We will use the subscript F to denote the final scattered particle states (i.e., f states) inside $d\Omega$ in a given angular direction, and whose 3-momentum lies in the range between $\mathbf{p}_f$ and $\mathbf{p}_f + d\mathbf{p}_f$. We designate the probability of measuring *any* scattered state inside $d\Omega$ by $d|S_{Fi}|^2$, so that $d|S_{Fi}|^2/T$ is the probability per unit time, where T is the total time of interaction (in our case, as we assumed in Chaps. 7 and 8, approaching infinity). So, (17-10) becomes

$$\frac{\text{transition rate}}{\text{unit solid angle}}\ (\text{at } \theta) = \frac{d\left|S_{Fi}\right|^2 / T}{d\Omega} \qquad (17\text{-}19)$$

$$= f_b \frac{d\sigma}{d\Omega} N_t = n_b v_b \frac{d\sigma}{d\Omega} N_t = \frac{\overbrace{N_b}^{1}}{V_b} v_b \frac{d\sigma}{d\Omega} \overbrace{N_t}^{1} = \frac{1}{V} v_b \frac{d\sigma}{d\Omega}.$$

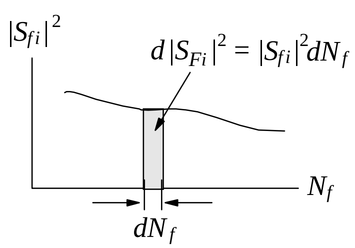
Transition rate in terms of transition amplitude S_{Fi} (for case of one beam and one target particle in V)

Thus,

$$\frac{d\left|S_{Fi}\right|^2}{T} = \frac{1}{V} v_b \, d\sigma\,. \qquad (17\text{-}20)$$

For dN_f = the number of states for the scattered particle between $\mathbf{p}_f$ and $\mathbf{p}_f + d\mathbf{p}_f$, where $\mathbf{p}_f = \mathbf{p}'_1$ in our prior examples, (see Fig. 17-12)

$$d\left|S_{Fi}\right|^2 = \left|S_{fi}\right|^2 dN_f\,. \qquad (17\text{-}21)$$



f = single final particle state

F = collection of final particle states (differential range)

$|S_{fi}|^2$ = probability of specific f state given i state

dN_f = number of final particle states (in differential range $d\mathbf{p}_f$)

$d\,|S_{Fi}|^2$ = probability of final state being in range F, given initial state i

Expressing (differential) probability as probability for an individual final state f times number of final states in differential range

Figure 17-12. Total Probability for dN_f Number of Scattered Particle States