You will get used to the complication in time. When you do, you should gain an appreciation for the beauty, as well. In the words of the equation's discoverer,

> *"The research worker, in his efforts to express the fundamental laws of Nature in mathematical form, should strive mainly for mathematical beauty. He should take simplicity into consideration in a subordinate way to beauty ... It often happens that the requirements of simplicity and beauty are the same, but where they clash, the latter must take precedence. "*
> — Paul A. M. Dirac

Common, short hand form of Dirac equation

You should do Prob. 4 to provide some practice with (4-14), and then note that the common way to write the Dirac equation is to hide the spinor space indices in κ and η, i.e.,

$$\boxed{\left(i\gamma^{\mu}\partial_{\mu} - m\right)\left|\psi\right\rangle = 0}\,, \tag{4-15}$$

where you have to be vigilant to remember the implicit 4X4 spinor space matrix/column nature of (4-15) as expressed explicitly in (4-14).

Slash notation also very common in Dirac equation

Another notation commonly used, which is the most streamlined of all, is

$$\partial\!\!\!/ = \gamma^{\mu}\partial_{\mu} \qquad \text{so, the Dirac equation} \rightarrow \left(i\partial\!\!\!/ - m\right)\left|\psi\right\rangle = 0\,. \tag{4-16}$$

We note in passing that

$$m \rightarrow \frac{mc}{\hbar} \qquad \text{in non-natural units in the Dirac equation}\,. \tag{4-17}$$

4.1.5 Solutions to the Dirac Equation

We can write out (4-15) fully as

$$i\gamma^{\mu}\partial_{\mu}\left|\psi\right\rangle = i\left(\gamma^{0}\partial_{0} + \gamma^{1}\partial_{1} + \gamma^{2}\partial_{2} + \gamma^{3}\partial_{3}\right)\left|\psi\right\rangle = m\left|\psi\right\rangle = \tag{4-18}$$

Writing out Dirac equation

$$i\left(\begin{bmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{bmatrix}\partial_0 + \begin{bmatrix}&&&1\\&&1&\\&-1&&\\-1&&&\end{bmatrix}\partial_1 + \begin{bmatrix}&&&-i\\&&i&\\&i&&\\-i&&&\end{bmatrix}\partial_2 + \begin{bmatrix}&&1&\\&&&-1\\-1&&&\\&1&&\end{bmatrix}\partial_3\right)\begin{vmatrix}\psi_1\\\psi_2\\\psi_3\\\psi_4\end{pmatrix}$$

$$= i\left(\begin{bmatrix}\partial_0 & 0 & \partial_3 & \partial_1 - i\partial_2\\ 0 & \partial_0 & \partial_1 + i\partial_2 & -\partial_3\\ -\partial_3 & -\partial_1 + i\partial_2 & -\partial_0 & 0\\ -\partial_1 - i\partial_2 & \partial_3 & 0 & -\partial_0\end{bmatrix}\right)\begin{vmatrix}\psi_1\\\psi_2\\\psi_3\\\psi_4\end{pmatrix} = m\begin{vmatrix}\psi_1\\\psi_2\\\psi_3\\\psi_4\end{pmatrix}. \tag{4-19}$$

Note that the numeric subscripts on the ∂ symbols refer to derivatives with respect to time and space, whereas the numeric subscripts on the components of $|\psi\rangle$ refer to the respective components of the ket in spinor space.

(4-19) is a 4X4 matrix problem, for which we can try solutions of form $\left|\psi\right\rangle = \left|u_{\alpha}e^{\pm kx}\right\rangle$, where u_{α} is a four component spinor space column matrix. Doing this and carrying out the derivatives in (4-19), we end up with an 4X4 eigenvalue problem. This has four solutions $|\psi^{(n)}\rangle$, where n = 1,2,3,4, with each such solution having four spinor space components. We will not go through the tedium of doing this. Rather, I will simply provide the solutions, and you will do Prob. 5 to prove to yourself, by substitution, that they are indeed valid solutions to (4-19).

The Dirac equation solutions in the Dirac-Pauli (standard) representation are