

# Horizons in Cosmology

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Refs:

Davis, T.M., and Lineweaver, C.H., Expanding Confusion: common misconceptions of cosmological horizons and the superluminal expansion of the universe, <https://arxiv.org/abs/astro-ph/0310808>

Melia, F., The apparent (gravitational) horizon in cosmology, *Am. J. Phys.*, **86** (8), Aug 2018, pgs. 585-593. <https://arxiv.org/abs/1807.07587>

## Wholeness Chart in Words: Cosmological Horizons

<u>Type Horizon</u>	<u>Decelerating Universe</u>	<u>Accelerating Universe</u>	<u>Note</u>	<u>Value Now</u> ( <u>light-years</u> <u>in billions</u> )
Hubble sphere	$L_H$ = radius at which galaxy recession velocity = $c$	As at left	Function of time.	14.3
Particle horizon	$L_P$ = distance to farthest light we can see	As at left	Function of time, but common meaning is now (our present time). This = observable universe	46
Event horizon	Not exist for eternally slowing expansion.	$L_E$ = distance from us now at which we will never see light emitted from there now Also = distance from us now at which light we emit now will never be seen by any observer there. Also = distance now to the farthest location we could ever reach if we left now at speed of light.	Function of time (i.e., function of what time we take as “now”). Acceleration moves distant space away faster than light can reach it.	16

## Mathematics of Cosmological Horizons

### 1 The Universe’s Metric and Physical Distances

The Friedmann–Lemaître–Robertson–Walker metric for a homogeneous, isotropic universe is

$$ds^2 = -c^2 dt^2 + (a(t))^2 (d\chi^2 + S_k^2(\chi) d\psi^2) \quad (1)$$

where  $a(t)$  is the expansion factor of the universe at time  $t$ ,  $\chi$  is the comoving radial coordinate where we can take Earth at  $\chi = 0$ , present time is  $t_0$ ,  $a(t_0) = 1$ , and  $S_k$  depends on the curvature of the 3D spatial universe, i.e.

Postitive curvature,  $k = +1$ ,  $S_k = \sin \chi$ ; Zero curvature,  $k = 0$ ,  $S_k = \chi$ ; Negative curvature,  $k = -1$ ,  $S_k = \sinh \chi$ . (2)

The physical distance  $dL$  (measured with meter sticks) between two points fixed in the comoving coordinate system (such as two galaxies) at the same time (i.e.,  $dt = 0$ ), known as the proper distance, is

$$dL = ds = \sqrt{(a(t))^2 (d\chi^2 + S_k^2(\chi) d\psi^2)}. \quad (3)$$

If we center our comoving coordinate system on the Earth and only concern ourselves with the radial (direct line) physical distance  $L$  to a galaxy with given comoving coordinate value  $\chi$ , at time  $t$ , it is

$$L(t) = a(t) \chi. \quad (4)$$

## 2 The Hubble Sphere Distance

The Hubble “constant” (which changes in time as the universe evolves) is

$$H(t) = \frac{\dot{L}(t)}{L(t)} \quad \left[ \begin{array}{l} \text{physical velocity increase per unit physical distance of} \\ \text{galaxies (which are fixed in comoving coordinate system)} \end{array} \right]. \quad (5)$$

$$\text{Current measured value } H(t_0) \approx 70 \frac{\text{km/sec}}{\text{megaparsec}} = 2.1 \times 10^{-5} \text{ km/sec/light-year.}$$

For  $L$  and its derivative using the same distance units (such as km),  $H$  has units of 1/sec.

At a distance where the galaxy recession velocity is the speed of light, the first row of (5) becomes

$$H(t) = \frac{c}{L(t)} = \frac{c}{L_H(t)} \quad \rightarrow \quad L_H(t) = \frac{c}{H(t)} \quad \xrightarrow{\text{present time}} \quad L_H(t_0) = \frac{c}{H(t_0)} = \frac{3 \times 10^5}{2.1 \times 10^{-5}} = 14.3 \text{ billion light-years.} \quad (6)$$

$L_H$ , also called the Hubble distance  $D_H$ , is the radius of what is known as the Hubble sphere. Galaxies outside this sphere recede from us at a speed faster than light. This does not violate the special relativity postulate for maximum speeds in the universe, because it is spacetime itself that is expanding faster than the speed of light, not objects within that spacetime.

Alternative names for the Hubble distance are the gravitational horizon (typical in cosmology) or the apparent horizon (typical in other applications of general relativity). The “gravitation horizon” in black hole theory is the Schwarzschild radius, which separates regions from which light traveling toward the outside universe can reach the outside universe from regions where light cannot. The cause of the Schwarzschild radius is gravity, hence the name “gravitational horizon”. In static black holes this radius/horizon does not change location. In our expanding, accelerating universe, however, it does.

The time dependence of the Hubble sphere varies with the model of the universe (different values for dark energy, matter, radiation densities). Depending on the model and  $t$ , its distance  $L_H$  could recede from us, or not. For the concordance model (our universe’s current values for the densities) at  $t_0$ , the Hubble sphere is receding, i.e.,  $L_H$  is increasing with time.

### An aside on the Hubble constant

From the first row of (5) and (4), we have

$$H(t) = \frac{\dot{L}(t)}{L(t)} = \frac{\dot{a}(t) \chi}{a(t) \chi} = \frac{\dot{a}(t)}{a(t)}, \quad (7)$$

which is a very common way to express the Hubble constant.

### End of aside

Different models of the universe lead to different  $a(t)$ , and thus from (7) different  $H(t)$ . This results, from (6), in different  $L_H(t)$ , i.e., different time evolution of the Hubble sphere.

## 3 The Particle Horizon

The particle horizon (also called the cosmological horizon, the comoving horizon, or the cosmic light horizon) is  $L_P$  distance away and represents the farthest reaches of the universe we can see today. The light from there has traveled for 13.8 billion years at local speed  $c$ , but because the universe was expanding while that light was traveling, the distance  $L_P$  it has traveled is more than 13.8 billion light-years.

But, during that entire travel time the light has been on a null spacetime path, i.e.,  $ds = 0$  all along the path. From (1), for a radial path towards us, this means

$$ds^2 = -c^2 dt^2 + (a(t))^2 d\chi^2 = 0 \quad \rightarrow \quad d\chi = c \frac{dt}{a(t)}. \quad (8)$$

To get the comoving coordinate distance traversed (which is not in meters or kilometers but simply a numerical value used for locations in the comoving system) at some time (in the history of the universe)  $t$ , we need to integrate (8) from the time of the Big Bang (or shortly thereafter when light could propagate through the universe), which we will designate as zero, to  $t$ .

$$\chi_P(t) = c \int_0^t \frac{dt'}{a(t')} \quad \xrightarrow{\text{present time}} \quad \chi_P(t_0) = c \int_0^{t_0} \frac{dt'}{a(t')}. \quad (9)$$

Using (4), we can then find the physical distance from us to the particle horizon at any time  $t$  in the history of the universe as

$$L_P(t) = a(t) \chi_P(t), \quad (10)$$

and particularly, at the present time (where we don't carry out the actual calculations),

$$L_P(t_0) = a(t_0) \chi_P(t_0) = 46 \text{ billion light-years}. \quad (11)$$

Note that everything we see from 14.3 light-years away (Hubble sphere distance) to 46 billion light-years away is presenting receding from us at speeds greater than light.

#### 4 The Event Horizon

In an accelerating universe, light leaving now from far enough away can never reach us because it travels locally at speed  $c$ , but the 3D part of spacetime there is receding from us at greater than  $c$ , and that recession is accelerating. Relative to us, the light photons have velocity away from, not towards, us. This region of space is called the event horizon, and we herein label the physical distance to it as  $L_E$ .

Light leaving the event horizon now, the distance  $L_E$  beyond which we will never see light from, is, of course, light-like, i.e.,  $ds = 0$  all along its path. So, such light obeys (8). But instead of (9), we now need to integrate from present time  $t_0$  to the end of the universe.

$$\text{Present time: } \chi_E(t_0) = c \int_{t_0}^{t_{end}} \frac{dt'}{a(t')} \quad \text{Any time in history of universe: } \chi_E(t) = c \int_t^{t_{end}} \frac{dt'}{a(t')} \quad (12)$$

Concordance (accelerating) universe:  $t_{end} = \infty$       Collapsing universe:  $t_{end} = \text{time of big crunch}$

Using (4) again, as we did in (10), we find the physical distance from us to the event horizon at any time  $t$  in the history of the universe is

$$L_E(t) = a(t) \chi_E(t), \quad (13)$$

and at the present time (where again we don't carry out the actual calculations)

$$L_E(t_0) = a(t_0) \chi_E(t_0) = 16 \text{ billion light-years}. \quad (14)$$

Note that  $L_E$  is the distance for each of the following.

1. The distance now at which we will never see light emitted from there now.
2. The distance now at which light we emit now will never be seen by any observer there.
3. The distance now to the farthest location we could ever reach if we left now at the speed of light.

#### Wholeness Chart Math: Cosmological Horizons

$\chi$  = comoving radial coordinate     $L$  = physical radial distance     $H$  = Hubble constant

<u>Type Horizon</u>	<u>Decelerating Universe</u>	<u>Accelerating Universe</u>	<u>Note</u>
Hubble sphere	$L_H = \frac{c}{H(t)}$	As at left	$t = t_0$ for value now
Particle horizon	$\chi_P(t) = c \int_0^t \frac{dt'}{a(t')}$ $L_P(t) = a(t) \chi_P$	As at left	Common meaning $t = t_0$ (now)
Event horizon	$\chi_E(t) = c \int_t^{t_{end}} \frac{dt'}{a(t')}$ $L_E(t) = a(t) \chi_E$	$\chi_E(t) = c \int_t^{\infty} \frac{dt'}{a(t')}$ $L_E(t) = a(t) \chi_E$	Common meaning $t = t_0$ (now) $t_{end} = \text{time of big crunch}$

## 5 A Note

One may sometimes hear the incorrect statement that light beyond the Hubble sphere can never reach us, since space beyond it is receding at greater than  $c$ . However, in the concordance model, as well as others, the Hubble sphere is expanding, so even though photons just beyond it are now traveling, relative to us, away from us, at some point in the future, the Hubble sphere will overtake those photons. When that happens, the photons will no longer be moving away from us, but toward us. And someday we would see them.

The event horizon, not the Hubble sphere, is the boundary point beyond which photons leaving now will never be seen by us. And as we have seen, at present time, the event horizon is larger than the Hubble sphere radius (16 billion light-years to 14.3). The math supports the conclusion of the prior paragraph.