Chirality vs Helicity Chart

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There is much confusion over the difference between chirality and helicity. This chart compares and contrasts their respective properties.

	Chirality	**Helicity**
Physical description	Related to weak charge	Related to handedness: thumb in velocity direction, fingers in spin direction. No direct relation to weak charge.
Operator form	γ^5	$\frac{1}{2}\frac{\mathbf{\Sigma}\cdot\mathbf{p}}{\lvert\mathbf{p}\rvert}$
Projection operator form	$P^{R}_{L}=\frac{1}{2}\left(1\pm\gamma^5\right)$	$\Pi^{R}_{L}(\mathbf{p})=\frac{1}{2}\left(1\pm\frac{\mathbf{\Sigma}\cdot\mathbf{p}}{\lvert\mathbf{p}\rvert}\right)$
Plus vs minus	– = LH ; + = RH	same as at left
Interpretation of RH/LH	Only a label, not real handedness.	Physical handedness via right hand rule
Verbal explanation	Function of γ^5, i.e., function of a spinor space entity	Function of spin $\mathbf{\Sigma}$ component along linear mom direct, i.e., function of phys space entities
Components	4X4 matrix in spinor space	same as at left
In limit $v \rightarrow c$ (or $m = 0$)		Equals chirality because $\frac{\mathbf{\Sigma}\cdot\mathbf{p}}{\lvert\mathbf{p}\rvert}\psi\rightarrow\gamma^5\psi$
Comment	Probably reason for defining chirality +/– as RH and LH	
Operation on a spinor field	$P^L\psi=\psi_L$; similar for RH Projects out L (or R for P^R) chirality component	$\Pi^L\psi=\psi_{L\text{ helicity}}$; similar for RH Projects out L (or R for Π^R) helicity component
Comment	Take care as some authors may use ψ_L for LH helicity field	
Effect of LH field ψ_L	ψ_L destroys LH chiral particle; creates RH chiral antiparticle	$\psi_{L\text{ helicity}}$ destroys LH helicity particle (& creates LH helicity antiparticle I think)
Effect of RH field ψ_R	ψ_R destroys RH chiral particle; creates LH chiral antiparticle	
Effect of $\bar{\psi}_L$	$\bar{\psi}_L$ creates LH chiral particle; destroys RH chiral antiparticle	
Effect of $\bar{\psi}_R$	$\bar{\psi}_R$ creates RH chiral particle; destroys LH chiral antiparticle	

Weak charge relation	Somehow nature has chosen to relate γ^5 to weak charge, such that only ψ_L "feels" that charge.	Unrelated to weak charge, unless at $v = c$, then essentially same as chirality.
Lorentz transf properties (change to diff frame)	Lorentz invariant γ^5 (and thus $P^{L,R}$) is 4D pseudo scalar	Not Lorentz invariant (for $v \neq c$) if frame velocity > $v_{\mathbf{p}}$, reverses **p** direction, but not spin.
Parity reversal	Changes sign, i.e., RH ↔ LH	Changes sign, i.e., RH ↔ LH
Mass term in Lagrangian	$m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L$	
Proof of above	Take def of $\psi_{L,R}$ at top and use gamma matrix algebra	
Conservation properties (change in time)	Not conserved. $m\bar{\psi}_R\psi_L$ term will destroy a LH (with weak charge) particle and create a RH (zero weak charge) particle.	Conserved for free particles. No external forces or toques leave linear momentum and angular momentum unchanged.
Weak charge non-conservation	Weak charge is chiral charge. "Vacuum eats weak charge."	
Weak charges	Only LH chiral particles charged Via SU(2) symmetry (LH sym) Weak charges e^-_L – ½ ν_L + ½ e^-_R and ν_R = 0.	
Weak charge vs weak 4-current under Lorentz transformation	Weak *4-current* ${}_wj^\mu$ is actually Lorentz *co-variant*. Total weak *charge* is invariant.	
comment	Like electric charge 4-current ${}_ej^\mu$ in which ${}_ej^0$component is elec charge density ρ. ρ changes by γfactor under Lor transf, but vol V changes by $1/\gamma$. The product ρV, tot charge q, is invariant.	