Alternative nomenclature: The transition amplitude U is sometimes called the propagator (though *not* the QFT Feynman propagator). It projects the wave function at $T + t_a$ that evolved from the initial state $|\psi_i\rangle$ at t_a onto the final state $|\psi_f\rangle$ at time $T + t_a$. It "propagates" the particle from i to f.

18.3.2 Position Eigenstates

When the particle has a definite position, e.g., x_i, the ket is an eigenstate of position, written $|x_i\rangle$. The transition amplitude for measuring a particle initially at x_i, and finally at x_f, would take the form

$$U\left(x_i, x_f; T\right) = \left\langle x_f \,\middle|\, \underbrace{e^{-iHT/\hbar}\,|x_i\rangle}_{\substack{\text{evolved state,}\\ \text{in } x \text{ space } = \psi}} \right. \quad . \tag{18-9}$$

$|x_i\rangle$, eigenstate of position, in x space rep, is a delta function

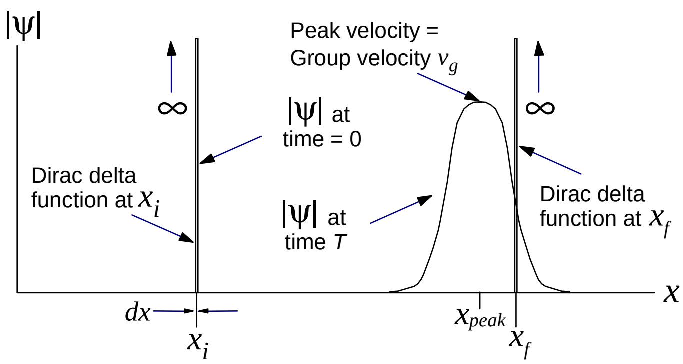

Figure 18-1. Propagation of an Effectively Initial Position Eigenstate Quantum Wave

In wave mechanics notation, $|x_i\rangle$ and $|x_f\rangle$ are, in x space, delta functions of forms[1] $\delta(x - x_i)$ and $\delta(x - x_f)$, the first of which is represented schematically on the left in Fig. 18-1. As the initial state evolves into ψ, however, it, like wave packets generally do, spreads, and its peak diminishes (wave function envelope at right in Fig. 18-1.) The amplitude for measuring the particle at time T at x_f, i.e., for measuring $|x_f\rangle$ that collapsed from ψ, is (18-9).

It spreads as it evolves

When measured at x_f, wave packet collapses to $|x_f\rangle$, eigenstate of position, i.e., a delta function

We can re-write (18-9), in wave mechanics notation, as

$$U(x_i, x_f; T) = \int_{-\infty}^{+\infty} \delta\left(x - x_f\right) \psi\left(x, T\right) dx = \psi(x_f, T) \tag{18-10}$$

So U for position eigenstate at x_f, $\rightarrow |U|^2 =$ probability density at x_f

Thus,

$$\left|U\left(x_i, x_f; T\right)\right|^2 = \left|\psi\left(x_f, T\right)\right|^2 = \psi^*(x_f, T)\,\psi(x_f, T) \;=\; \begin{matrix}\text{probability density of measuring}\\ \text{particle at } x_f \text{ at time } T.\end{matrix} \quad . \tag{18-11}$$

Modification to definition: Hence, from (18-10), the square of the absolute value of the transition amplitude for eigenstates of position, with the chosen normalization, is *probability density*, *not probability*, as was the case for energy eigenstate wave functions of form (18-5).

As we will see, the value found using the RHS of (18-9), i.e., that of the Schrödinger approach, is the same as the value found using Feynman's many paths approach.

[1] There are different ways to normalize position eigenstates. Here we use the most common one, which is also the easiest to understand for our purposes. Also, practically, a position measurement is over finite Δx, not dx, so our initial delta function represents a very narrow, very high real world wave packet.