$$\frac{1}{(2\pi)^4}\int\frac{1}{\left(p^2-m^2\right)^2}\,d^4p \rightarrow \frac{i}{(4\pi)^2}\left(ln\frac{\Lambda^2}{m^2}\right)-\frac{i}{(4\pi)^2}\frac{\Lambda^2}{\infty^2}+...=\frac{i\,2}{(4\pi)^2}\left(ln\,\Lambda-ln\,m-\frac{1}{2}\frac{\Lambda^2}{\infty^2}\right)+... \quad (15\text{-}59)$$

**SEE NEXT PAGE ALSO**

where $\gamma$ here is the Euler-Mascheroni constant ≈ 0.5772, which will always cancel, or be negligible, in observable quantities. We also use the standard relation

$$a^x = 1 + x\ln a + \frac{(x\ln a)^2}{2!} + \frac{(x\ln a)^3}{3!} + .... \qquad (15\text{-}76)$$

*Expansion of $a^x$*

with $a = 1/m^2$ and $x = \eta/2$ to obtain

$$\left(\frac{1}{-m^2}\right)^{2-\frac{D}{2}} = (-1)^{\frac{\eta}{2}}\left(\frac{1}{m^2}\right)^{\frac{\eta}{2}} \underset{\substack{\eta\to 0 \\ D\to 4}}{\rightarrow} = 1 + \frac{\eta}{2}\ln\frac{1}{m^2} + \mathcal{O}\left(\eta^2\right) = 1 - \frac{\eta}{2}\ln m^2 + \mathcal{O}\left(\eta^2\right). \qquad (15\text{-}77)$$

*Using limiting values as $D \to 4$*

(15-74), with Γ(2) = 1, is then

$$\frac{1}{(2\pi)^D}\int\frac{1}{\left(p^2-m^2\right)^2}d^Dp \underset{\substack{\eta\to 0 \\ D\to 4}}{\rightarrow} \frac{i}{(4\pi)^2}\left(\frac{2}{\eta} - \gamma + \mathcal{O}(\eta)\right)\left(1 - \frac{\eta}{2}\ln m^2\right) = \frac{i2}{(4\pi)^2}\left(\frac{1}{\eta} - \frac{\gamma}{2} - \ln m\right). \qquad (15\text{-}78)$$

Of course, in the full limit $\eta \to 0$ and $D \to 4$, (15-74) then becomes

$$\frac{1}{(2\pi)^4}\int\frac{1}{\left(p^2-m^2\right)^2}d^4p \underset{\substack{\eta\to 0 \\ D\to 4}}{\rightarrow} \frac{i2}{(4\pi)^2}\frac{1}{\eta}, \qquad (15\text{-}79)$$

*We get integral for D = 4*

found via dimensional regularization.

### *15.4.4 Important Conclusion*

We can compare our dimensional regularization result (15-78)-(15-79) to that for the same integral found via Pauli-Villars regularization (15-59)-(15-60), and if we assume they must give us the same result, we can conclude that,

$$\text{for finite } \Lambda \text{ and } \eta \text{ small, } \ln\Lambda = \frac{1}{\eta} - \frac{\gamma}{2}. \qquad \text{For } \Lambda\to\infty \text{ and } \eta\to 0, \quad \ln\Lambda = \frac{1}{\eta}. \qquad (15\text{-}80)$$

*Comparing result with Pauli-Villars to relate Λ to $\eta$*

For this integral at least, $(1/\eta - \gamma/2)$ plays the role of *ln* Λ. Both go to infinity in the limiting condition, where the Euler-Mascheroni constant $\gamma$ becomes negligible. This conclusion is true in general for regularization of any integral, though we won't prove that here. Hopefully, this one example will provide some justification for adopting (15-80) as an identity in what follows.