mu\quad\Big\}\,\mathcal{L}_0\\&\qquad-(\lambda v)\sigma^3-\tfrac{\lambda}{4}\sigma^4+\tfrac{q^2}{2}A^\mu A_\mu\sigma^2+q^2vA^\mu A_\mu\sigma\quad\Big\}\,\mathcal{L}_I.\end{aligned} \tag{13}$$

HIGGS MECHANISM IN WEINBERG/SALAM ELECTROWEAK THEORY

(PURE LEPTONIC PROCESSES ONLY)

	AT FALSE VACUUM: COMPLEX REP	AT FALSE VACUUM: REAL REP	AT TRUE VACUUM: COMPLEX REP	AT TRUE VACUUM: REAL REP
HIGGS FIELD (ISOS/W DOUBLET)	$\phi(x) = \begin{pmatrix}\phi_a(x)\\ \phi_b(x)\end{pmatrix} = \begin{pmatrix}\phi_1 + i\phi_2\\ \phi_3 + i\phi_4\end{pmatrix}$	$\underset{\sim}{\phi} = \begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\\ \phi_4\end{pmatrix}$	$\Phi = \begin{pmatrix}\eta_1 + i\eta_2\\ \sigma + i\eta_3\end{pmatrix}$ WHERE $\eta_1 = \phi_1/\sqrt{2}$, $\eta_2 = \phi_2/\sqrt{2}$, $\sigma = (\phi_3 - v)/\sqrt{2}$, $\eta_3 = \phi_4/\sqrt{2}$	$\underset{\sim}{\Phi} = \begin{pmatrix}\eta_1\\ \eta_2\\ \sigma\\ \eta_3\end{pmatrix} = \begin{pmatrix}\phi_1/\sqrt{2}\\ \phi_2/\sqrt{2}\\ (\phi_3 - v)/\sqrt{2}\\ \phi_4/\sqrt{2}\end{pmatrix}$ $v/\sqrt{2}$ IS MIN POINT OF V, FOUND BY $\frac{\partial V}{\partial \phi_3} = 0$ TO BE $\sqrt{\frac{-\mu^2}{2\lambda}}$

$\mathcal{L}$

At false vacuum, complex rep:

$\mathcal{L} = \mathcal{L}^L + \mathcal{L}^B + \mathcal{L}^H + \mathcal{L}^{LH}$ (MASSLESS)

$\mathcal{L}^L = i\left[\bar{\Psi}^L_\ell \not{D} \Psi^L_\ell + \bar{\Psi}^R_\ell \not{D}\Psi^R_\ell + \bar{\Psi}^R_{\nu_\ell}\not{D}\Psi^R_{\nu_\ell}\right]$

$\mathcal{L}^B = -\frac{1}{4} W_a^{\mu\nu} W_{a\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$

$\mathcal{L}^H = (D^\mu\phi)^\dagger (D_\mu\phi) - \mu^2 \phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2$

$\mathcal{L}^{LH} = -g_\ell \bar{\Psi}^R_\ell \phi^\dagger \Psi^L_\ell + \text{h.c.}$
$\quad - g_{\nu_\ell} \bar{\Psi}^R_{\nu_\ell} \tilde{\phi}^\dagger \Psi^L_{\nu_\ell} + \text{h.c.}$ } ASSUMES NEUTRINO MASS

Yukawa couplings to yield lepton masses

At false vacuum, real rep: USE ALGEBRA ON STUFF AT LEFT TO OBTAIN $\mathcal{L}$ IN TERMS OF ϕ_i REAL

D^μ subst here gives rise to $\frac{1}{2}\left(\frac{vg}{2}\right)^2 W_{\mu\pm} W^\mu_\pm$ terms when ϕ takes a VEV. Due to non-Abelian $T_a T_b I$ mult

At true vacuum, complex rep: NOT SO ILLUMINATING

At true vacuum, real rep:

FOR $\mathcal{L}_{TRUE}$:

1. SUBSTIT $\phi_1 = \sqrt{2}\eta_1$, $\phi_2 = \sqrt{2}\eta_2$, $\phi_3 = \sqrt{2}\sigma + v$, $\phi_4 = \sqrt{2}\eta_3$ IN FAR LEFT $\mathcal{L}_{FALSE}$ (THIS TRANSLATES AXES TO V MIN)
2. SUBSTITUTE $W_3^\mu = A^\mu \sin\theta_W + Z^\mu \cos\theta_W$, $B^\mu = A^\mu\cos\theta_W - Z^\mu \sin\theta_W$, $W_1 = \frac{W^+ + W^-}{\sqrt{2}}$, $W_2 = \frac{i(W^+ - W^-)}{\sqrt{2}}$

COULD DO IN EITHER ORDER

3. RESULT: COMPLICATED $\mathcal{L}_{TRUE}$ WITH (LIKE IN HIGGS MODEL)
   i) NON INDEP NORMAL FIELDS
   ii) MORE DEGS OF FREEDOM THAN $\mathcal{L}_{FALSE}$ (3 MORE) $\to \therefore$ NON PHYS FIELDS
4. TO RESOLVE, USE UNITARY GAUGE BELOW

SYM TRANSF LOCAL U(2)xU(1)

At false vacuum:

$\delta\phi = i(\epsilon_a(x) T_a + \alpha(x) S)\phi$

$\delta X^\mu = -\frac{1}{g'}\partial^\mu \alpha$

$\delta \underset{\sim}{W}^\mu = -\frac{1}{g}\partial^\mu \underset{\sim}{\epsilon} + i\left[\underset{\sim}{\epsilon}, \underset{\sim}{W}^\mu\right]$

$\delta \Psi^{L,R}_{\ell\,(\nu_\ell)} = i(\epsilon_a(x) T_a + \alpha(x) S)\Psi^{L,R}_\ell$

$\therefore \mathcal{L}$ SYM IN ABOVE FIELDS (UNBROKEN SYM)

At true vacuum: $\mathcal{L}$ NOT SYM IN η_1, η_2, η_3 (BROKEN SYM)

UNITARY GAUGE

At true vacuum: CHOOSING $\eta_1 = \eta_2 = \eta_3 = 0 \Rightarrow \mathcal{L}_{TRUE}$ THEN EXPRESSED IN TERMS OF INDEP NORMAL FIELDS WITH 3 FEWER D.O.F = SAME NUM D.O.F. AS $\mathcal{L}_{FALSE}$. ALSO ALL FIELDS EXPRESSED AS INDEP & NORMAL.

$\mathcal{L}_{TRUE}$ IN UNITARY GAUGE

VIA (1), (2), (4) ABOVE: $\mathcal{L}$ BECOMES

$$\mathcal{L}_0 \begin{cases} \bar{\Psi}_\ell\left(i\not{\partial} - \frac{v g_\ell}{\sqrt{2}}\right)\Psi_\ell + \bar{\Psi}_{\nu_\ell}\left(i\not{\partial} - \frac{v g_{\nu_\ell}}{\sqrt{2}}\right)\Psi_{\nu_\ell} & (\leftarrow \text{LEPTONS}) \\ -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} & (\leftarrow \text{PHOTONS}) \\ -\frac{1}{2}(\partial^\mu W^\nu_\pm - \partial^\nu W^\mu_\pm)(\partial_\mu W_{\nu\pm} - \partial_\nu W_{\mu\pm}) + \frac{1}{2}\left(\frac{1}{2}vg\right)^2 W_{\mu\pm}W^\mu_\pm & (\leftarrow W \text{ PARTICLES}) \\ -\frac{1}{4} Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}\left(\frac{vg}{2\cos\theta_W}\right)^2 Z_\mu Z^\mu & (\leftarrow Z\text{'s}) \\ +\frac{1}{2}(\partial^\mu\sigma)(\partial_\mu\sigma) - \frac{1}{2}(-2\mu^2)\sigma^2 & (\leftarrow \text{HIGGS}, \sigma) \end{cases}$$

$$\mathcal{L}_I \left\{ + \mathcal{L}_I^{LB} + \mathcal{L}_I^{BB} + \mathcal{L}_I^{HH} + \mathcal{L}_I^{HB} + \mathcal{L}_I^{HL} \quad (\leftarrow \text{HIGHER ORDER INTERACTION TERMS}) \right.$$

DETERMINING MASSES

At true vacuum: BY INSPECTION OF $\mathcal{L}_0$ ABOVE

$m_\ell = \frac{v g_\ell}{\sqrt{2}}$ $\quad m_{\nu_\ell} = \frac{v g_{\nu_\ell}}{\sqrt{2}}$ $\quad m_\gamma = 0$

$m_{W_\pm} = \frac{1}{2} v g$ $\quad m_Z = \frac{m_W}{\cos\theta_W}$ $\quad m_H = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$

NOTE:

At false vacuum: $-\mu^2$ SHOWS UP IN $\mathcal{L}_I^{HH}$ & CAN ONLY BE MEASURED IN HIGGS SELF COUPLING

At true vacuum:

1. GAGE INVAR OF A^μ UNBROKEN, $\therefore m_\gamma = 0$ IN BOTH CASES
2. CAN FIND $v = (G_F\sqrt{2})^{-1/2}$ & HENCE GET m_W & m_Z IN TERMS OF e, G_F, & θ_W.
3. KNOWING m_ℓ (m_{ν_ℓ}) CAN FIND COUPLING g_ℓ (g_{ν_ℓ}). IF $m_{\nu_\ell} = 0$, NEUTRINO NOT COUPLED TO HIGGS FIELD.
4. CAN'T PREDICT m_H FROM KNOWN DATA. m_H MUST BE FOR RENORMALIZABILITY. HIGHER ORDER CORRECTION THEORY $\to$ 7 GEV $< m_H < 10^3$ GEV

Weinberg Salam Mechanism True Vacuum Calculation

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1 Weinberg/Salam Model vs Goldstone and Higgs Models

The Weinberg/Salam model is based on

1) a complex scalar Higgs field <u>doublet</u> $\Phi(x)=\begin{bmatrix}\phi_a\\ \phi_b\end{bmatrix}=\begin{bmatrix}\phi_1+i\phi_2\\ \phi_3+i\phi_4\end{bmatrix}$ $\quad\phi_1,\phi_2,\phi_3,\phi_4$ real (1)

2) a Lagrangian density with local U(2) X U(1) symmetry.

The difference between the Goldstone, Higgs, and Weinberg/Salam models is shown in Wholeness Chart 2 below.

	<u>Goldstone Model</u>	<u>Higgs Model</u>	<u>Weinberg/Salam Model</u>
Higgs fields	complex scalar singlet ϕ	same as at left	complex scalar doublet ϕ
Interactions?	No (global U(1) sym of $\mathcal{L}$)	Yes (local U(1) sym of $\mathcal{L}$)	Yes (local U(2) X U(1) sym of $\mathcal{L}$)
Normalization?	$\phi=\frac{1}{\sqrt{2}}\left(\phi_1+i\phi_2\right)$	Same as at left	$1/\sqrt{2}$ factor left out of Φ definition (see (1))

Wholeness Chart 2. Difference Between Goldstone, Higgs, and Weinberg/Salam Models

2 Lagrangian at False Vacuum

At the false vacuum, the total Lagrangian in terms of the ϕ field doublet, the three high energy W_i^μ SU(2) vector boson fields (i = 1,2,3), the one high energy U(1) vector boson B^μ field, and the high energy lepton fields, where all fields are massless (needed for local symmetry to hold and thus, for renormalization) is (where we define symbols further below)

$$
\begin{aligned}
\mathcal{L}=\mathcal{L}^L+\mathcal{L}^B+\mathcal{L}^H+\mathcal{L}^{LH}\qquad &\left(LH=\text{Lepton-Higgs coupled terms}\right)\\
\mathcal{L}^L&=i\left(\bar{\Psi}_l^L\not{D}\Psi_l^L+\bar{\psi}_l^R\not{D}\psi_l^R+\bar{\psi}_{\nu_l}^R\not{D}\psi_{\nu_l}^R\right)\\
\mathcal{L}^B&=-\frac{1}{4}G_i^{\mu\nu}G_{i\,\mu\nu}-\frac{1}{4}B^{\mu\nu}B_{\mu\nu}\\
\mathcal{L}^H&=\left(D^\mu\Phi\right)^\dagger\left(D_\mu\Phi\right)-\mu^2\Phi^\dagger\Phi-\lambda\left(\Phi^\dagger\Phi\right)^2\\
\mathcal{L}^{LH}&=-g_l\left(\bar{\Psi}_l^L\psi_l^R\Phi+\Phi^\dagger\bar{\psi}_l^R\Psi_l^L\right)-g_{\nu_l}\left(\bar{\Psi}_{\nu_l}^L\psi_{\nu_l}^L\tilde{\Phi}+\tilde{\Phi}^\dagger\bar{\psi}_{\nu_l}^R\Psi_{\nu_l}^L\right)
\end{aligned}
\tag{2}
$$

<u>For $\mathcal{L}^L$:</u>

There are three lepton families (l = 1,2,3 for e, μ, τ). g is the coupling constant between the leptons and the W_i^μ ; g' is the coupling constant between the leptons and B^μ. The τ_j are 2X2 Pauli matrices.

$$
\Psi_l^L=\begin{pmatrix}\psi_{\nu_l}^L\\ \psi_l^L\end{pmatrix}\qquad \bar{\Psi}_l^L=\left(\bar{\psi}_{\nu_l}^L,\bar{\psi}_l^L\right)
\tag{3}
$$

$$\not{D}\Psi_l^L = \gamma_\mu D^\mu \Psi_l^L = \gamma_\mu\left(\partial^\mu + \tfrac{i}{2} g\tau_j W_j^\mu + ig'YB^\mu\right)\Psi_l^L$$
$$\not{D}\psi_l^R = \gamma_\mu D^\mu \psi_l^R = \gamma_\mu\left(\partial^\mu + ig'YB^\mu\right)\psi_l^R \qquad (4)$$
$$\not{D}\psi_{\nu_l}^R = \gamma_\mu D^\mu \psi_{\nu_l}^R = \gamma_\mu\left(\partial^\mu + ig'YB^\mu\right)\psi_{\nu_l}^R$$

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \left[\tau_i, \tau_j\right] = 2i\varepsilon_{ijk}\tau_k \qquad (5)$$

$$\text{weak hypercharge } Y = -\tfrac{1}{2} \text{ for } \Psi_l^L \qquad -1 \text{ for } \psi_l^R \qquad 0 \text{ for } \psi_{\nu_l}^R \qquad (6)$$

For $\mathcal{L}^B$:

In parallel with a similar relation from QED, where $F^{\mu\nu} = \partial^\nu A^\nu - \partial^\mu A^\nu$,

$$G_i^{\mu\nu} = \underbrace{\partial^\nu W_i^\nu - \partial^\mu W_i^\nu}_{F_{W\,i}^{\mu\nu}} + g\varepsilon_{ijk} W_j^\mu W_k^\nu \qquad B^{\mu\nu} = \partial^\nu B^\nu - \partial^\mu B^\nu . \qquad (7)$$

Note above, that the LHS expression for $G_i^{\mu\nu}$ is the most general form, since for a U(1) generated gauge boson like A^μ (or B^μ), there is only one such boson (i=j=k) so the ε_{ijk} will equal zero.

For $\mathcal{L}^H$:

$$D^\mu\Phi = \left(\partial^\mu + \tfrac{i}{2} g\tau_j W_j^\mu + ig'YB^\mu\right)\Phi \qquad Y = \tfrac{1}{2} \text{ for } \Phi \qquad (8)$$

For $\mathcal{L}^{HB}$:

g_l and g_{ν_l} are the coupling constants between the Higgs field and the three lepton families (l = 1,2,3 for e, μ, τ), and

$$\tilde{\Phi} = \begin{pmatrix} \phi_b^* \\ -\phi_a^* \end{pmatrix} = \begin{bmatrix} \phi_3 - i\phi_4 \\ -\phi_1 + i\phi_2 \end{bmatrix} \qquad (9)$$

3 Why This Form for $\mathcal{L}$?

The Lagrangian takes the form (2) because that form is symmetric. Recall from QED, that if the Lagrangian is symmetric under some set of local transformations of its fields, then the interactions that show up in that particular Lagrangian mirror those in the real world. See Klauber[1], pgs. 293-298.

The transformation set under which (2) is symmetric (invariant) is shown below. We will not show how this is so here. Perhaps another day.

3.1 Local Finite Transformations

3.1.1 SU(2)

$$\Psi_l^L(x) \to \Psi_l^{L\prime} = SU(2)\Psi_l^L = e^{i\omega_i(x)\tau_i/2}\Psi_l^L = \left(1 + \tfrac{i}{2}\omega_i(x)\tau_i + ...\right)\Psi_l^L = \Psi_l^L + \delta\Psi_l^L$$
$$\bar{\Psi}_l^L(x) \to \bar{\Psi}_l^{L\prime} = \bar{\Psi}_l^L\left(SU(2)\right)^\dagger = \bar{\Psi}_l^L e^{-i\omega_i(x)\tau_i/2} = \bar{\Psi}_l^L\left(1 - \tfrac{i}{2}\omega_i(x)\tau_i + ...\right) = \bar{\Psi}_l^L + \delta\bar{\Psi}_l^L \qquad (10)$$
$$\psi_l^R(x) \to \psi_l^{R\prime} = \psi_l^R$$
$$\bar{\psi}_l^R(x) \to \bar{\psi}_l^{R\prime} = \bar{\psi}_l^R$$

$$W_i^\mu(x) \to W_i^{\mu\prime} = \text{ complicated for finite case (see infinitesimal case) } = W_i^\mu + \delta W_i^\mu \qquad (11)$$

$$B^{\mu} \to B^{\mu\prime} = B^{\mu} \tag{12}$$

$$\begin{aligned} \Phi(x) \to \Phi' &= e^{i\omega_i(x)\tau_i/2}\Phi = \left(1+\tfrac{i}{2}\omega_i(x)\tau_i + ...\right)\Phi = \Phi + \delta\Phi \\ \tilde{\Phi}(x) \to \tilde{\Phi}' &= e^{i\omega_i(x)\tau_i/2}\tilde{\Phi} = \left(1+\tfrac{i}{2}\omega_i(x)\tau_i + ...\right)\tilde{\Phi} = \tilde{\Phi} + \delta\tilde{\Phi} \end{aligned} \tag{13}$$

3.1.2 U(1)

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l^{L\prime} &= U(1)\Psi_l^L = e^{ig'Yf(x)}\Psi_l^L = \left(1+ig'Yf(x)+....\right)\Psi_l^L = \Psi_l^L + \delta\Psi_l^L \\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l^{L\prime} &= \bar{\Psi}_l^L\left(U(1)\right)^{\dagger} = \bar{\Psi}_l^L e^{-ig'Yf(x)} = \bar{\Psi}_l^L\left(1-ig'Yf(x)+....\right) = \bar{\Psi}_l^L + \delta\bar{\Psi}_l^L \\ \psi_l^R(x) \to \psi_l^{R\prime} &= U(1)\psi_l^R = e^{ig'Yf(x)}\psi_l^R = \left(1+ig'Yf(x)+....\right)\psi_l^R = \psi_l^R + \delta\psi_l^R \\ \bar{\psi}_l^R(x) \to \bar{\psi}_l^{R\prime} &= \bar{\psi}_l^R\left(U(1)\right)^{\dagger} = \bar{\psi}_l^R e^{-ig'Yf(x)} = \bar{\psi}_l^R\left(1-ig'Yf(x)+....\right) = \bar{\psi}_l^R + \delta\bar{\psi}_l^R \end{aligned} \tag{14}$$

$$W_i^{\mu}(x) \to W_i^{\mu\prime} = \text{ complicated for finite case (see infinitesimal case) } = W_i^{\mu} + \delta W_i^{\mu} \tag{15}$$

$$B^{\mu} \to B^{\mu\prime} = B^{\mu} - \partial^{\mu} f \tag{16}$$

$$\begin{aligned} \Phi(x) \to \Phi' &= e^{ig'Yf(x)}\Phi = \left(1+ig'Yf(x)+...\right)\Phi = \Phi + \delta\Phi \\ \tilde{\Phi}(x) \to \tilde{\Phi}' &= e^{-ig'Yf(x)}\tilde{\Phi} = \left(1-ig'Yf(x)+...\right)\tilde{\Phi} = \tilde{\Phi} + \delta\tilde{\Phi} \end{aligned} \tag{17}$$

3.2 Local Infinitesimal Transformations

It is far easier to examine the invariance of $\mathcal{L}$ for infinitesimal transformations, i.e., for the above where ω_i and f are very small. One can then, in principle, integrate to obtain the finite transformation case. But, if $\mathcal{L}$ is symmetric under the infinitesimal transformation, then it should also be so under the finite transformation. So, we take the easy (most efficient) way out, and examine the Lagrangian symmetry under the infinitesimal transformation set corresponding to the finite set (10) through (17). The relevant relations are then as follows.

3.2.1 SU(2)

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l^{L\prime} &\approx \left(1+\tfrac{i}{2}\omega_i(x)\tau_i\right)\Psi_l^L = \Psi_l^L + \delta\Psi_l^L & \delta\Psi_l^L &= \tfrac{i}{2}\omega_i(x)\tau_i\Psi_l^L \\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l^{L\prime} &\approx \bar{\Psi}_l^L\left(1-\tfrac{i}{2}\omega_i(x)\tau_i\right) = \bar{\Psi}_l^L + \delta\bar{\Psi}_l^L & \delta\bar{\Psi}_l^L &= -\tfrac{i}{2}\omega_i(x)\bar{\Psi}_l^L\tau_i \\ \psi_l^R(x) \to \psi_l^{R\prime} &= \psi_l^R & \delta\psi_l^R &= 0 \\ \bar{\psi}_l^R(x) \to \bar{\psi}_l^{R\prime} &= \bar{\psi}_l^R & \delta\bar{\psi}_l^R &= 0 \end{aligned} \tag{18}$$

$$W_i^{\mu}(x) \to W_i^{\mu\prime} \approx W_i^{\mu} - \partial^{\mu}\omega_i = W_i^{\mu} + \delta W_i^{\mu} \qquad \delta W_i^{\mu} = -\partial^{\mu}\omega_i \tag{19}$$

$$B^{\mu} \to B^{\mu\prime} = B^{\mu} \qquad \delta B^{\mu} = 0 \tag{20}$$

$$\begin{aligned} \Phi(x) \to \Phi' &\approx \left(1+\tfrac{i}{2}\omega_i(x)\tau_i\right)\Phi = \Phi + \delta\Phi & \delta\Phi &= \tfrac{i}{2}\omega_i(x)\tau_i\Phi \\ \tilde{\Phi}(x) \to \tilde{\Phi}' &\approx \left(1+\tfrac{i}{2}\omega_i(x)\tau_i\right)\tilde{\Phi} = \tilde{\Phi} + \delta\tilde{\Phi} & \delta\tilde{\Phi} &= \tfrac{i}{2}\omega_i(x)\tau_i\tilde{\Phi} \end{aligned} \tag{21}$$

3.2.2 U(1)

$$\begin{aligned} \Psi_l^L(x) \to \Psi_l^{L\prime} &\approx \left(1+ig'Yf(x)\right)\Psi_l^L = \Psi_l^L + \delta\Psi_l^L & \delta\Psi_l^L &= ig'Yf(x)\Psi_l^L \\ \bar{\Psi}_l^L(x) \to \bar{\Psi}_l^{L\prime} &\approx \bar{\Psi}_l^L\left(1-ig'Yf(x)\right) = \bar{\Psi}_l^L + \delta\bar{\Psi}_l^L & \delta\bar{\Psi}_l^L &= -ig'Yf(x)\bar{\Psi}_l^L \\ \psi_l^R(x) \to \psi_l^{R\prime} &\approx \left(1+ig'Yf(x)\right)\psi_l^R = \psi_l^R + \delta\psi_l^R & \delta\psi_l^R &= ig'Yf(x)\psi_l^R \\ \bar{\psi}_l^R(x) \to \bar{\psi}_l^{R\prime} &\approx \bar{\psi}_l^R\left(1-ig'Yf(x)\right) = \bar{\psi}_l^R + \delta\bar{\psi}_l^R & \delta\bar{\psi}_l^R &= -ig'Yf(x)\bar{\psi}_l^R \end{aligned} \tag{22}$$

$$W_i^\mu(x) \to W_i^{\mu\prime} \approx W_i^\mu - g\varepsilon_{ijk}\omega_j W_k^\mu = W_i^\mu + \delta W_i^\mu \qquad \delta W_i^\mu = -g\varepsilon_{ijk}\omega_j W_k^\mu \tag{23}$$

$$B^\mu \to B^{\mu\prime} = B^\mu - \partial^\mu f \qquad \delta B^\mu = -\partial^\mu f \tag{24}$$

$$\begin{aligned} &\Phi(x) \to \Phi' \approx \left(1 + ig'Yf\right)\Phi = \Phi + \delta\Phi && \delta\Phi = ig'Yf\,\Phi \\ &\tilde{\Phi}(x) \to \tilde{\Phi}' \approx \left(1 - ig'Yf\right)\tilde{\Phi} = \tilde{\Phi} + \delta\tilde{\Phi} && \delta\tilde{\Phi} = -ig'Yf\,\tilde{\Phi} \end{aligned} \tag{25}$$

3.3 Local SU(2)XU(1) Infinitesimal Transformations

Under SU(2)XU(1) transformation combining the local, infinitesimal SU(2) and U(1) transformations, the differences in the fields (the terms with δ in front of them) add. Thus,

$$\begin{aligned} &\Psi_l^L \to \Psi_l^{L\prime} \approx \left(1 + \tfrac{i}{2}\omega_i\tau_i + ig'Yf\right)\Psi_l^L = \Psi_l^L + \delta\Psi_l^L && \delta\Psi_l^L = \tfrac{i}{2}\omega_i\tau_i\Psi_l^L + ig'Yf\Psi_l^L \qquad Y = -\tfrac{1}{2} \\ &\bar{\Psi}_l^L \to \bar{\Psi}_l^{L\prime} \approx \bar{\Psi}_l^L\left(1 - \tfrac{i}{2}\omega_i\tau_i - ig'Yf\right) = \bar{\Psi}_l^L + \delta\bar{\Psi}_l^L && \delta\bar{\Psi}_l^L = -\tfrac{i}{2}\omega_i\bar{\Psi}_l^L\tau_i - ig'Yf\bar{\Psi}_l^L \\ &\psi_l^R \to \psi_l^{R\prime} = \psi_l^R + ig'Yf\psi_l^R && \delta\psi_l^R = ig'Yf\psi_l^R \qquad Y = -1 \\ &\bar{\psi}_l^R \to \bar{\psi}_l^{R\prime} = \bar{\psi}_l^R - ig'Yf && \delta\bar{\psi}_l^R = -ig'Yf\bar{\psi}_l^R \end{aligned} \tag{26}$$

$$W_i^\mu \to W_i^{\mu\prime} \approx W_i^\mu - \partial^\mu W_i^\mu - g\varepsilon_{ijk}\omega_j W_k^\mu = W_i^\mu + \delta W_i^\mu \qquad \delta W_i^\mu = -\partial^\mu W_i^\mu - g\varepsilon_{ijk}\omega_j W_k^\mu \tag{27}$$

$$B^\mu \to B^{\mu\prime} = B^\mu - \partial^\mu f \qquad \delta B^\mu = -\partial^\mu f \tag{28}$$

$$\begin{aligned} &\Phi \to \Phi' \approx \left(1 + \tfrac{i}{2}\omega_i\tau_i + ig'Yf\right)\Phi = \Phi + \delta\Phi && \delta\Phi = \tfrac{i}{2}\omega_i\tau_i\Phi + ig'Yf\,\Phi \qquad Y = \tfrac{1}{2} \\ &\tilde{\Phi} \to \tilde{\Phi}' \approx \left(1 + \tfrac{i}{2}\omega_i\tau_i - ig'Yf\right)\tilde{\Phi} = \tilde{\Phi} + \delta\tilde{\Phi} && \delta\tilde{\Phi} = \tfrac{i}{2}\omega_i\tau_i\tilde{\Phi} - ig'Yf\,\Phi \end{aligned} \tag{29}$$

3.4 Showing $\mathcal{L}$ is Symmetric

Plugging the primed fields of (26) to (29) in for the unprimed fields in (2) should give us (2) back again in terms of unprimed fields. In other words, $\mathcal{L}$ of (2) is invariant under the set of transformations (26) to (29). As noted, we will not be doing this step-by-step at this time.

4 Returning to Breaking of Higgs Field Symmetry

We now turn back to our original focus (before justifying the form of $\mathcal{L}$ for the false vacuum as we did in Sect. 3) of breaking the Higgs field (in our present case represented by Φ) symmetry.

4.1 Higgs Fields in Terms of Other Real Fields

The Higgs field doublet Φ at false vacuum is represented in (1) (and (9)) by the real fields $\phi_1, \phi_2, \phi_3, \phi_4$. Just as we did earlier in the Goldstone and Higgs models, we examine only the potential part of the free Φ field of (2), i.e.,

$$\mathcal{V} = \mu^2\Phi^\dagger\Phi + \lambda\left(\Phi^\dagger\Phi\right)^2, \tag{30}$$

which is symmetric in Φ (and in $\phi_1, \phi_2, \phi_3, \phi_4$). We then, again similar to earlier procedures, designate new fields that are functions of the old ones and. That is, where η_1, η_2, σ, and η_3, are those new (real) fields, normalized by convention with a $\sqrt{2}$ factor,

$$\Phi(x) = \begin{bmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \eta_1 + i\eta_2 \\ (\sigma + v) + i\eta_3 \end{bmatrix} \quad . \tag{31}$$

It turns out (I am pressed for time right now, and hope to return to explain this in greater depth later) that when we substitute (31) into (2) we end up with 1) $\mathcal{L}$ being not symmetric in η_1, η_2, σ, and η_3, and 2) additional degrees of freedom (independent components of fields) in $\mathcal{L}$ from what we had in $\mathcal{L}$ for $\phi_1, \phi_2, \phi_3, \phi_4$. The additional degrees of freedom are three.

Recall that we cannot measure fields directly, and our measurables cannot be affected by how we designate our fields. That is, our fields are gauge fields. See Klauber[1], pgs 177-178. So, our having extra degrees we don't need means we are free to constrain those 3 extra degrees of freedom in any way we like. That is, we can designate a gauge condition for each, as is convenient, without affecting any measurables.

So, of course, we want to pick the gauge conditions that give us the simplest way to analyze the case at hand. That turns out to be what is called the unitary gauge, i.e.,

$$\eta_1 = 0 \qquad \eta_2 = 0 \qquad \eta_3 = 0 \qquad\qquad \text{unitary gauge .} \tag{32}$$

and thus,

$$\Phi(x) = \begin{bmatrix} 0 \\ \phi_3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ \sigma + v \end{bmatrix} \quad \text{in unitary gauge .} \tag{33}$$

4.2 Higgs Field at True Vacuum

To find the true vacuum ($\mathcal{V}$ is a minimum), we use (33) in (30) and take the derivative of (30) with respect to ϕ_3.

$$\begin{gathered} 0 = \frac{\partial \mathcal{V}}{\partial \phi_3} = \frac{\partial}{\partial \phi_3}\left(\mu^2 \Phi^\dagger \Phi + \lambda\left(\Phi^\dagger \Phi\right)^2\right) = \frac{\partial}{\partial \phi_3}\left(\mu^2 \phi_3^2 + \lambda \phi_3^4\right) = 2\mu^2 \phi_3 + 4\lambda \phi_3^3 \\ \rightarrow -\mu^2 = 2\lambda \phi_3^2 \qquad \rightarrow \phi_3 = \sqrt{\frac{-\mu^2}{2\lambda}} . \end{gathered} \tag{34}$$

The last expression in (34) is the location on the ϕ_3 axis where the Higgs field potential is a minimum.

With an eye to what comes later, we want to define σ as zero at the Higgs potential minimum. Thus in (33), we take

$$v = \sqrt{\frac{-\mu^2}{2\lambda}} \quad . \tag{35}$$

Note from (35), that if v is real (which is assumed, since we took ϕ_1 as positive at that location), then μ is imaginary. From the next to last line in (2), one might consider this the mass of the Φ field, which may seem strange in the context of how we normally think of mass.

5 One More Wrinkle in the Weinberg/Salam Model

One might now naively think (and early researchers probably did) that we simply need to plug (33) and (35) into (2), to get our electroweak theory at the true vacuum where we reside today. Not. When one does that, the result has different fields than we find in experiment. For one example, we get one vector field, which one might expect to be a photon, to have mass.

The answer was found by Steven Weinberg, and it is this. The B^μ and W_i^μ fields are really linear combinations of the experimentally detectable photon A^μ and intermediate vector bosons (weak gauge fields) Z^μ and $W_\pm^\mu$. That is, the former are transformations of the latter. The relation between the two sets of fields is given by

$$\begin{bmatrix} B_\mu \\ W_{3\mu} \end{bmatrix} = \begin{bmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{bmatrix} \begin{bmatrix} A_\mu \\ Z_\mu \end{bmatrix} \qquad W_{1\mu} = \frac{W_\mu + W_\mu^\dagger}{\sqrt{2}} \qquad W_2^\mu = i\frac{W_\mu - W_\mu^\dagger}{\sqrt{2}} , \tag{36}$$

where Weinberg modestly calls θ_W the weak mixing angle, but others call it the Weinberg mixing angle. Note that the LHS of (36) is much like a rotation in a real 2D space. Since the determinant of the matrix there is unity, from matrix theory, we know the "lengths" of the vectors on each side of the equal sign remain unchanged. Since, in QM, lengths of such vectors reflect probabilities, the total probability of measuring a component (any component at all, not a given component) in either vector is unchanged. This is the hallmark of a unitary transformation.

For future reference, we note, from the second two relations in (36),

$$W_\mu = \frac{W_{1\mu} - iW_2^\mu}{\sqrt{2}} \qquad W_\mu^\dagger = \frac{W_{1\mu} + iW_2^\mu}{\sqrt{2}} \tag{37}$$

and we define

$$F_W^{\mu\nu} = \partial^\nu W^\mu - \partial^\mu W^\nu \ . \tag{38}$$

6 Finally, the Lagrangian at True Vacuum

Now if we substitute (33), (35), *and* (36) into (2), we will get the present day electro-weak Lagrangian. I do not have time to do this step-by-step right now (but will one day), but the final result is, where

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_I \\
\mathcal{L}_0 &= \bar{\psi}_l \big(i\partial\!\!\!/ - \underbrace{\tfrac{vg_l}{\sqrt{2}}}_{m_l} \big)\psi_l + \bar{\psi}_{\nu_l}\big(i\partial\!\!\!/ - \underbrace{\tfrac{vg_{\nu_l}}{\sqrt{2}}}_{m_{\nu_l}} \big)\psi_{\nu_l} && (\text{leptons}) \\
&\quad -\tfrac{1}{4} F_{\mu\nu}F^{\mu\nu} && (\text{photons}) \\
&\quad -\tfrac{1}{2} F^\dagger_{W\,\mu\nu}F_W^{\mu\nu} + \underbrace{\left(\tfrac{1}{2}vg\right)^2}_{m_W^2}\left(W^\dagger_{+\mu}W_+^\mu + W^\dagger_{-\mu}W_-^\mu\right) && (W \text{ particles}) \\
&\quad -\tfrac{1}{4} Z_{\mu\nu}Z^{\mu\nu} + \tfrac{1}{2}\underbrace{\left(\tfrac{1}{2}\frac{vg}{cos\,\theta_W}\right)^2}_{m_Z^2} Z_\mu Z^\mu && (Z \text{ particle}) \\
&\quad +\tfrac{1}{2}\left(\partial_\mu\sigma\right)\left(\partial^\mu\sigma\right) - \tfrac{1}{2}\underbrace{\left(-2\mu^2\right)}_{m_H^2}\sigma^2 && (\text{Higgs} = \sigma) \\
\mathcal{L}_I &= \mathcal{L}^{LB} + \mathcal{L}^{BB} + \mathcal{L}^{HH} + \mathcal{L}^{HB} + \mathcal{L}^{HL} && (\text{interaction terms}),
\end{aligned} \tag{39}$$

where we will wait to another day to write down the interaction terms.

Note that for a *complex* boson field (W_+^μ and W_-^μ here) mass is, as has always been the case before, the square root of the factor in front of the bilinear expression in the Lagrangian of that single type of complex boson field. For a *real* boson field (Z^μ and σ here), mass is the square root of that same factor aside from a factor of ½. The photon field has no such term, meaning its mass is zero, one indication that the methodology used by Weinberg and Salam does indeed give us a valid theory.

So, we started in (2) with all massless fields (because massive fields mean the Lagrangian cannot be symmetric), and after symmetry breaking (of the Φ field, as easily seen in the Mexican hat figure, but also other massless fields), we now have fields with mass terms. The symmetry breaking bore the fruit of massive particles, as in our present day universe.

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[1] R. D. Klauber, *Student Friendly Quantum Field Theory* (Sandtrove Press 2013)