Chapter 1 *Version: 10/29/2021 copyright of Robert D. Klauber*

Bird's Eye View: The Standard Model

In teaching, start with wholeness and end with wholeness. In between, do the parts.

1.0 Purpose of this Chapter

In *Student Friendly Quantum Field Theory: Basic Principles and Quantum Electrodynamics* (hereinafter "Vol. 1"), Chapter 1, we started with a grand, but much oversimplified, schematic vision of the entire scope of quantum field theory (QFT) and the subset within it contained in that book. Here we do the same thing.

1.1 This Book's Approach and Content

This book is structured in two ways, which are, figuratively speaking, orthogonal. These are via

1) interaction type (quantum electrodynamic, weak, or strong), and
2) quantization type (canonical vs path integral).

Two-way structure of our path:
1) interaction type
2) quantization type

By "orthogonal", we mean that our development of theory at any time, like a node point in a 2D coordinate grid, can have any "component" (quantum electrodynamic, weak, or strong) of 1) along with any independent "component" (canonical approach vs path integral approach) of 2).

For example, we develop quantum electrodynamics (QED) herein via the path integral approach, whereas, in Vol. 1, we did QED via the canonical quantization approach. We will also fully develop weak interaction theory via canonical quantization, and after that, briefly look at how it can be developed using path integrals. Strong interaction theory is most easily developed using path integrals, and that is what we will do here, though as with electroweak theory, we will discuss how it can alternatively be done with canonical quantization.

In other words, any one of 1) above can be done via either approach of 2). We will focus on the easiest-to-learn approach for a given interaction type, at least as an introduction to the concomitant theory (which is what we did in Vol. 1 for QED.)

1.1.1 Interaction Types: The Standard Model

If you are reading this book, you are well aware of the three interactions (electrodynamic, weak, and strong) found in nature that are encompassed by contemporary QFT. The field of theoretical physics encompassing these three interactions is known as the standard model (SM). You are also no doubt aware that a fourth interaction, gravity, is not, as of this writing, a part of the standard model. There is no adequate theory of quantum gravity, though that is not due to lack of trying. Most researchers in the field believe that a viable theory of quantum gravity, if one exists, lies beyond the structure of the SM, as we know it.

Wholeness Chart 1-1. The Standard Model and Gravity

Standard Model			Odd Man Out
↙	↓	↘	↓
QED	Weak	Strong	Gravity

Nature's 4 forces, 3 of which are handled quantum mechanically by the SM

As we will see, weak interaction theory is actually closely meshed with electromagnetic interaction theory, and one commonly finds use of the term "electroweak" interactions. Weak interactions are responsible for particle decays, and thus, radioactivity. Weak interactions affect both leptons (electrons, muons, taus, three types of neutrinos, and antiparticles for all of them) and quarks. Strong interactions hold quarks together inside hadrons such as protons or neutrons, and mesons such as the pi meson). As a residual effect, strong forces associated with baryons hold nuclei together. Wholeness Chart 1-2 summarizes the various interactions, their associated charge types, virtual bosons that mediate them, and the type of fundamental fermions affected by each. (For a summary of bosons vs fermions, see Vol. 1, pg. 65, Wholeness Chart 3-1.)

Charges, bosons, and fermions associated with the 4 interactions

Wholeness Chart 1-2. The Fundamental Interactions

Interaction	Charge Type	Mediating Boson(s)	Fundamental Fermions Affected
Electromagnetic	electric	photon	all quarks and charged leptons (all fermions except neutrinos)
Weak	weak	3 intermediate vector bosons (IVBs) W^+, W^-, Z	all quarks and leptons
Strong	color	8 gluons	all quarks
Gravity	mass-energy	graviton (conjectured)	all

Note that QFT can, in fact, handle the effect of weak (as in feeble, not as in weak force) gravitational fields on fundamental particles (quarks, leptons, photons, other bosons) by modeling the usual quantum fields as behaving in a classical spacetime background. That is, gravity is treated in the classical general relativistic way such that it bends the spacetime within which the quantum fields interact[1]. This is not quantum gravity as gravity in such a theory is classical, not quantized. In quantum gravity, the gravitational force should be mediated by a boson, such as the conjectured graviton, rather than effected by curved classical spacetime.

No quantum theory of gravity, but can model 3 SM forces in weak gravity limit

1.1.2 Canonical vs Path Integral Approaches

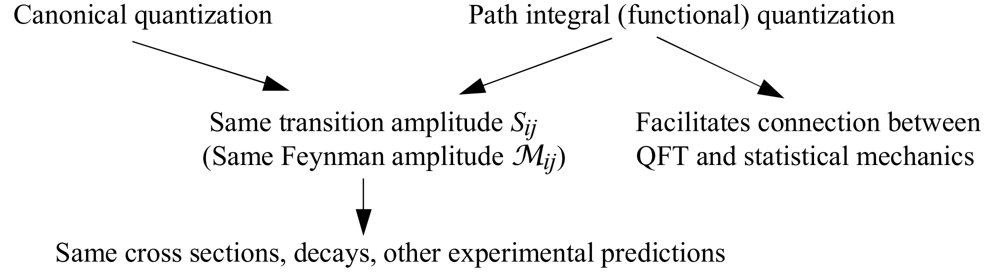
In Vol. 1, we developed QFT primarily via the canonical quantization approach, which as mentioned therein, I and many others consider the easier and more intuitive way to be introduced to the subject. In Chap. 18 of that volume, we investigated the path integral (also known as functional quantization, many paths, or sum over histories) approach to non-relativistic quantum mechanics (NRQM) and relativistic quantum mechanics (RQM), and from there, extrapolated to the first few steps of its use in QFT.

Two approaches to QFT: canonical and path integral

In Part Two of this volume, we will start path integration (PI) from the point where we left off in Vol. 1, and use it to re-derive QED. Later in the book, we will note its application in electroweak theory and then use it in vigor to develop strong interaction theory.

The two approaches differ primarily in the manner in which one arrives at transition amplitudes. From there, calculations of cross sections (see Vol. 1, Chap. 17) and other experimental results (Vol. 1, Chap. 16) remain the same. PI does, however, make deducing the partition function of statistical mechanics easier (which we will not study herein). All this is illustrated in Wholeness Chart 1-3.

Wholeness Chart 1-3. Both Quantization Methods Yield Same Transition Amplitude



Both approaches yield the same transition amplitude

PI also very helpful connecting QFT and thermodynamics

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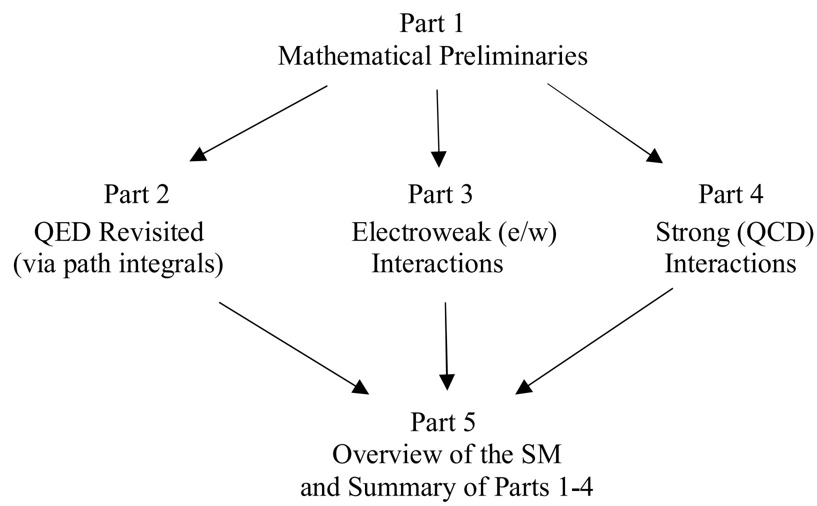
[1] When you are ready for it (probably not now), see Mukhanov, V. F., and Winitzki, S., *Introduction to Quantum Effects in Gravity* (Cambridge, 2007) as well as the relevant notes on the website for the present text (URL on opposite page from pg. 1 herein.)

1.2 Structure of this Book

1.2.1 Major Parts

After this first chapter, this book has five major parts, as shown in Wholeness Chart 1-4.

Wholeness Chart 1-4. The Major Parts of this Book



The five major parts of this book

Part 1 includes introductions to group theory, the mathematical structure underlying the SM, and other areas of mathematics that play key roles in advanced QFT.

Part 2 rederives QED (which we originally derived in Vol. 1 using canonical quantization) via the path integral quantization formulation for fields.

Part 3 develops electroweak theory (sometimes abbreviated as e/w theory) starting from basic principles and covers symmetry breaking, the Higgs particle, weak interactions including decay, neutrino behavior, and more.

Part 4 derives strong interaction theory, more formally, quantum chromodynamics (QCD), and investigates the manner in which quarks form composite particles (hadrons).

Part 5 puts all the pieces together into the wholeness of the SM and presents a final summary of the prior four parts. It is an overview of the whole book.

1.2.2 Chapter and Part Content

Wholeness Chart 1-5 outlines the part and chapter content of Vol. 1 (shaded) and this Vol. 2 (not shaded).

Wholeness Chart 1-5. Book Topic vs Volume, Part, and Chapter

Topic	**Canonical Quantization**	**Functional Quantization**
Basic principles	**Vol. 1** Prep, Parts 1 & 2 Chaps. 1-7	**Vol. 1** Addenda Chap. 18
QED	**Vol. 1** Parts 2, 3 & 4 Chaps. 8-17	**Vol. 2** Part 2 Chap. 4
Electroweak	**Vol. 2** Parts 1 & 3 Chaps. 3, 5-11	**Vol. 2** Part 3 Chap. 11*
QCD	**Vol. 2** Parts 1 & 4 Chap. 15*	**Vol. 2** Part 4 Chap. 15
Whole SM	**Vol. 2** Part 5 Chap. 18	

Where particular content is located in Vol. 1 and this book (Vol.2)

* We will only briefly discuss this alternative approach. The other approach is more common in the literature.
Vol. 2 chapter numbers not shown cover material involved in both quantization methods.

1.3 Units, Notation, and Field Dimensions

1.3.1 Units

As we did in Vol. 1, and as is common practice, we will use natural units where $c = 1$ and $\hbar = 1$, both dimensionless. For a complete explanation and description of natural units, see Chap. 2 of Vol. 1, Sect. 2.1, pgs. 11-15.

Natural units in this book

1.3.2 Notation

We will follow the same notation as used in Vol. 1, which is also common (though not universal) practice. See that volume, Sect. 2.2, pgs. 15-16, plus Appendix A therein, pgs. 32-36.

Notation and dimensions as in Vol. 1

1.3.3 Dimensions

Dimensions for various quantities in physics in general, and QFT in particular, in the natural units system are equal to the value of the symbol M (M=[mass]=[energy]=[length]$^{-1}$=[time]$^{-1}$) in Vol. 1, Chap. 2, Wholeness Chart 2-1, pg. 14.

Natural unit dimensions for different field types

1.3.4 Dimensions of Quantum Fields

Spin 0 Field Dimension

In Chap. 2 of Vol. 1, problem 3, we deduced the dimension of a scalar field ϕ as equal to one. I should have had similar problems in Chaps. 4 and 5 to deduce the dimensions of spinor fields ψ and vector fields such as the photon field A^μ. These are things we should know for the future, so we examine them here.

Spin ½ Field Dimension

The Lagrangian L (not density) has units of energy, which in natural units has dimension $M = 1$. (See Vol. 1, pg. 14.) Length has dimension $M = -1$, so volume has dimension –3. Energy density (energy per unit volume) thus has dimension 4, and so, the Lagrangian density $\mathcal{L}$ has a dimension M = 4.

The free spinor Lagrangian (density) has form (where each term has dimension 4)

$$\mathcal{L}_0^{1/2} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi \; . \tag{1-1}$$

Picking the second of these terms on the RHS, we see that m has dimension 1 and both of $\bar{\psi}$ and ψ have the same dimension. For the term to have total dimension 4, ψ must have dimension 3/2.

Spin 1 Field Dimension

The free photon Lagrangian (density) (Vol. 1, pg. 294, equation (11-36)) is

$$\mathcal{L}_0^1 = -\tfrac{1}{2}\left(\partial_\beta A_\nu \partial^\beta A^\nu - \partial_\nu A_\beta \partial^\beta A^\nu\right). \tag{1-2}$$

Each term in (1-2) has two derivatives, each such derivative having length in the denominator (ct is spatial), with dimension $M = -1$, i.e., the equivalent of $M = 1$ in the numerator. Each photon factor in the term must then have the dimension 1 in order for the entire term to be dimension 4.

Do **Problem** 1 for a little practice in determining field dimensions.

Wholeness Chart 1-6 summarizes the results of this section.

Wholeness Chart 1-6. Dimensions of Quantum Fields for Natural Units

Summary of dimensions for different field types

Field Type	**Examples**	**Symbols**	**Dimension *M***
Scalar	Higgs	ϕ, H	1
Spinor	leptons, quarks	ψ	3/2
Vector	photon, IVBs, gluons	A^μ, W^μ, Z^μ, $A^\mu{}_a$ (gluons)	1

1.4 Chapter Summary: Our Goal in this Book

With our foundational basis in basic principles and QED from Vol. 1 (or other sources), we seek, in this book, to extend our understanding of QFT to the entire range of the SM, i.e., to weak and strong interactions and to the path integral approach. The way in which this endeavor will be structured is shown in Wholeness Charts 1-3 (pg. 2), 1-4 (pg. 3), and 1-5 (pg. 3).

1.5 Suggestions?

If you have suggestions to make the material anywhere in this book easier to learn, or if you find any errors, please let me know via the web site address for this book posted herein opposite pg.1. Thank you.

1.6 Problem

1. Using the first term after the equal sign in (1-1), show, as we did with the second term, that the dimension of ψ is 3/2. Then, show that a massive vector boson field (spin 1, by definition), symbolized by W^{μ} and having a term in the Lagrangian of form $m_W^2 W_\mu^\dagger W^\mu$, where m_W is its mass, has dimension M = 1. After that, note that since the fine structure constant has dimension zero, as we showed in Prob. 2, Chap. 2 of Vol. 1, the constant *e*, the absolute value of the charge on the electron, has dimension zero. With that knowledge, show that the interaction term in the QED Lagrangian $e\bar{\psi}\gamma^{\nu}\psi A_{\nu}$ (Vol. 1, pg. 294, equation (11-36)) has dimension 4, and thus is consistent dimension-wise with the free Lagrangian terms.

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