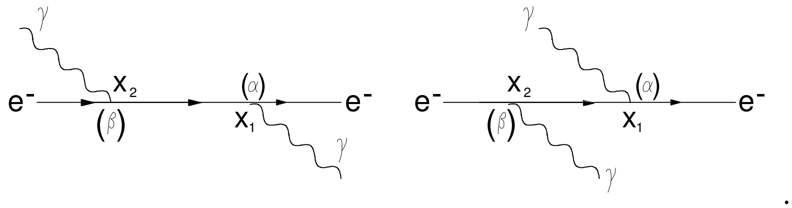
Compton Scattering Transition Amplitude Derivation

Two ways (same particles in and out) for Compton scattering



$S_{B1}^{(2)}$ $S_{B2}^{(2)}$

All terms in $S^{(n)}$ where n ≠ 2 go to zero below since the initial state (the ket) will be acted on by operators which will result in bras and kets which are unequal (non-orthogonal eignenstates) and whose inner products are therefore zero. Similarly, only the $S_B^{(2)}$ term of all the n = 2 terms will result in the same particle states in the bra and ket. The S matrix transition amplitude for Compton scattering is thus (with incoming particles unprimed, outgoing primed)

$$S_{fi} = \langle f | S | i \rangle = \left\langle e^-_{\mathbf{p}',s'}, \gamma_{\mathbf{k}',r'} \middle| \sum_n S^{(n)} \middle| e^-_{\mathbf{p},s}, \gamma_{\mathbf{k},r} \right\rangle = \left\langle e^-_{\mathbf{p}',s'}, \gamma_{\mathbf{k}',r'} \middle| S_B^{(2)} \middle| e^-_{\mathbf{p},s}, \gamma_{\mathbf{k},r} \right\rangle \tag{B3.1-1}$$

$$= \left\langle e^-_{\mathbf{p}',s'}, \gamma_{\mathbf{k}',r'} \middle| \left(-e^2\right) \int d^4x_1 d^4x_2 : \left\{ \left(\bar{\psi} A\!\!\!/ \psi\right)_{x_1} \left(\bar{\psi} A\!\!\!/ \psi\right)_{x_2} \right\} \middle| e^-_{\mathbf{p},s}, \gamma_{\mathbf{k},r} \right\rangle \tag{B3.1-2}$$

$$\begin{aligned} = -e^2 \Big\langle e^-_{\mathbf{p}',s'}, \gamma_{\mathbf{k}',r'} \Big| \int d^4x_1 d^4x_2 : \Big\{ & \left(\bar{\psi}^+ + \bar{\psi}^-\right)_{x_1} \left(A\!\!\!/^+ + A\!\!\!/^-\right)_{x_1} \\ & \times \left(iS_F\left(x_1 - x_2\right)\right) \left(A\!\!\!/^+ + A\!\!\!/^-\right)_{x_2} \left(\psi^+ + \psi^-\right)_{x_2} \Big\} \Big| e^-_{\mathbf{p},s}, \gamma_{\mathbf{k},r} \Big\rangle \end{aligned} . \tag{B3.1-3}$$

After the operators raise and lower the ket, only two terms in (B3.1-3) remain (i.e., have identical bra and ket). They are

$$\begin{aligned} = -e^2 \Big\langle e^-_{\mathbf{p}',s'}, \gamma_{\mathbf{k}',r'} \Big| \int d^4x_1 d^4x_2 : \Big\{ & \underbrace{\bar{\psi}^-_{x_1} A\!\!\!/^-_{x_1} \left(iS_F(x_1 - x_2)\right) A\!\!\!/^+_{x_2} \psi^+{}_{x_2}}_{\text{will result in } S_{B1}^{(2)} \text{ term}} \\ + & \underbrace{\bar{\psi}^-_{x_1} A\!\!\!/^+_{x_1} \left(iS_F(x_1 - x_2)\right) A\!\!\!/^-_{x_2} \psi^+{}_{x_2}}_{\text{will result in } S_{B2}^{(2)} \text{ term}} \Big\} \Big| e^-_{\mathbf{p},s}, \gamma_{\mathbf{k},r} \Big\rangle . \end{aligned} \tag{B3.1-4}$$

Continuing with <u>only the first term in (B3.1-4)</u>, we have (with Dirac indices explicitly shown)

$$\langle f | S_{B1}^{(2)} | i \rangle =$$

$$\underbrace{\langle f \mid f \rangle}_{=1} \left(-e^2\right) \int d^4x_1 d^4x_2 \left\{ \begin{aligned} & \left(\frac{m}{VE_{\mathbf{p}'}}\right)^{1/2} \bar{u}_{s'\alpha}(\mathbf{p}') e^{ip'x_1} \left(\frac{1}{2V\omega_{\mathbf{k}'}}\right)^{1/2} \varepsilon^\mu_{r'}(\mathbf{k}') \gamma^{\alpha\beta}_\mu e^{ik'x_1} \\ & \times \frac{1}{(2\pi)^4} \int d^4q \, iS_{F\beta\delta}(q) e^{-iq(x_1 - x_2)} \\ & \times \left(\frac{1}{2V\omega_{\mathbf{k}}}\right)^{1/2} \varepsilon^\nu_r(\mathbf{k}) \gamma^{\delta\eta}_\nu e^{-ikx_2} \left(\frac{m}{VE_{\mathbf{p}}}\right)^{1/2} u_{s\eta}(\mathbf{p}) e^{-ipx_2} \end{aligned} \right\} . \tag{B3.1-5}$$

Re-arranging factors in the above, we have

$$\langle f|S_{B1}^{(2)}|i\rangle =$$
$$\left(-e^2\right)\left(\frac{m}{VE_{\mathbf{p}'}}\right)^{1/2}\left(\frac{1}{2V\omega_{\mathbf{k}'}}\right)^{1/2}\left(\frac{1}{2V\omega_{\mathbf{k}}}\right)^{1/2}\left(\frac{m}{VE_{\mathbf{p}}}\right)^{1/2}\overline{u}_{s'\alpha}(\mathbf{p}')\varepsilon_{r'}^{\mu}(\mathbf{k}')\gamma_{\mu}^{\alpha\beta}\varepsilon_{r}^{\nu}(\mathbf{k})\gamma_{\nu}^{\delta\eta}u_{s\eta}(\mathbf{p}) \quad \text{(B3.1-6)}$$
$$\times\frac{1}{(2\pi)^4}\int d^4q\, iS_{F\beta\delta}(q)\left\{\int d^4x_1 e^{-iqx_1}e^{ip'x_1}e^{ik'x_1}\int d^4x_2 e^{iqx_2}e^{-ikx_2}e^{-ipx_2}\right\}$$

Noting that

$$\int d^4x_1 e^{ix_1(p'+k'-q)}\int d^4x_2 e^{ix_2(q-p-k)} = (2\pi)^4\,\delta^{(4)}\left(p'+k'-q\right)(2\pi)^4\,\delta^{(4)}\left(q-(p+k)\right), \quad \text{(B3.1-7)}$$

we find thus,

$$q = p + k = p' + k' \quad \text{(B3.1-8)}$$

and

$$\frac{1}{(2\pi)^4}\int d^4q\, iS_{F\beta\delta}(q)\delta^{(4)}\left(q-(p+k)\right) = iS_{F\beta\delta}(p+k)\,. \quad \text{(B3.1-9)}$$

Then

$$\langle f|S_{B1}^{(2)}|i\rangle = (2\pi)^4\,\delta^{(4)}\left(p'+k'-p-k\right)\left(\frac{m}{VE_{\mathbf{p}'}}\right)^{1/2}\left(\frac{m}{VE_{\mathbf{p}}}\right)^{1/2}\left(\frac{1}{2V\omega_{\mathbf{k}'}}\right)^{1/2}\left(\frac{1}{2V\omega_{\mathbf{k}}}\right)^{1/2}M_{B1}, \quad \text{(B3.1-10)}$$

where the Feynman amplitude is

$$M_{B1} = -e^2\overline{u}_{s'\alpha}(\mathbf{p}')\varepsilon_{r'}^{\mu}(\mathbf{k}')\gamma_{\mu}^{\alpha\beta}iS_{F\beta\delta}(q=p+k)\varepsilon_{r}^{\nu}(\mathbf{k})\gamma_{\nu}^{\delta\eta}u_{s\eta}(\mathbf{p}) \quad . \quad \text{(B3.1-11)}$$

Similarly, for the <u>second term in (B3.1-4)</u>, one gets the same relation for $\langle f|S_{B2}^{(2)}|i\rangle$ as (B3.1-10) except that M_{B1} is replaced with

$$M_{B2} = -e^2\overline{u}_{s'\alpha}(\mathbf{p}')\varepsilon_{r'}^{\mu}(\mathbf{k})\gamma_{\mu}^{\alpha\beta}iS_{F\beta\delta}(q=p-k')\varepsilon_{r}^{\nu}(\mathbf{k}')\gamma_{\nu}^{\delta\eta}u_{s\eta}(\mathbf{p}) \quad . \quad \text{(B3.1-12)}$$

Thus,

$$\text{Compton's }\; S_{fi} = \langle f|S|i\rangle_{Comp} = \langle f|S_{B}^{(2)}|i\rangle = \langle f|S_{B1}^{(2)}|i\rangle + \langle f|S_{B2}^{(2)}|i\rangle$$
$$= (2\pi)^4\,\delta^{(4)}\left(p'+k'-p-k\right)\left(\frac{m}{VE_{\mathbf{p}'}}\right)^{1/2}\left(\frac{m}{VE_{\mathbf{p}}}\right)^{1/2}\left(\frac{1}{2V\omega_{\mathbf{k}'}}\right)^{1/2}\left(\frac{1}{2V\omega_{\mathbf{k}}}\right)^{1/2}\left(M_{B1}+M_{B2}\right). \quad \text{(B3.1-13)}$$