Wholeness Chart 6-3. Summary of Effect of Lorentz Transformation on Fields

	x^μ system	**x'^μ system**	**Comment**
	$x^\mu \longrightarrow$ Lorentz transformation $\Lambda \longrightarrow x'^\mu$		Shorthand symbols defined: $x'^\mu = \Lambda x^\mu$ x'^μ and x^μ represent the same event
Scalar field	$S(x^\mu)$	$S'(x'^\mu) = S(x^\mu)$ always $S'(x'^\mu) = S(x'^\mu)$ if sym form	↙ If S is symmetric function under Λ, S' has same functional form as S
Vector field	$V^\alpha(x^\mu)$	$V'^\alpha(x'^\mu) = \Lambda V^\alpha(x^\mu)$ always $V'^\alpha(x'^\mu) = V^\alpha(x'^\mu)$ if sym	$V'^\alpha V'_\alpha = V^\alpha V_\alpha$ invariant, V^α covariant 2 vectors, $V'^\alpha W'_\alpha = V^\alpha W_\alpha$ invariant ← If V^α components sym under Λ
Tensor field	$T^{\alpha\beta}(x^\mu)$	$T'^{\alpha\beta}(x'^\mu) = \Lambda\Lambda T^{\alpha\beta}(x^\mu)$ always $T'^{\alpha\beta}(x'^\mu) = T^{\alpha\beta}(x'^\mu)$ if sym	$T'^{\alpha\beta}T'_{\alpha\beta} = T^{\alpha\beta}T_{\alpha\beta}$ invar, $T^{\alpha\beta}$ covar Other invariants exist such as trace $T^\alpha{}_\alpha$ ← If $T^{\alpha\beta}$ components sym under Λ
Spinor field	$\psi(x^\mu)$	$\psi'(x'^\mu) = D\psi(x^\mu)$ $\bar{\psi}\psi$ invariant $\left(\bar{\psi}\gamma^\alpha\psi\right)' = \Lambda^\alpha{}_\beta\,\bar{\psi}\gamma^\beta\psi$	D = Lorentz group rep for spinors $\bar{\psi}\psi$ transforms like world scalar $\bar{\psi}\gamma^\alpha\psi$ transforms like 4-vector
Law of nature	$(\partial^\alpha\partial_\alpha + m)\,\phi(x^\mu) = 0$	$(\partial'^\alpha\partial'_\alpha + m)\,\phi'(x'^\mu) = 0$	Same form under Λ Example is Klein-Gordon field equation
Euler-Lagrange equation	$\frac{\partial}{\partial x^\alpha}\left(\frac{\partial\mathcal{L}}{\partial\phi^r{}_{,\alpha}}\right) - \frac{\partial\mathcal{L}}{\partial\phi^r} = 0$	$\frac{\partial}{\partial x'^{\alpha'}}\left(\frac{\partial\mathcal{L}'}{\partial\phi'^r{}_{,\alpha'}}\right) - \frac{\partial\mathcal{L}'}{\partial\phi'^r} = 0$	Same form under Λ
Lagrangian density	$\mathcal{L}(\phi^r(x^\mu))$ $\mathcal{L} = \partial_\alpha\phi^\dagger\partial^\alpha\phi - \mu^2\phi^\dagger\phi$	$\mathcal{L}'(\phi'^r(x'^\mu)) = \mathcal{L}(\phi'^r(x'^\mu))$ $\mathcal{L} = \partial'_\alpha\phi'^\dagger\partial'^\alpha\phi' - \mu^2\phi'^\dagger\phi'$	For Euler-Lagrange eq to be covariant (same form under Λ), $\mathcal{L}$ must keep same form under Λ, as well Example is Klein-Gordon $\mathcal{L}$