Distinguishable Particles, 2 Particle Systems

(Two Different Bosons, Two Different Fermions, or a Boson and a Fermion)

	Example	Not interacting with each other	Interacting with each other
Individual particle wave functions	At right, $1 = e^-$ and $2 = e^+$	$\Psi_1(1,t) = \Psi_1(\vec{r}_1,t)$ *and* $\Psi_2(2,t) = \Psi_2(\vec{r}_2,t)$	Can't write wave functions individually
Total sys wave function	As above		
Symbolically		$\Psi(1,2,t)$ *or* $\Psi(2,1,t)$ whichever we like.	$\Psi(1,2,t)$ *or* $\Psi(2,1,t)$ whichever we like.
Math expression		Can express by multiplying individual wave functions. $\Psi(1,2,t) = \Psi_1(1,t)\Psi_2(2,t)$ *or* $\Psi(2,1,t) = \Psi_1(2,t)\Psi_2(1,t)$	Cannot express in terms of individual wave functions multiplied. Must determine separately for each case using H_{total} and the Schroedinger eq.

Identical Particles, 2 Particle Systems

	Bosons	Fermions	Compare
Individual particle w.f. if not interacting	$\Psi_1(1,t) = \Psi_1(\vec{r}_1,t)$ *and* $\Psi_2(2,t) = \Psi_2(\vec{r}_2,t)$	$\Psi_1(1,t) = \Psi_1(\vec{r}_1,t)$ *and* $\Psi_2(2,t) = \Psi_2(\vec{r}_2,t)$	same
System w.f., pretending we can label particles	$\Psi(1,2,t)$ *or* $\Psi(2,1,t)$ whichever we like.	$\Psi(1,2,t)$ *or* $\Psi(2,1,t)$ whichever we like.	same
	For special case of not interacting with each other $\Psi(1,2,t) = \Psi_1(1,t)\Psi_2(2,t)$ $\Psi(2,1,t) = \Psi_1(2,t)\Psi_2(1,t)$	For special case of not interacting with each other $\Psi(1,2,t) = \Psi_1(1,t)\Psi_2(2,t)$ $\Psi(2,1,t) = \Psi_1(2,t)\Psi_2(1,t)$	same
Actual system w.f. where we can't label parts	$\Psi_+(1,2,t) = \Psi(1,2,t) + \Psi(2,1,t)$	$\Psi_-(1,2,t) = \Psi(1,2,t) - \Psi(2,1,t)$	different
Can identical particles be in same state?	Yes. If so, $\Psi_+(1,1,t) = 2\Psi(1,1,t)$ (We can normalize to get rid of the "2" factor.)	No. If so, $\Psi_-(1,1,t) = 0$ (And if there is no wave function, there are no particles.)	different
Particle exchange operator	$P_{12}\Psi_+(1,2,t) = \Psi_+(2,1,t)$	$P_{12}\Psi_-(1,2,t) = \Psi_-(2,1,t)$	same
Eigenvalue of particle exchange operator P_{12}	$P_{12}\Psi_+(1,2,t) = \Psi_+(2,1,t)$ $= \Psi(2,1,t) + \Psi(1,2,t)$ $= \Psi_+(1,2,t)$ *eigval* $= 1$	$P_{12}\Psi_-(1,2,t) = \Psi_-(2,1,t)$ $= \Psi(2,1,t) - \Psi(1,2,t)$ $= -\Psi_-(1,2,t)$ *eigval* $= -1$	different

by Robert D. Klauber www.quantumfieldtheory.info