7.4.2 The S Operator

You should now go back and read pgs. 2-3 in Chap.1. Note that (1-6) is the transition probability for the initial state of Fig. 1-1 to transition/scatter into the final state. It is the square of the absolute value of the particular component of the S Matrix connecting those two states.

Review of scattering example from Chap. 1

For simplicity at this point, consider having final $e^- e^+$ in Fig. 1-1 instead of final $\mu^- \mu^+$. $e^- e^+ \to e^- e^+$ is known as <u>Bhabha scattering</u>, and Fig. 1-1 with the above change represents one type of Bhabha scattering. Note that we got to (1-6) from (1-1), which is similar to (7-45) below. In both, operators act on a ket and step-by-step convert that initial ket state into the final ket state.

$$\text{Trans amplit} = \left\langle e^+ e^- \right| \iint \underbrace{\left(\bar{\psi} \not{A} \psi\right)_{x_1} \left(\bar{\psi} \not{A} \psi\right)_{x_2}}_{\substack{\text{operators that convert} \\ \text{initial ket into final ket} \\ = \text{same state as bra}}} dx_1 dx_2 \left| e^+ e^- \right\rangle = \left\langle e^+ e^- \right| \underbrace{S_{\substack{operator \\ Bhabha}}}_{\substack{\text{one operator} \\ \text{symbol} = \text{several} \\ \text{sub-operators}}} \left| e^+ e^- \right\rangle \qquad (7\text{-}45)$$

$S_{Bhabbha}$ operator changes initial eigenstate ket to final eigenstate ket

In Chap. 1, we alluded to, but for simplicity, effectively ignored the integration shown in (7-45). (We will see how this comes in along with constants and subtleties not shown in (7-45).) You can begin to suspect that the ψ and A operators shown in (7-45) are the fields we have been dealing with for several chapters now. As in (1-1) to (1-6), these operators destroy the initial particles, generate a virtual photon that propagates until it is destroyed, at which point the final particles are created. The bra represents the final state. When, after all the operator fields in (7-45) have operated on the ket, the ket has changed to match the bra, and there are no more operators left, only a number sandwiched between the bra and ket, which we call S_{Bhabha}. The bra and ket inner product =1, so we are left with the number S_{Bhabha} as our transition amplitude (our associated S Matrix component).

Thus, where we express the spin and momenta explicitly,

$$\text{Trans amplit} = \left\langle e^+_{r^3,\mathbf{p}^3}\, e^-_{r^4,\mathbf{p}^4} \right| \underbrace{S_{\substack{operator \\ Bhabha}}}_{\substack{\text{comprised of} \\ \text{sub operators}}} \left| e^+_{r^1,\mathbf{p}^1}\, e^-_{r^2,\mathbf{p}^2} \right\rangle$$

$$= \left\langle e^+_{r^3,\mathbf{p}^3}\, e^-_{r^4,\mathbf{p}^4} \right| \underbrace{S_{Bhabha}}_{\substack{\text{number left} \\ \text{after opers act}}} \left| e^+_{r^3,\mathbf{p}^3}\, e^-_{r^4,\mathbf{p}^4} \right\rangle$$

$$= S_{Bhabha} \underbrace{\left\langle e^+_{r^3,\mathbf{p}^3}\, e^-_{r^4,\mathbf{p}^4} \middle\| e^+_{r^3,\mathbf{p}^3}\, e^-_{r^4,\mathbf{p}^4} \right\rangle}_{=1} = S_{fi} \quad \text{for this interaction}.$$

Scattering transition amplitude S_{Bhabha} found from operator $S_{\substack{operator \\ Bhabha}}$, initial ket, and final bra