Closed vs Open Relativistic String Solutions

Light-Cone Gauge & Light-Cone Coordinates: Classical Mechanics

As an aid for Zwiebach (which equation numbers below reference) Robert D Klauber March 2, 2023

	Open String Field $\beta = 2$		**Closed String Field** $\beta = 1$	
Indep variables	$X^I, x_0^-, \mathcal{P}^{\tau I}, p^+$	(12.5) $\mathcal{P}^\tau$ = momentum density	Same $X^I, x_0^-, \mathcal{P}^{\tau I}, p^+$	
Motion descrip	$X^\mu(\tau,\sigma) = (X^+, X^-, X^I)$		Same $X^\mu(\tau,\sigma) = (X^+, X^-, X^I)$	
Eq motion	$\ddot{X}^\mu - X''^\mu = 0$	(9.39)	Same $\ddot{X}^\mu - X''^\mu = 0$	
General sol	$X^\mu =$ $x_0^\mu + \underbrace{\sqrt{2\alpha'}\alpha_0^\mu}_{2\alpha' p^\mu}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-in\tau}\cos n\sigma$	(9.56), p^μ from (9.52) Neumann B.C.s Wave motion of string with no external force	$X^\mu = X_L^\mu(u) + X_R^\mu(v) = \downarrow \quad (u=\tau+\sigma \quad v=\tau-\sigma)$ $x_0^\mu + \underbrace{\sqrt{2\alpha'}\alpha_0^\mu}_{\alpha' p^\mu}\tau + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}\frac{e^{-in\tau}}{n}\left(\alpha_n^\mu e^{in\sigma} + \bar{\alpha}_n^\mu e^{-in\sigma}\right)$	(13.9), (13.24), & (13.22) No B.C. but 2π identification Wave motion of string with no external force
Transverse	X^I = (9.56) above with $\mu = I$	(9.69)	X^I = (13.24) above with $\mu = I$	
Dependent variables	$X^+ = \beta\alpha' p^+\tau = 2\alpha' p^+\tau = \sqrt{2\alpha'}\alpha_0^+\tau$ $\left(\mu = +,\ x_0^+ = \alpha_n^+ = 0 \text{ in (9.56) above}\right)$ X^- = (9.56) above with $\mu = -$	←Light-cone gauge (9.70) (9.72)	$X^+ = \beta\alpha' p^+\tau = \alpha' p^+\tau = \sqrt{2\alpha'}\alpha_0^+\tau$ $\left(\mu = +,\ x_0^+ = \alpha_n^+ = 0 \text{ in (13.24) above}\right)$ X^- = (13.24) above with $\mu = -$	← Light-cone gauge (9.70) (Closed α_0^+ = half of open α_0^+. Same symbol, diff meaning)
Auxiliary for ↑	$\sqrt{2\alpha'}\alpha_n^- = \frac{1}{p^+}L_n^\perp \qquad L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_{n-p}^I\alpha_p^I$ $\sqrt{2\alpha'}\alpha_0^- = 2\alpha' p^- = \frac{1}{p^+}L_0^\perp \qquad L_0^\perp = 2\alpha' p^+ p^-$ $L_0^\perp = \underbrace{\frac{1}{2}\alpha_0^I\alpha_0^I}_{\alpha' p^I p^I} + \sum_{p=1}\alpha_p^{I\dagger}\alpha_p^I = \alpha'\left(p^I p^I + M^2\right)$	(9.77) Derived [189] $L_n^\perp$ = transverse Viasoro modes (9.78) M^2 from (9.83)	$\sqrt{2\alpha'}\alpha_n^- = \frac{2}{p^+}L_n^\perp \qquad \text{Same } L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_{n-p}^I\alpha_p^I$ $\sqrt{2\alpha'}\bar{\alpha}_n^- = \frac{2}{p^+}\bar{L}_n^\perp \qquad \bar{L}_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}}\bar{\alpha}_{n-p}^I\bar{\alpha}_p^I$ $\sqrt{2\alpha'}\alpha_0^- = \alpha' p^- = \frac{2}{p^+}L_0^\perp \qquad L_0^\perp = \frac{1}{2}\alpha' p^+ p^- = \bar{L}_0^\perp$ $L_0^\perp = \frac{\alpha'}{4}p^I p^I + \sum_{p=1}\alpha_p^{I\dagger}\alpha_p^I$	or in (13.24) $\sqrt{\frac{\alpha'}{2}}\alpha_n^- = \frac{1}{p^+}L_n^\perp$ (13.40) & (13.37) (13.40) and (13.41) $\left(\bar{L}_0^\perp = L_0^\perp\right)$ (13.43 in (13.42), same form for $\bar{L}_0^\perp$
Hamiltonian	$H = 2\alpha' p^+ p^- = L_0^\perp$	(12.16) (classical version)	$H = \alpha' p^+ p^- = L_0^\perp + \bar{L}_0^\perp$	(13.49) (classical version)
Valuable relation	$\dot{X}^- \pm X^{-\prime} = \frac{1}{\beta\alpha'}\frac{1}{2p^+}\left(\dot{X}^I \pm X^{I\prime}\right)^2$ This yields $L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_{n-p}^I\alpha_p^I$	(9.65) $\beta = 2$ for open (9.77) above	$\dot{X}^- \pm X^{-\prime} = \frac{1}{\beta\alpha'}\frac{1}{2p^+}\left(\dot{X}^I \pm X^{I\prime}\right)^2$ This yields $L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}}\alpha_{n-p}^I\alpha_p^I \qquad \bar{L}_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}}\bar{\alpha}_{n-p}^I\bar{\alpha}_p^I$	(13.35) $\beta = 1$ for closed (13.37) above