Elaboration on Klauber, Vol. 1, Sept 2021 version, Chap 8 Appendix, pg. 247.

Replace (8-109) and the paragraph after it with the following:

$$S_F(x-y)=\frac{1}{(2\pi)^4}\int\frac{(\not p+m)e^{-ip(x-y)}}{p^2-m^2+i\varepsilon}d^4p \underset{\text{in (8-108)}}{\overset{x=y}{\longrightarrow}} A_\mu\gamma^\mu S_F(0)=\frac{A_\mu\gamma^\mu}{(2\pi)^4}\int\frac{(p_\nu\gamma^\nu+m)}{p^2-m^2+i\varepsilon}d^4p \qquad (8\text{-}109)$$

S matrix term for this interaction = 0

because trace of one gamma matrix = 0 *and* p_ν *term is odd*

The amplitude for both interactions in the figure is zero, as we can see by first using Feynman rule #7, where for each closed fermion loop, we take the spinor space trace of the resulting matrix. The trace of an odd number of gamma matrices is zero, so the term with m (having one gamma matrix γ^μ) drops out. Since p_ν is odd, and the denominator is even, the term with p_ν also drops out.

Further notes:

In theories of weak and strong interactions, where the tadpole loop could be a boson, we can't use Feynman rule #7 (for fermion loops) to get a term to drop out, and loops in those theories typically have even integrands. But, tadpoles are exorcised therein by judicious choice of renormalizations.

Bottom line: Unless we are working with renormalization in non-Abelian (weak and strong interaction) theories, we take tadpole amplitudes equal to zero and ignore them.

Another way to the same conclusion.

For each diagram of Fig. 8-13, we actually have a second diagram where the fermion loop has arrows in the opposite direction. That is, one diagram has a fermion in for a loop and the other, an anti-fermion. The two amplitudes (really sub-amplitudes) must be added to get the total tadpole amplitude.

But, then, as noted in Klauber (2013), Sect. 14.1.2, pg. 341, the antifermion propagator has the same magnitude but opposite sign of the fermion propagator. So, the two sub-amplitudes cancel, leaving no contribution from the tadpole. This is actually an application of Furry's theorem (see cited reference) to the case of a single vertex fermion "polygon" (a one-sided "polygon").

Again, if we had a boson loop (which we never get in QED), the antiparticle propagator would have the same sign as the particle propagator, so we don't get a sub-amplitude cancellation.

Additionally, tadpoles have always seemed impossible to me on physical grounds. They comprise closed timelike loops (i.e., part of the loop path goes backward in time) and the boson leading to the loop has to have $k^\mu = 0$ (no energy or 3-momentum). Even for virtual particles, these constraints seem enough to justify elimination of tadpoles from any viable theory.

Regardless, the analyses and processes delineated above do remove these pesky critters from the theory math.

On this topic, see also Peskin and Schroeder, pgs 317-318; Schwartz, pgs. 323-324; Coleman, Pg. 315; Srednicki, pg. 67; Ticciati, pgs. 220 and 291. Some of these authors seem (to me) to take a bit more obscure route to our conclusion than we do here.