Addendum to Chap 8 Appendix
Returning to Equal-Times-Contractions and Wick's Theorem

With what we have learned in this chapter, we can understand why equal-times-contractions terms in Wick's theorem for QED equal zero.

Consider the following QED S matrix element with an equal time contraction at x_1, which would arise via Wick's theorem.

$$S_X^{(2)} = -e^2 \int d^4x_1 d^4x_2 N\left\{ \left(\bar{\psi} \not{A} \psi\right)_{x_1} \left(\bar{\psi} \not{A} \psi\right)_{x_2} \right\} \tag{8-108}$$

(8-108) represents certain interactions, two or which are shown by the Feynman diagrams below.

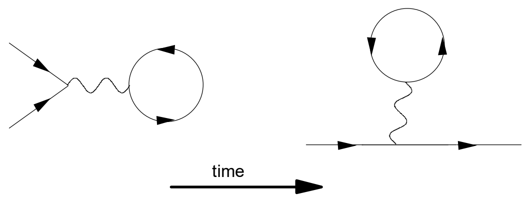

Figure 8-13. Feynman Diagrams for Equal Times Contractions of Fermions

In the LH figure the outgoing energy must be zero (as we only have the vacuum outgoing), so, the incoming energy must also be zero. But this is only true if the incoming particles have zero energy. If those particles are real, this means they don't exist, so the diagram doesn't exist.

For virtual particles in the LH figure, and for the RH figure, things are a little more problematic. (8-108) entails a contraction (fermion propagator) whose energy is integrated from + ∞ to – ∞, even if the virtual photon has zero energy (since, for the RHS, the electron energy out must equal the electron energy in). Mathematically, the loop propagator doesn't have to equal zero, at least at first blush.

But note that since the fermion propagator starts and ends at the same spacetime point, the propagator

$$S_F(x-y) = \frac{1}{(2\pi)^4} \int \frac{(\not{p}+m)e^{-ip(x-y)}}{p^2 - m^2 + i\varepsilon} d^4p = \frac{1}{(2\pi)^4} \int \frac{e^{-ip(x-y)}}{\not{p} - m + i\varepsilon} d^4p \tag{8-109}$$

has $x = x_1$ and $y = x_1$, so it becomes

$$S_F(0) = \frac{1}{(2\pi)^4} \int \frac{(\not{p}+m)}{p^2 - m^2 + i\varepsilon} d^4p = \frac{1}{(2\pi)^4} \int \frac{1}{\not{p} - m + i\varepsilon} d^4p\,. \tag{8-110}$$

Equal time propagator integral = 0 due to Feynman rule #7

The amplitude for both interactions in the figure is zero, as we can see using Feynman rule #7, where for each closed fermion loop, we take the spinor space trace of the resulting matrix. But for a single loop with one vertex, this results in a single gamma matrix, and traces of an odd number of those matrices are zero.

Loop has trace of one gamma matrix, which equals zero

Each side of Fig. 8-13 is called a <u>tadpole diagram</u>, for an obvious reason: part of it looks like a tadpole. Its transition amplitude equals zero in QED, and since we can only have tadpole diagrams in QED with fermion propagator loops, we don't have to worry about it in that theory.

Called tadpole diagram

Its transition amplitude equals zero in QED, but there are cases in other theories, like electroweak interaction theory, where such contributions do not vanish, so in such theories we will need to take care in evaluating tadpole diagrams.