With this value for F_B inserted into the general form of our amplitude, we find, in addition to the first order magnetic moment interaction term, an additional term of similar form (except for the factor in front), $\frac{\alpha}{\pi}\mu_B \mathbf{\Sigma} \bullet \mathbf{B}^e$. When we add this to the first order term, we find $g = 2 + \alpha/\pi = 2.0023$.

Lamb Shift

The Lamb shift comprises a shift in atomic energy levels from their RQM values that was found experimentally before it was evaluated theoretically. QED, using higher order radiative corrections, postdicts this shift.

Addendum

We should keep in mind the distinction between the commutation and anti-commutation relations for the gamma matrices (see (16-102), repeated below), each of which we use at different points in our development of QFT.

$$\left[\gamma^\mu,\gamma^\nu\right]_+ = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \qquad \left[\gamma^\mu,\gamma^\nu\right] = \gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu = -2i\sigma^{\mu\nu}. \tag{16-120}$$

16.8 Appendix: Deriving Feynman Rules for Static, External (Potential) Field

16.8.1 Comparing Four-Momenta in Different Types of Interactions

Before deriving the transition amplitude for the RHS of Fig. 16-3 (repeated below as Diagram (F) of Fig. 16-5), we point out its salient (four-momentum) characteristics by comparing it to other types of interactions. In Fig. 16-5 we show purely elastic types of classical interactions in the upper part, and each corresponding quantum elastic interactions below its classical counterpart.

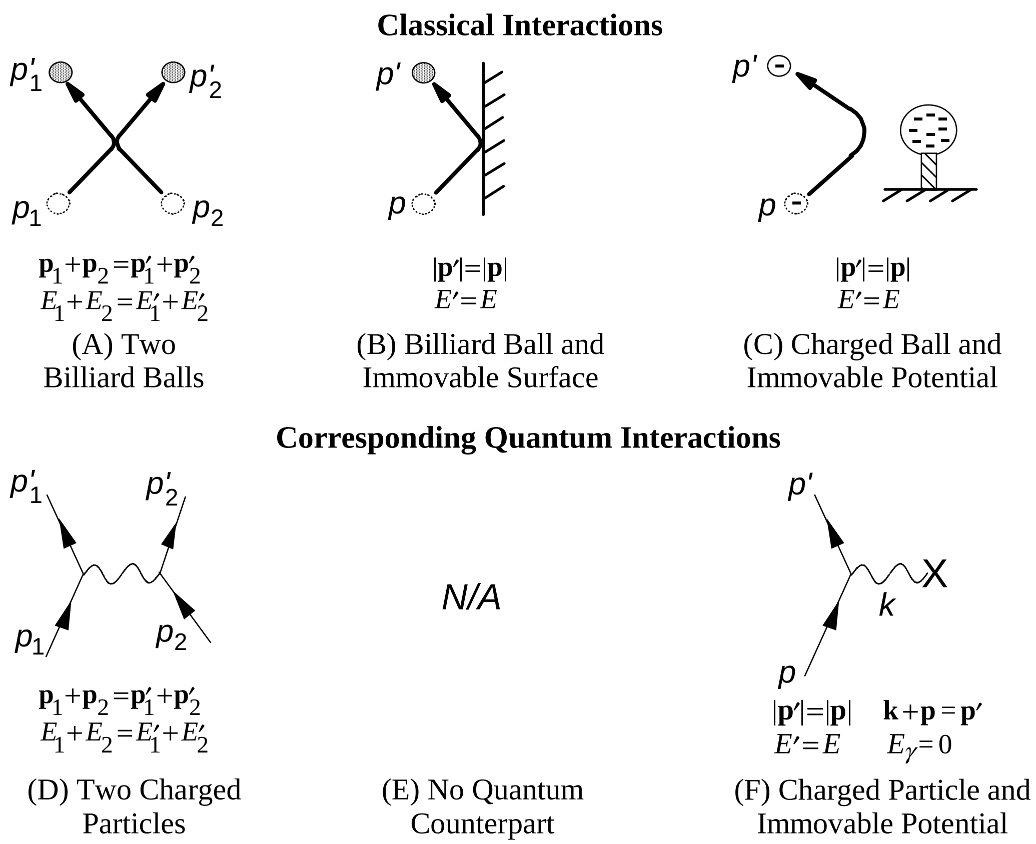

Figure 16-5. Comparing Different Types of Elastic Classical Interactions with Their Quantum Counterparts

Diagrams (A) and (D)

The energy and 3-momentum balances in Diagrams (A) and (D) should not need explanation. No external forces (potentials) act, so total initial 4-momentum of the particles equals total final 4-momentum.

Diagrams (B) and (E)

In the classical case of an object (billiard ball of Diagram (B) in Fig. 16-5) rebounding (fully elastically) off of a solid (immovable) surface, the surface picks up no energy. It doesn't begin to move as a result of the collision, so it has no kinetic energy. There is no stored compressive energy because the interaction is elastic. Hence, the surface neither gains nor loses energy. Thus, the outgoing particle must have the same kinetic energy it came in with. $E' = E$. The object changed direction so there was a change in velocity direction (and thus 3-momentum direction). Since $E = (\mathbf{p})^2 / 2m$ and $E' = (\mathbf{p}')^2 / 2m$, we must have $|\mathbf{p}| = |\mathbf{p}'|$, i.e., the magnitude of the 3-momentum is unchanged. No energy was transferred to the object. The surface transmitted 3-momentum, but not energy. It exerted a force on the object for a time (leading to 3-momentum change) but no force acting over a distance (which would lead to kinetic energy change) since it moved zero distance.

There are no solid surfaces in the quantum realm, so there is no quantum counterpart to Diagram (B).

Diagrams (C) and (F)

The classical interaction (scattering) of a charged object (ball in Diagram (C)) by an immovable, static potential (arising from the collection of charges on the stationary ball in Diagram (C)) parallels the case of Diagram (B). That is, the stationary source of the field does not move, so it picks up no kinetic energy. No energy is absorbed by the field from the interaction with the object. So neither the stationary source nor its field gain or lose energy. Hence there is no energy transfer in the process.

Jumping to the quantum realm (Diagram (F)), the same principles remain. The source of the potential (static, external) field does not move and so neither it nor its field gains kinetic energy (or any other kind of energy). But the potential field can exert a force on a charged particle and thus change the particle's 3-momentum. Since the particle energy is purely kinetic (ignoring rest mass-energy, which is always the same for a given particle), and that energy does not increase or decrease, then no energy is transmitted from the field to the particle. That is, the energy of the virtual photon E_γ equals zero. But the virtual photon must carry 3-momentum since it results in the change in direction (but not magnitude) of the charged particle 3-momentu. $E' = E$, $E_\gamma = 0$, $|\mathbf{p}'| = |\mathbf{p}|$, $\mathbf{k} = \mathbf{p}' - \mathbf{p}$.

Bottom line: In our QFT transition amplitude, we would expect the virtual photon of Diagram (F), which is off shell, to have $\mathbf{k} = \mathbf{p}' - \mathbf{p}$ and $E_\gamma = 0$.

16.8.2 The Transition Amplitude for Diagram (F)

We need to start from scratch to find $S_{fi,}$ for Diagram (F), as we have no ready-made Feynman rules for this case. This will parallel the development of (8-12) to ((8-18) on pgs. 217-218 (see Fig. 8-1 there), except that we cannot represent the photon here by the free field solution $A_\mu(x)$,which we have grown familiar with, and which we used there. The photon there was a free field, but in Diagram (F) above it is not free, but related to the external force the potential source supplies. In the present case, A_μ is virtual (it must be off shell, for the initial and final particles to be on shell) and *not* a free field. Additionally, it is static, so has no dependence on time, and thus contains no factor of $e^{\pm i\omega_k t}$. That is, a static, external, potential e/m field must have generic form $A_\mu(\mathbf{x})$, where $\mathbf{x}$ is the 3D position vector. Further, we cannot specify the precise dependence of A_μ on $\mathbf{x}$, as that will vary with the characteristics of the potential source (shape, number of charges, etc.)

Using (8-12), pg. 217, with our initial multiparticle state containing a real electron and a virtual photon emanating from the source of the potential along with a final real electron, we have a transition amplitude

$$S_{fi}=\langle f\,|S^{(1)}|\,i\rangle=\left\langle e^{-}_{\mathbf{p}',r'}\right|(-i)\int d^4x_1 N\left\{-e\overline{\psi}(x)A^e_\mu(\mathbf{x})\psi(x)\right\}_{x_1}\left|e^{-}_{\mathbf{p},r},\gamma_{\mathbf{k},s}\right\rangle \quad \left(\gamma_{\mathbf{k},s}\text{ off shell}\right),\quad (16\text{-}121)$$

where the fermion fields are the usual free fields we have been working with since Chap. 4, and

$$A^e_\mu(\mathbf{x})=\sum_{s'',\mathbf{k}''}\left(a^e_{s''}(\mathbf{k}'')A^e_{\mathbf{k}''\mu}(\mathbf{x})+a^{e\dagger}_{s''}(\mathbf{k}'')A^{\dagger e}_{\mathbf{k}''\mu}(\mathbf{x})\right), \qquad (16\text{-}122)$$

Note the numerical coefficients $1/\sqrt{2V\omega_{\mathbf{k}}}$ and the $e^{\pm ikx}$ factors we usually see for terms in $A_\mu(x)$ are replaced by the $A^e_{\mathbf{k}''\mu}(\mathbf{x})$ factors, whose precise form is not specified and unknown at this point. The construction and destruction operators in (16-122) create and destroy virtual photons emitted from the external field source. If we were to express these photon states mathematically, they would have the same dependence on **x** as $A^e_{\mathbf{k}''\mu}(\mathbf{x})$.

Using (16-122) in (16-121), we get (by a process that hopefully is getting somewhat familiar to you, i.e., the ket particles are destroyed and a sum of final particle kets are created, but only the final particle ket that matches the bra final particle gives a non-zero result)

$$\begin{aligned}S_{fi}=\underset{ext\ pot}{S_{static}}&=ie\left\langle e^{-}_{\mathbf{p}',r'}\right|\sum_{r'',\mathbf{p}''}\int\left(\sqrt{\frac{m}{VE_{\mathbf{p}''}}}\sqrt{\frac{m}{VE_{\mathbf{p}}}}A^e_{\mathbf{k}\mu}(\mathbf{x})\overline{u}_{r''}(\mathbf{p}'')\gamma^\mu u_r(\mathbf{p})e^{ip''x}e^{-ipx}\right)d^4x\left|e^{-}_{\mathbf{p}'',r''}\right\rangle\\&=0+\ldots+ie\int\left(\sqrt{\frac{m}{VE_{\mathbf{p}'}}}\sqrt{\frac{m}{VE_{\mathbf{p}}}}A^e_{\mathbf{k}\mu}(\mathbf{x})\overline{u}_{r'}(\mathbf{p}')\gamma^\mu u_r(\mathbf{p})e^{ip'x}e^{-ipx}\right)d^4x\underbrace{\left\langle e^{-}_{\mathbf{p}',r'}\middle|\middle|e^{-}_{\mathbf{p}',r'}\right\rangle}_{=1}+0+\ldots\end{aligned}\qquad(16\text{-}123)$$

Re-arranging, we have

$$\begin{aligned}\underset{ext\ pot}{S_{static}}&=ie\sqrt{\frac{m}{VE_{\mathbf{p}'}}}\sqrt{\frac{m}{VE_{\mathbf{p}}}}\,\overline{u}_{r'}(\mathbf{p}')\gamma^\mu u_r(\mathbf{p})\int A^e_{\mathbf{k}\mu}(\mathbf{x})e^{i\mathbf{p}'\cdot\mathbf{x}}e^{-i\mathbf{p}\cdot\mathbf{x}}\underbrace{\left(\int e^{iE't}e^{-iEt}dt\right)}_{2\pi\delta(E'-E)}d^3x\\&=2\pi\delta(E'-E)ie\sqrt{\frac{m}{VE_{\mathbf{p}'}}}\sqrt{\frac{m}{VE_{\mathbf{p}}}}\,\overline{u}_{r'}(\mathbf{p}')\gamma^\mu u_r(\mathbf{p})\underbrace{\underbrace{\int A^e_{\mathbf{k}\mu}(\mathbf{x})e^{i\mathbf{p}'\cdot\mathbf{x}}e^{-i\mathbf{p}\cdot\mathbf{x}}\,d^3x}_{\int A^e_{\mathbf{k}\mu}(\mathbf{x})e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}}\,d^3x}}_{\text{Fourier transform with }\mathbf{k}=\mathbf{p}'-\mathbf{p}}\ .\end{aligned}\qquad(16\text{-}124)$$

Using the Fourier transform relation (16-11), we end up with the transition amplitude for Diagram (F),

$$\underset{ext\ pot}{S_{static}}=2\pi\delta(E'-E)ie\sqrt{\frac{m}{VE_{\mathbf{p}'}}}\sqrt{\frac{m}{VE_{\mathbf{p}}}}\,\overline{u}_{r'}(\mathbf{p}')\gamma^\mu u_r(\mathbf{p})A^e_\mu(\mathbf{k}). \qquad (16\text{-}125)$$

16.8.3 Feynman Rules for Static, External (Potential) Field

We can see that (16-125) can be arrived at by using the usual Feynman rules except that we substitute rule #11 ((16-49)(a) and (b)) for the delta function and photon parts. One can then generalize ((16-49)(a) and (b) to apply to any Feynman diagram with a static, external (potential) field.

16.9 Problems

1. Use (16-28) to show that at low speeds, the adjoint anti-fermion factor in (16-27) have the following possible forms.

$$\overline{v}_{r_1=1}(\mathbf{p}_1)=v^\dagger_{r_1=1}(\mathbf{p}_1)\gamma^0\Rightarrow\begin{pmatrix}0&0&-1&0\end{pmatrix}\qquad \overline{v}_{r_1=2}(\mathbf{p}_1)=v^\dagger_{r_1=2}(\mathbf{p}_1)\gamma^0\Rightarrow\begin{pmatrix}0&0&0&-1\end{pmatrix}$$

Then assuming r_1 = 1 and r_2 = 1, use (16-14), (16-16), and (16-17) to get (16-29).