Clarifications for Chap. 7

(Should help the learning process)

Sect. 7.7.2, Pg. 203, before heading "Notation modification", insert the following

Non-adjacent Contraction Operators

We will encounter terms like $\underbrace{A(x_1)B(x_2)C(x_3)}_{}$, where an operator ($B$ in this example) is sandwiched between two operators that form a contraction. Because contractions are Feynman propagators they are *c*-numbers and not operators (even though they can be expressed, as in the LHS of (7-75), in terms of two operator fields.) Hence in a term they share with operator fields, contractions can be placed anywhere without affecting the term. That is, contractions commute with operators. For example,

$$\underbrace{A(x_1)B(x_2)C(x_3)}_{} = \underbrace{A(x_1)C(x_3)}_{}B(x_2) = B(x_2)\underbrace{A(x_1)C(x_3)}_{}. \qquad (7\text{-}76 + 1)$$

However, in what is about to come, it will prove efficient for our symbolism if we define normal ordering of terms including contractions so that the following holds true.

$$\begin{aligned}
&\text{For } B \text{ a destruction operator} \rightarrow \quad N\left\{\underbrace{A(x_1)B^d(x_2)C(x_3)}_{}\right\} = \underbrace{\pm\underbrace{A(x_1)C(x_3)}_{}B^d(x_2)}_{\substack{\text{minus sign for } C \text{ and } B^d \\ \text{both fermionic}}} \\
&\text{For } B \text{ a creation operator} \rightarrow \quad N\left\{\underbrace{A(x_1)B^c(x_2)C(x_3)}_{}\right\} = \underbrace{\pm B^c(x_2)\underbrace{A(x_1)C(x_3)}_{}}_{\substack{\text{minus sign for } A \text{ and } B^c \\ \text{both fermionic}}}
\end{aligned} \qquad (7\text{-}76 + 2)$$

In other words, normal ordered terms with contractions in them will take on whatever sign would occur if we treated the contraction as two operators rather than a *c*-number, and exchanged operators to put creation operators on the LHS and destruction operators on the RHS of the contraction.

This definition extends to terms with more than one operator (such as B above) and more than one contraction. Simply make the successive exchanges needed to get all operators originally sandwiched inside the contraction outside of it, and count the sign changes needed for those exchanges. Only fermions exchanged with adjacent fermions entail a sign change.

Sect. 7.8.2, pg. 204

Just after (7-80), change "the operator T_c as time ordering" to "the symbol T_c to represent time ordering"

Also change "the operator N_c as ordering" to "the symbol N_c to represent an ordering of".

These changes are made because a reader (Holger Teutsch) noted that confusion results because T_c and N_c are not really operators, which change the thing operated on, but rather represent identities. We change the order of operators such as *ABC*.. but what we end up with is the same as what we started with, just re-ordered in a proper manner such that the re-ordered entity equals the original.

N and T, on the other hand, when operating on an entity (a product of multiple operators), generally change the entity to something that no longer is identical to the original.

Sect. 7.8.5, pg. 209

In the 2nd paragraph, 3rd line, change “re-ordering operation” to “re-ordering procedure”

In the 3rd paragraph, 3rd line, change “re-ordering operation” to “re-ordering procedure”

In the last paragraph of the section, change “N_c operations” to “N_c procedures”

Sect. 7.8.6, pg. 209

3rd line, change “T_c operator” to “T_c procedure”

Appendix Sect. 7.11.1, pg. 211.

Change the text just before (7-112) by inserting parenthetical remark, as shown below.

“…… by the N operator (see (7-76 + 2)) as”

Change (7-112) to the following (i.e., insert the middle part below).

$$\pm B_2^c \underbrace{\left[A_1^d, C_3^c\right]_\mp}_{\underset{\sqcup}{A_1C_3} \text{ for } t_3<t_1} \pm \underbrace{\left[A_1^d, C_3^c\right]_\mp}_{\underset{\sqcup}{A_1C_3} \text{ for } t_3<t_1} B_2^d = \pm B_2^c \underset{\sqcup}{A_1C_3} \pm \underset{\sqcup}{A_1C_3} B_2^d = N\left\{\underset{\sqcup}{A_1 B_2 C_3}\right\} \qquad \text{(revised 7-112)}$$

Change line before (7-113) and (7-113) to the following (i.e., insert extra words before equation and in equation add under bracket notes.)

“So finally, where terms noted in under brackets refer to eq. (7-111), and we use (revised 7-112)

$$\downarrow\ A_1B_2C_3 \text{ re-arranged using full commutation relations } \downarrow$$

$$T\left\{A_1B_2C_3\right\} = \underbrace{N\left\{A_1B_2C_3\right\}}_{\text{1st 8 terms}} + \underbrace{N\{\underset{\sqcup}{A_1B_2}C_3\}}_{\text{1st term in last row}} + \underbrace{N\{A_1\underset{\sqcup}{B_2C_3}\}}_{\text{2nd term in last row}} + \underbrace{N\{\underset{\sqcup}{A_1B_2C_3}\}}_{\text{last 2 terms}} \text{ for } t_3 < t_2 < t_1 \qquad \text{(revised 7-113)”}$$