16.4

16.4.1 Sophomore Physics Review

As a refresher, an elementary review of the electron magnetic moment is hereby provided.

Review of basic physics

Consider a circular loop of current *I* encompassing an area *A*, which acts like a magnetic dipole, i.e., acts just as if fictitious positive and negative "magnetic charges" were separated by a small distance. If the current loop is placed in an external magnetic field $\mathbf{B}^e$, the torque $\boldsymbol{\tau}$ it experiences (which can be visualized as equal magnitude, opposite direction forces on the two fictitious "magnetic charges") is (where **A** has magnitude of area *A* and direction normal to the plane of the loop aligned with the thumb of the right hand when the fingers point in the direction of the current)

Torque on classical magnetic moment $\boldsymbol{\mu} = I\mathbf{A}$

$$\boldsymbol{\tau} = I\mathbf{A}\times\mathbf{B}^e = \boldsymbol{\mu}\times\mathbf{B}^e , \qquad (16\text{-}32)$$

where $\boldsymbol{\mu} = I\mathbf{A}$ is called the magnetic moment of the current loop. The energy of the loop/external field, with θ the angle between $\boldsymbol{\mu}$ and $\mathbf{B}^e$, where we define the $\theta = \pi/2$ position as zero potential energy, is

$$E_{loop/\,field} == \int_{\pi/2}^{\theta} T d\theta' = \int_{\pi/2}^{\theta} \mu B^e \sin\theta' d\theta' = -\mu B^e \cos\theta = -\boldsymbol{\mu}\bullet\mathbf{B}^e . \qquad (16\text{-}33)$$

If the current is composed of a single particle of charge – *e* (as one would have, for example, in the Bohr theory orbit of an electron in an atom), its speed is *v*, its time for one orbit is T_{orbit}, and the circular area *A* has radius *r*, then the orbital angular momentum of that charge is

$\boldsymbol{\mu}$ in terms of angular momentum **L**

$$\boldsymbol{\mu} = I\mathbf{A} = \frac{-e}{T_{orbit}}\pi r^2 \mathbf{e}_\perp = \frac{-ev}{2\pi r}\pi r^2 \mathbf{e}_\perp = -\frac{1}{2}evr\,\mathbf{e}_\perp \xrightarrow{L=mvr} \boldsymbol{\mu} = -\frac{1}{2}\frac{e}{m}\mathbf{L} , \qquad (16\text{-}34)$$

where $\mathbf{e}_\perp$ is a unit vector pointing in the direction of the right hand thumb above, and **L** is orbital angular momentum. In an atomic orbit, angular momentum magnitude is

Magnitude of angular mom L of atomic orbit

$$L = \hbar m_l \qquad (16\text{-}35)$$

with m_l an orbital quantum number. So with (16-34), we can define the Bohr magneton μ_B via

Above used to define Bohr magneton μ_B

$$(\text{orbital})\quad \boldsymbol{\mu} = -\frac{1}{2}\frac{e}{m}\mathbf{L} = -\frac{1}{2}\frac{e\hbar}{m}m_l\,\mathbf{e}_\perp = -\mu_B m_l\,\mathbf{e}_\perp \qquad \mu_B = \frac{1}{2}\frac{e\hbar}{m} . \qquad (16\text{-}36)$$

As to the electron itself, one can view it classically as a charge that is distributed internally, rather than being pointlike, and that charge rotates, or "spins" around some internal axis. So, in effect, we would have a circular current loop of sorts similar to that described above for an atom. In quantum theory, that spin of the electron is quantized, and the intrinsic (ignoring orbital contribution) angular momentum is spin angular momentum $\mathbf{S} = \hbar m_s \mathbf{e}_\perp = \pm\hbar/2\,\mathbf{e}_\perp$ (m_S = ± ½ is spin quantum number). So, one might consider

For electron, we know angular momentum (spin), but not internal charge distribution

$$(\text{spin})\quad \boldsymbol{\mu} = \pm\mu\,\mathbf{e}_\perp \xrightarrow[\text{loop?}]{\text{current}} \overset{?}{=} -\frac{1}{2}\frac{e}{m}\mathbf{S} = \pm\frac{1}{2}\frac{e\hbar}{m}\frac{1}{2}\mathbf{e}_\perp = \mu_B m_s \mathbf{e}_\perp = \pm\frac{\mu_B}{2}\mathbf{e}_\perp \qquad \left(\mu = |\boldsymbol{\mu}|\right) . \qquad (16\text{-}37)$$

However, the RHS of (16-37) is derived assuming charge is distributed as a neat current loop, as in (16-36), which is naïve. Given the unknown nature of this distribution, researchers introduced a constant *g*, called the gyromagnetic ratio[3] or the g-factor, which could be determined by experiment. So, the most general form for the magnetic moment $\boldsymbol{\mu}$ of the electron and its magnitude μ would be

The unknown charge distribution contribution labeled as g, gyromagnetic ratio

$$\boldsymbol{\mu} = \pm\mu\,\mathbf{e}_\perp = \pm g\frac{\mu_B}{2}\mathbf{e}_\perp \qquad \mu = g\frac{\mu_B}{2} = g\frac{e\hbar}{4m} \left(= g\frac{e}{4m}\text{ in natural units}\right). \qquad (16\text{-}38)$$

If our naïve analysis (current distributed in a neat loop) were correct, then the gyromagnetic ratio *g* would be found equal to 1.

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[3] This term is used in the literature for two things, the *g*-factor described herein (which is dimensionless) and the ratio of magnetic dipole moment to angular momentum (which is often denoted by the symbol γ, and which has SI dimensions of radians per second per tesla). In this book, the term gyromagnetic ratio will be used as equivalent to *g*, as shown above.