Chapter 14 Zwiebach: Summary and Helpful Notes

Section 14.3, pgs 309-312

Instead of employing ψ_1 and ψ_2 of (14.10) to (14.23) to deduce Ψ of (14.23) to (14.26) and the rest of the chapter, start with an action for Ψ, rather than one for both ψ_1 and ψ_2 as in (14.10).

$$S_\Psi = \frac{1}{2\pi}\int d\tau \int_{-\pi}^{+\pi} d\sigma\, \Psi^I \left(\partial_\tau + \partial_\sigma\right)\Psi^I \,. \tag{1}$$

Demanding a stationary action in the usual way, we get the same thing as (14.15), but with ψ_2 =0, $\psi_1 = \Psi$, and our boundary limits from $-\pi$ to $+\pi$. The equation of motion and its solution form are thus

$$\left(\partial_\tau + \partial_\sigma\right)\Psi^I = 0 \quad \rightarrow \quad \Psi^I = \Psi^I\left(\tau - \sigma\right). \tag{2}$$

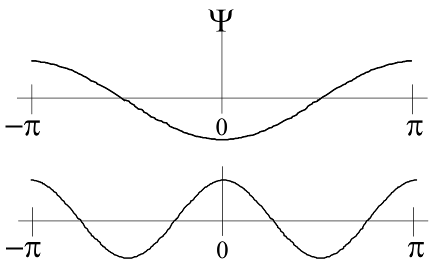
The boundary condition is

$$\Psi^I \delta\Psi^I \Big|_{-\pi}^{+\pi} = 0 \qquad \rightarrow \qquad \Psi^I\left(\tau,\pi\right)\delta\Psi^I\left(\tau,\pi\right) = \Psi^I\left(\tau,-\pi\right)\delta\Psi^I\left(\tau,-\pi\right). \tag{3}$$

Since $\delta\Psi$ and Ψ have the same sign, as shown in (14.25) and (14.26),

$$\Psi^I\left(\tau,\pi\right) = \pm\Psi^I\left(\tau,-\pi\right). \tag{4}$$

The meaning of (14.26) [(4) above] can be understood pictorially, as shown below.



etc. ...

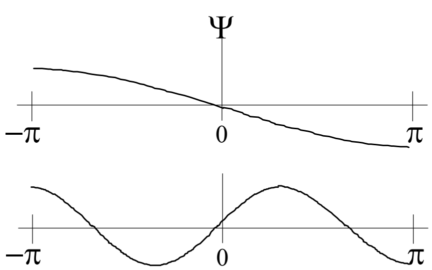
Ramond boundary condition

$\Psi^I\left(-\pi\right) = +\Psi^I\left(\pi\right)$ $\quad$ $\tau = 0$ for illustration

wavelengths $2\pi, \frac{2\pi}{2}, \frac{2\pi}{3}, \ldots$

frequencies f proportional to inverse of above $\frac{1}{2\pi}, \frac{2}{2\pi}, \frac{3}{2\pi}, \ldots$

mode numbers $n = 0,1,2,3,\ldots$



etc. ...

Neveu-Schwartz boundary condition

$\Psi^I\left(-\pi\right) = -\Psi^I\left(\pi\right)$

wavelengths $4\pi, \frac{4\pi}{3}, \frac{4\pi}{5}, \ldots$

frequencies f proportional to inverse of above $\frac{\frac{1}{2}}{2\pi}, \frac{\frac{3}{2}}{2\pi}, \frac{\frac{5}{2}}{2\pi}, \ldots$

mode numbers $r = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$

World Sheet Bosons/Fermions vs Spacetime Bosons/Fermions

It is key to keep in mind the distinction between world-sheet fermions (bosons) and spacetime fermions (bosons).

$|NS+\rangle$ are world sheet bosons and spacetime bosons
$|NS-\rangle$ are world sheet fermions and spacetime bosons (including tachyon)
$|R+\rangle$ are world sheet bosons and spacetime fermions
$|R-\rangle$ are world sheet fermions and spacetime fermions

BUT ALL employ anti-commuting creation and destruction operators on the world sheet.

(continued on next page)

Spacetime States for Open and Closed Strings

Open strings: For spacetime, truncate both NS and R to |NS+⟩ (bosons) and |R–⟩ (fermions) → same number at each M^2.

Massless: 8 bosons and 8 fermions.

Don't use |NS–⟩, since it has a tachyon. Other combinations not SUSY.

Closed strings:

Type IIA – L sector $\begin{Bmatrix} NS+ \\ R- \end{Bmatrix}$, R sector $\begin{Bmatrix} NS+ \\ R+ \end{Bmatrix}$;

(NS+, NS+) (R–,R+) = spacetime bosons, (NS+,R+)(R–,NS+) = spacetime fermions

Massless: 8X8 = 64 of each of above combinations → 128 bosons and 128 fermions

Same result if exchange R– and R+ above.

Type IIB – L sector $\begin{Bmatrix} NS+ \\ R- \end{Bmatrix}$, R sector $\begin{Bmatrix} NS+ \\ R- \end{Bmatrix}$;

(NS+, NS+) (R–,R–) = spacetime bosons, (NS+,R–)(R–,NS+) = spacetime fermions

Massless: 8X8 = 64 of each of above combinations → 128 bosons and 128 fermions

Sane result if use R+ instead of R– above.

Heterotic O(32) Summary: Problem 14-5.

Note: I believe R´+ is a spacetime boson as stated below, though not said explicitly in Zwiebach. That sector comes from 26D spacetime bosonic strings, so this should be correct. It is the only way the tensor products below make sense.

NS´+ and R´+ (left moving) are spacetime bosons (because they come from 26D bosonic string)
NS+ (right moving) are spacetime bosons (as they are in open superstring theory)
R– (right moving) are spacetime fermions (as they are in open superstring theory)

NS´+ ⊗ NS+ (spacetime boson times spacetime boson) is a spacetime boson
R´+ ⊗ NS+ (spacetime boson times spacetime boson) is a spacetime boson

NS´+ ⊗ R– (spacetime boson times spacetime fermion) is a spacetime fermion
R´+ ⊗ R– (spacetime boson times spacetime fermion) is a spacetime fermion