8.12 Summary of Where We Have Been: Chaps. 7 and 8

Wholeness Chart 8-4 on the next four pages summarizes all of interaction theory to here, i.e., Chaps. 7 and 8. However, the following is a briefer summary of how we got to this point.

Overview of our progress

<u>Summary of the Ground We Have Covered</u>

Free field theory via H_0 → Interaction Picture → State evolution via $|\Phi(t)\rangle = S_{oper}\,|\,i\,\rangle$ (we seek S_{oper})

↓ Free fields ↓ State eq of motion via H_I^I (H_I^I in terms of free fields) → ↓ use $|\Phi\rangle$ above in state eq

→ solution $S_{oper}(t)$ in terms of H_I^I, time ordered → $S = S_{oper}(t=\infty)$ → Wick's theorem

→ S normal ordered + contractions → transition amplitude $S_{fi} = \langle f\,|S|\,i\rangle$ → algebra → S_{fi}

→ probability of interaction = $|S_{fi}|^2$

<u>Or the Shortcut (Based on the Above) Instead</u>

Feynman rules → S_{fi} → $|S_{fi}|^2$

8.13 Appendix: Returning to Equal-Times-Commutators and Wick's Theorem

Recall that at the end of Sect. 7.8.4 on pg. 209, we noted that had we included equal-times contractions in Wick's theorem, it would have given rise to non-physical situations. Now that we have digested the current chapter, we can investigate just what that means.

Consider if the following term, containing an equal-times contraction, were in the S operator.

$$S_X^{(2)} = -e^2 \int d^4x_1 d^4x_2\, N\left\{ \left(\bar{\psi} A\!\!\!/ \psi\right)_{x_1} \left(\bar{\psi} A\!\!\!/ \psi\right)_{x_2} \right\} \tag{8-108}$$

(8-108) represents, in one case, an initial real electron and positron at x_2 annihilated into a virtual photon that then travels to x_1. There it becomes a very strange virtual fermion, one that begins and ends at the same spacetime point, and thus can hardly be said to exist. Essentially, there are no real final particles, i.e., the incoming particles disappear.

We could plug all the mathematical expressions for the fields and propagators into (8-108), in the same manner we have done repeatedly in this chapter to evaluate the transition amplitude

$$S_{EqTim} = \left\langle 0 \middle| S_X^{(2)} \middle| e^-_{\mathbf{p}_1,r_1}, e^+_{\mathbf{p}_2,r_2} \right\rangle . \tag{8-109}$$

We get the same results via the Feynman rules short cut, so (8-109) becomes

$$S_{EqTim} = \sqrt{\frac{m}{VE_{\mathbf{p}_1}}}\sqrt{\frac{m}{VE_{\mathbf{p}_2}}}\,(2\pi)^4\,\delta^{(4)}(p_1+p_2)\,\mathcal{M}_{EqTim} . \tag{8-110}$$

From the delta function, we see that the only time the transition amplitude is non-zero is when the energies of the two incoming fermions total zero. That means each incoming fermion must have zero energy, i.e., neither can exist and the interaction doesn't happen.

Similar kinds of issues result for other interactions with equal-times contractions.

Thus, we can conclude, as stated in Chap. 7, that equal-times contractions, if included in our theory, would give rise to non-physical situations. Physical reality and our theory match if equal-times contractions are not part of the application of Wick's theorem to QFT, which we justified by other means in Chap. 7.