coefficient commutation relations. Even if we chose to use these terms directly, without employing the commutation relations, the $a(\mathbf{k})a^{\dagger}(\mathbf{k})$ term is not coupled to the $b(\mathbf{k})b^{\dagger}(\mathbf{k})$term so both terms together would not represent a vertex in a Feynman diagram. In that interpretation, one might think of the $a(\mathbf{k})a^{\dagger}(\mathbf{k})$ as representing creation of a particle and destruction of the same particle at the same event, i.e., nothing would happen as time evolves. No evanescence. No pair popping.

In summary, for <u>free fields</u>

- Terms in the free Hamiltonian density containing two creation operators that might create a particle/antiparticle pair at an event drop out of the full (not density) Hamiltonian.
- The only terms surviving in the full Hamiltonian have creation and destruction operators paired. These would create and destroy the same particle at the same event, i.e., nothing would effectively happen.

We conclude that the free field components of the Hamiltonian do not lead to particle/antiparticle pairs popping in and out of the vacuum.

10.12 Appendix E: Considerations for Finite Volume Interactions

All of the foregoing material in this chapter related to "standard" QFT, in which fields/particles are considered to extend over infinite volume *V* and infinite time *T*. That assumption, as we will see in Part 4 of this book, leads to accurate real world predictions for real world fields/particles of finite extensions in *V* and *T*.

In developing our theory, this assumption gave rise to delta functions (see (8-30), pg. 222) because we integrated over unbounded space and time. These delta functions, arising in each transition amplitude, led to strict conservation of 4-momentum at every vertex. Had *V* and *T* been finite instead of unbounded, integration would not have led to delta functions, and so one might question if, with finite *V* and *T*, if the resulting relation would lead to uncertainty in outgoing 4-momentum. Presumably, for large *V* and *T*, the relation would approximate a delta function implying approximate, but not exact, conservation of 4-momenta. And thus, smaller *V* and *T* would mean 4-momenta would be less constrained to be conserved.

This would give rise to an uncertainty in outgoing 4-momentum at any vertex for which the fields did not have infinite extension in *V* and *T*. Smaller *V* and *T* means greater uncertainty, and this correlates with the familiar uncertainty principle.

To examine this more closely, consider the delta function shown in (8-30), pg. 222, where $k = P_f$ is the 4-momentum leaving the vertex and $P_i = p_1 + p_2$ is the incoming 4-momentum,

$$(2\pi)^4\,\delta^{(4)}\left(P_f - P_i\right) = \int_{-\infty}^{\infty} e^{i\left(P_f - P_i\right)x_2}\,d^4x_2\,. \tag{10-32}$$

Now consider the RHS of (10-32) integrated over finite, instead of infinite, *V* and *T*, where, to keep things simple, we use the 1D correlate of the 4D integral, and represent that with the symbol *I*,

$$\int_{-V,-T}^{V,T} e^{i\left(P_f - P_i\right)x_2}\,d^4x_2 \;\xrightarrow{\;1D\;}\; \int_{-L/2}^{L/2} e^{i\left(P_f - P_i\right)x_2}\,dx_2 = I\left(P_f - P_f\right) \tag{10-33}$$

The integral is easy to evaluate, and *I* is found to be

$$I\left(P_f - P_f\right) = \left.\frac{e^{i\left(P_f - P_i\right)x_2}}{i\left(P_f - P_i\right)}\right|_{-L/2}^{L/2} = \left.\frac{\cos\left(P_f - P_i\right)x_2 + i\sin\left(P_f - P_i\right)x_2}{i\left(P_f - P_i\right)}\right|_{-L/2}^{L/2} = 2\,\frac{\sin\left(\left(P_f - P_i\right)\frac{L}{2}\right)}{\left(P_f - P_i\right)}\,. \tag{10-34}$$

In the development of NRQM, RQM, and QFT (see (3-24) to (3-25), pg 46 and Sect. 3.4.1, pgs. 53-54), we typically assume

$$P_i = \frac{2\pi n_i}{L} \quad P_f = \frac{2\pi n_f}{L} \quad \rightarrow \quad P_f - P_i = \frac{2\pi\left(n_f - n_i\right)}{L} \qquad n_i, n_f \text{ integers}, \tag{10-35}$$

because (10-35) results in orthogonal functions of e^{iPx} and zero values for quantities like probability density in NRQM for particles at $L/2$ and $-L/2$, as well as certain terms in the probability of RQM and in the Hamiltonian of QFT that must be zero. (See above references.)

<u>n_i and n_f as integers</u>

For (10-35) in (10-34), we find

$$I\left(P_f - P_f\right) = 2\frac{sin\left(\frac{2\pi\left(n_f - n_i\right)}{L}\frac{L}{2}\right)}{\frac{2\pi\left(n_f - n_i\right)}{L}} = 2L\frac{sin\left(\pi\left(n_f - n_i\right)\right)}{2\pi\left(n_f - n_i\right)}. \tag{10-36}$$

Due to the numerator, this is zero except for $n_f = n_i$. Then

$$I\left(P_f - P_f\right)_{n_f = n_i} = \lim_{n_f \to n_i}\left(2L\frac{sin\left(\pi\left(n_f - n_i\right)\right)}{2\pi\left(n_f - n_i\right)}\right) = 2L\frac{\pi\left(n_f - n_i\right)}{2\pi\left(n_f - n_i\right)} = L \tag{10-37}$$

So, I is zero, except when $n_f = n_i$, i.e., when $P_f = P_i$. That behaves like a delta function for argument $P_f \neq P_i$. However when $P_f = P_i$, I is not ∞, as a delta function is, as long as L is finite.

Looking again at our transition amplitude calculation in (8-30), pg. 222, we see that the finite L (V there for 3D case; V and T, for 4D) will still leave us with zero value unless $P_f = P_i$ ($k = p_1 + p_2$ there.) The value of the transition amplitude will change because we now have L (V for 3D, VT for 4D) as finite when $P_f = P_i$, but other values for P_f are prohibited (have zero probability of occurring.)[1]

Bottom line: For n_i and n_f as integers, and finite volume and time, we still must have strict 4-momentum conservation at a vertex. That is, there is no uncertainty principle at play giving rise to evanescent energy and 3-momentum "popping in and out" of the vacuum.

<u>n_i and n_f as non-integers</u>

If, however, n_i and n_f could be non-integers, then I of (10-36) can have non zero values when $n_f \neq n_i$ (and thus when $P_f \neq P_i$). Analogous results hold for 4D, so for finite V and T, we could have non-zero probability (due to a non-zero value in (10-32)) for $P_f \neq P_i$ and not have strict conservation of 4-momentum[2].

Bottom line: For n_i and n_f as non-integers, and finite volume and time, we do not have strict 4-momentum conservation at a vertex. That is, there is an uncertainty principle of sorts at play, which could give rise to evanescent energy and 3-momentum "popping in and out" of the vacuum. For infinite volume and time, strict conservation exists.

<u>Impact of n_i and n_f as non-integers on various kinds of "vacuum fluctuations"</u>

If non-integer values for n_i and n_f manifest in nature, then the following may be surmised for each type of "vacuum fluctuation" in QFT.

"Pair Popping"

The functional form of the transition amplitude and thus questions involving the delta function found therein are not relevant to the pair popping story, as there are no transition amplitudes having vertices with only two (not three, as for vacuum bubbles) particles. (See "Virtual Bubbles" section below.)

Zero Point Energy

The non-integer n_i and n_f condition would not modify anything we have said herein about the ZPE ½ quanta, as they represent free fields, with no vertices, i.e., no interactions. However, it does relate to virtual vacuum bubbles and radiative corrections, which are manifestations of interacting fields. (See Wholeness Chart 10-1, pg. 278.)

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1 In the limit where $L \to \infty$, (10-37) becomes $2\pi\delta\,(P_f = P_{i.})$. When $V \to \infty$, we get the the 3D delta function, and for $T, V \to \infty$, the 4D relation.

[2] Additional analysis, which we mention but not do here, leads to the conclusion that for n_i and n_f as non-integers, we do get a delta function in (10-33) (and (10-32) as $L \to \infty$ ($V,T \to \infty$).

Virtual Bubbles

A 3-particle virtual bubble has zero initial 4-momentum, but as noted above for finite V and T, it could then, after the first vertex, have a non-zero total 4-momentum (solely for non-integer n_i and n_f). And this then starts to look like the pair popping scenario (even though there are three, not two particles.)

However, we have seen that negative energy virtual particles are as likely as positive energy ones. So, the sum total energy of the bubble could be positive or negative. The sum over large numbers of such bubbles would be effectively zero energy. In other words, even for small values of V and T, there would be no net global contribution to the energy of the vacuum from virtual bubbles. It is conceivable, however, that tiny black holes could exist for positive energy bubbles, and possibly "white holes", we could call them, for the negative ones. We could have quantum foam, but zero total vacuum energy.

Radiative Corrections

As noted, radiative corrections do not arise alone in the vacuum and make no direct contribution to vacuum energy. This is true for finite, or infinite, V and T. Additionally, variations in energy from uncertainty at each vertex would go in both directions (positive and negative) and cancel globally, over many interactions.

BUT remember

Integer values for n_i and n_f in (10-35) seem to be required by nature. If this were not true, we would not have orthogonal functions as our solutions to the RQM/QFT wave equations and certain of our derivations, such as that for the number operator form of the Hamiltonian, would no longer be valid.

Bottom line:

Thus, vacuum energy, carried by particles popping in and out of the vacuum (for virtual 3 particle bubbles), appears inconsistent with the rest of our theory. To my knowledge, this issue (regarding non-integer n_i and n_f in transition amplitudes) has not been explored in great depth and might make a good research topic for someone. If any reader does pursue this, please apprise me of the results (via the website for this book, the address of which is found on pg.$xvi, opposite pg. 1.)

10.13 Problem

1. Show that for the single particle state $\left|\phi\right\rangle = \left|\int \frac{A(\mathbf{k}')e^{-ik'x}}{\sqrt{(2\pi)^3}}\,d^3k'\right\rangle$ having unit norm, i.e. $\left\langle\phi|\phi\right\rangle = 1$, then $\int \left|A(\mathbf{k}')\right|^2 d^3k' = 1$. Hint: The bra is $\left\langle\phi\right| = \left\langle\int \frac{A^\dagger(\mathbf{k}'')e^{ik''x}}{\sqrt{(2\pi)^3}}\,d^3k''\right|$, the norm implies integration of the bra times the ket over $\mathbf{x}$, and $\int e^{i(\mathbf{k}''-\mathbf{k}')\cdot\mathbf{x}}\,d^3x = (2\pi)^3\,\delta^{(3)}(\mathbf{k}''-\mathbf{k}')$.