Corrections for Chapter 10 in *Student Friendly QFT*

September 20, 2018

You can simply print this page out, then cut and paste the material below in the appropriate places.

<u>Page 275, (10-11) and line above it.</u> Replace with the following.

The expectation value of the number density operator (10-8) for the wave packet state (10-9) is

$$\langle\phi|\mathcal{N}_a(\mathbf{k})|\phi\rangle=\left\langle\int\frac{A(\mathbf{k}'')e^{-ik''x}}{\sqrt{(2\pi)^3}}\,d^3k''\right|\mathcal{N}_a(\mathbf{k})\left|\int\frac{A(\mathbf{k}')e^{-ik'x}}{\sqrt{(2\pi)^3}}\,d^3k'\right\rangle=|A(\mathbf{k})|^2 \qquad (10\text{-}11)$$

Continuous field Hamiltonian EV for single particle wave packet state

<u>Same page, replace (10-12)</u> with

$$\langle 2\phi|\mathcal{N}_a(\mathbf{k})|2\phi\rangle=2\left|A(\mathbf{k})\right|^2. \qquad (10\text{-}12)$$

Continuous field Hamiltonian EV for two particle wave packet state

<u>Page 276, equation (10-13)</u>

In the second line, delete the bracket that has " = 1" underneath it.

<u>Page 282, third row up from bottom</u>. Replace with blocks below.

N_a (k) Acting on General State	$N_a(\mathbf{k})\left\|\sum_{\mathbf{k}'}A_{\mathbf{k}'}\frac{e^{-ik'x}}{\sqrt{V}}\right\rangle$ $=A_{\mathbf{k}}\left\|\frac{e^{-ikx}}{\sqrt{V}}\right\rangle$	$\mathcal{N}_a(\mathbf{k})\left\|\int A(\mathbf{k}')\frac{e^{-ik'x}}{\sqrt{(2\pi)^3}}d^3k'\right\rangle$ $=A(\mathbf{k})\left\|\frac{e^{-ikx}}{\sqrt{(2\pi)^3}}\right\rangle$

<u>Page 283, third row up from bottom.</u> Replace with the blocks below.

N_a (k) Acting on Multi General Particle States	$N_a(\mathbf{k})\left\|\phi_q,2\phi_r,..\right\rangle$ $=N_a(\mathbf{k})\left\|\sum_{\mathbf{k}'}A_{q\mathbf{k}'}\frac{e^{-ik'x_q}}{\sqrt{V}},2\sum_{\mathbf{k}''}A_{r\mathbf{k}''}\frac{e^{-ik''x_r}}{\sqrt{V}},..\right\rangle$ $=A_{q\,\mathbf{k}}\left\|\phi_{q\mathbf{k}},2\phi_r,..\right\rangle+2A_{r\,\mathbf{k}}\left\|\phi_q,2\phi_{r\mathbf{k}},..\right\rangle+\ldots$	$\mathcal{N}_a(\mathbf{k})\left\|\phi_q,2\phi_r,..\right\rangle=\mathcal{N}_a(\mathbf{k})\times$ $\left\|\int A_q(\mathbf{k}')\frac{e^{-ik'x_q}}{\sqrt{(2\pi)^3}}d^3k',2\int A_r(\mathbf{k}'')\frac{e^{-ik''x_r}}{\sqrt{(2\pi)^3}}d^3k'',..\right\rangle$ $=A_q(\mathbf{k})\left\|\phi_{q\mathbf{k}},2\phi_r,..\right\rangle+2A_r(\mathbf{k})\left\|\phi_q,2\phi_{r\mathbf{k}},..\right\rangle$

The reason for these changes is shown on the next page.

Reason for Changes

The creation operator C relation on page 282, in the second column, third row, for a general state $|\phi\rangle$ is

$$|\phi\rangle = C|0\rangle = \sum_{\mathbf{k}'} A_{\mathbf{k}'} a^\dagger(\mathbf{k}')|0\rangle = \sum_{\mathbf{k}'} A_{\mathbf{k}'} \left| \frac{e^{-ik'x}}{\sqrt{V}} \right\rangle = \sum_{\mathbf{k}'} A_{\mathbf{k}'} |\phi_{\mathbf{k}'}\rangle \; . \tag{1}$$

Now consider the operation of the number operator on (1).

$$\begin{aligned} N_a(\mathbf{k})|\phi\rangle &= N_a(\mathbf{k})C|0\rangle = N_a(\mathbf{k})\sum_{\mathbf{k}'} A_{\mathbf{k}'} a^\dagger(\mathbf{k}')|0\rangle = \sum_{\mathbf{k}'} A_{\mathbf{k}'} N_a(\mathbf{k}) a^\dagger(\mathbf{k}')|0\rangle \\ &= \sum_{\mathbf{k}'} A_{\mathbf{k}'}\left(a^\dagger(\mathbf{k})a(\mathbf{k})\right)a^\dagger(\mathbf{k}')|0\rangle = \sum_{\mathbf{k}'} A_{\mathbf{k}'} a^\dagger(\mathbf{k})\left(a(\mathbf{k})a^\dagger(\mathbf{k}')\right)|0\rangle \\ &= \sum_{\mathbf{k}'} A_{\mathbf{k}'} a^\dagger(\mathbf{k})\left(a^\dagger(\mathbf{k}')a(\mathbf{k}) + \delta_{\mathbf{k}\mathbf{k}'}\right)|0\rangle = \sum_{\mathbf{k}'} A_{\mathbf{k}'} a^\dagger(\mathbf{k})\delta_{\mathbf{k}\mathbf{k}'}|0\rangle \\ &= A_{\mathbf{k}}\, a^\dagger(\mathbf{k})|0\rangle = A_{\mathbf{k}}\, |\phi_{\mathbf{k}}\rangle. \end{aligned} \tag{2}$$

In the text, this was written incorrectly as $N_a(\mathbf{k})|\phi\rangle = |A_{\mathbf{k}}|^2|\phi\rangle$. The $A_{\mathbf{k}}$ should not have had absolute value squared, and the ϕ on the RHS should have been $\phi_{\mathbf{k}}$. See pg. 282, second column, third row up from bottom.

This means, of course, that a general state $|\phi\rangle$ is not an eigenstate of $N_a(\mathbf{k})$.

Note that expectation values come out the same as shown in the text.

$$\langle\phi|\underbrace{N_a(\mathbf{k})|\phi\rangle}_{\text{see (2) above}} = \underbrace{\sum_{\mathbf{k}''} \langle\phi_{\mathbf{k}''}|A^\dagger_{\mathbf{k}''}}_{\substack{\text{bra form of}\\ \text{ket in (1)}}} \underbrace{A_{\mathbf{k}}\,|\phi_{\mathbf{k}}\rangle}_{\substack{\text{from (2)}\\ \text{above}}} = \sum_{\mathbf{k}''} A^\dagger_{\mathbf{k}''} A_{\mathbf{k}} \underbrace{\langle\phi_{\mathbf{k}''}||\phi_{\mathbf{k}}\rangle}_{\delta_{\mathbf{k}''\mathbf{k}}} = |A_{\mathbf{k}}|^2 \tag{3}$$

These results carry over to wave packets, so (10-11) and (10-12) in the text should have had $A(\mathbf{k})$ wherever $|A(\mathbf{k})|^2$ appears with concomitant modification to each ket on the RHS (no integration and $\mathbf{k}$ instead of $\mathbf{k}'$). I made the simpler change of just making those relations into expectation values for number operators, as there wasn't space in the text for the explanations and derivations shown in (1) to (2) above, as well as their extension to the continuous solutions. Related changes were made in the wholeness chart on pages 282 and 283 as shown on the prior page.

Acknowledgement

Thanks to Ezequiel Lozano for finding and reporting this.