

# Chapter 1

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## Bird's Eye View

*Well begun is half done.*

Old Proverb

### 1.0 Purpose of the Chapter

Before starting on any journey, thoughtful people study a map of where they are going. This allows them to keep their bearings as they progress, and not get lost en route. This chapter is like such a map, a schematic overview of the terrain of quantum field theory (QFT) without the complication of details. You, the student, can get a feel for the theory, and be somewhat at home with it, even before delving into the “nitty-gritty” mathematics. Hopefully, this will allow you to keep sight of the “big picture”, and minimize confusion, as you make your way, step-by-step, through this book.

### 1.1 This Book's Approach to QFT

There are two main branches to (ways to do) quantum field theory called

- the canonical quantization approach, and
- the path integral approach (also called the many paths or sum over histories approach).

This book is primarily devoted to the first of these. It is considered by many, and certainly by me, as the easiest way to be introduced to the subject. I have, however, provided a simplified, brief introduction to the path integral approach at the end of the book. Students wishing to make a career in field theory will eventually need to become well versed in both approaches.

### 1.2 Why Quantum Field Theory?

The quantum mechanics (QM) courses students take prior to QFT generally treat a single particle such as an electron in a potential (e.g., square well, harmonic oscillator, etc.), and the particle retains its integrity (e.g., an electron remains an electron throughout the interaction.) There is no general way to treat interactions between particles, such as that of a particle and its antiparticle annihilating one another to yield neutral particles such as photons (e.g.,  $e^- + e^+ \rightarrow 2\gamma$ .) Nor is there any way to describe the decay of an elementary particle such as a muon into other particles (e.g.  $\mu^- \rightarrow e^- + \nu + \bar{\nu}$ , where the latter two symbols represent neutrino and antineutrino, respectively).

Here is where QFT comes to the rescue. It provides a means whereby particles can be annihilated, created, and transmigrated from one type to another. In so doing, its utility surpasses that provided by ordinary QM.

There are other reasons why QFT supersedes ordinary QM. For one, it is a relativistic theory, and thus more all encompassing. Further, as we will discuss more fully later on, the straightforward extrapolation of non-relativistic quantum mechanics (NRQM) to relativistic quantum mechanics (RQM) results in states with negative energies, and in the early days of quantum theory, these were quite problematic. We will see in subsequent chapters how QFT resolved this issue quite nicely.

*Limitation of original QM: no transmutation of particles*

*QFT: transmutation included*

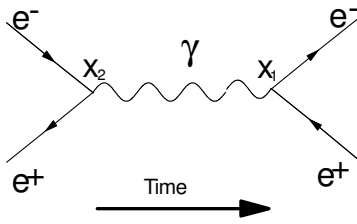
*Energies <0  
RQM yes  
QFT no*

### 1.3 How Quantum Field Theory?

As an example of the type of problem QFT handles well, consider the interaction between an electron and a positron known as Bhabha scattering shown in Fig. 1-1. At event  $x_2$ , the electron and

*QFT example: Bhabha scattering*

positron annihilate one another to produce a photon. At event  $x_1$ , this photon is transmuted back into an electron and a positron. Antiparticles like positrons are represented by lines with arrows pointing opposite their direction of travel through time. The seemingly strange, reverse order of numbering here, i.e.,  $2 \rightarrow 1$ , is standard in QFT.



**Figure 1-1. Bhabha Scattering**

Note that we can think of this interaction as an annihilation (destruction) of the electron and the positron at  $x_2$  accompanied with creation of a photon, and that followed by the destruction of the photon accompanied by creation of an electron and positron at  $x_1$ . Unlike the electrons and positrons in this example, the photon here is not a “real” particle, but transitory, short-lived, and undetectable, and is called a “virtual” particle (which mediates the interaction between real particles.)

What we seek and what, as students eventually see, QFT delivers, is a mathematical relationship, called a transition amplitude, describing a transition from an initial set of particles to a final set (i.e., an interaction) of the sort shown pictorially via the Feynman diagram of Fig. 1-1. It turns out that the square of the absolute value of the transition amplitude equals the probability of finding (upon measurement) that the interaction occurred. This is similar to the square of the absolute value of the wave function in QM equaling the probability density of finding the particle.

QFT employs creation and destruction operators acting on states (i.e., kets), and these creation/destruction operators are part of the transition amplitude. We illustrate the general idea with the following grossly oversimplified transition amplitude, reflecting the interaction process of Fig. 1-1. Be cautioned that we have omitted a few more formal, and ultimately essential, ingredients in (1-1), in order to make it simpler, and easier, to grasp the fundamental concept.

$$\text{Transition amplitude} = \underbrace{\langle e^+ e^- | (\bar{\psi}_c \mathcal{A}_d \psi_c)_{x_1} (\bar{\psi}_d \mathcal{A}_c \psi_d)_{x_2} | e^+ e^- \rangle}_{(1-1)}$$

In (1-1), the ket  $|e^+ e^- \rangle_{x_2}$  represents the incoming electron and positron at  $x_2$ . The bra represents the outgoing electron and positron at  $x_1$ .  $\psi_d$  is an operator that destroys an electron (at  $x_2$ );  $\bar{\psi}_d$  an operator that destroys a positron (at  $x_2$ );  $\psi_c$  creates a positron (at  $x_1$ ); and  $\bar{\psi}_c$  creates an electron (at  $x_1$ ). The  $\mathcal{A}_c$  is a photon operator that creates (at  $x_2$ ) a virtual photon, and  $\mathcal{A}_d$  is an operator that destroys (at  $x_1$ ) that virtual photon, with the lines underneath indicating that the photon is virtual and propagates from  $x_2$  to  $x_1$ . The mathematical procedure and symbolism (lines underneath) representing this virtual particle (photon here) process, as shown in (1-1), is called a contraction. When the virtual particle is represented as a mathematical function, it is known as the Feynman propagator or simply, the propagator, because it represents the propagation of a virtual particle from one event to another.

*Feynman propagator*

Note what happens to the ket part of the transition amplitude as we proceed, step-by-step, through the interaction process. At  $x_2$ , the incoming particles (in the ket) are destroyed by the destruction operators, so at an intermediate point, we have

$$\text{transition amplitude} = \underbrace{\langle e^+ e^- | (\bar{\psi}_c \mathcal{A}_d \psi_c)_{x_1} (\mathcal{A}_c)_{x_2} K_2 | 0 \rangle}_{(1-2)}$$

*Destruction operators leave vacuum ket times a numeric factor*

where the destruction operators have acted on the original ket to leave the vacuum ket  $|0 \rangle$  (no particles left) with a purely numeric factor  $K_2$  in front of it. The value of this factor is determined by the formal mathematics of QFT.

In the next step after (1-2), the virtual photon propagator, due to the creation operator  $\mathcal{A}_c$ , creates a virtual photon at  $x_2$  that then propagates from  $x_2$  to  $x_1$ , where it is annihilated by  $\mathcal{A}_d$ . This process leaves the vacuum ket still on the right along with an additional numeric factor, which comes out of the formal mathematics, and which we designate below as  $K_\gamma$ .

$$\text{transition amplitude} = {}_{x_1} \langle e^+ e^- | (\bar{\psi}_c \psi_c)_{x_1} K_\gamma K_2 | 0 \rangle \quad (1-3)$$

*Propagator action leaves only another numeric factor*

The remaining creation operators then create an electron and positron out of the vacuum at  $x_1$ . This leaves us with the newly created ket  $|e^+ e^- \rangle_{x_1}$  plus a factor  $K_1$  in front. The ket and the bra now represent the same state, i.e., the same particles at the same time and place  $x_1$ , so their inner product (the bracket) is not zero (as it would be if they were different states). Nor are there any operators left, but only numeric quantities, so we can move them outside the bracket without changing anything. Thus, at  $x_1$  and thereafter, we have

*Creation operators leave final state ket plus one more numeric factor*

$$\begin{aligned} \text{transition amplitude} &= {}_{x_1} \langle e^+ e^- | K_1 K_\gamma K_2 | e^+ e^- \rangle_{x_1} = {}_{x_1} \langle e^+ e^- | \underbrace{S_{Bhabha}}_{\substack{\text{just a number} \\ \text{without operators}}} | e^+ e^- \rangle_{x_1} \\ &= S_{Bhabha} \underbrace{{}_{x_1} \langle e^+ e^- | e^+ e^- \rangle_{x_1}}_1 = S_{Bhabha} \end{aligned} \quad (1-4)$$

*Calculated amplitude*

where we note the important point that in QFT the bracket of a multiparticle state (inner product of multiparticle state with itself) such as that shown in (1-4) is defined so it *always equals unity*. Note, that if we had ended up with a ket different than the bra (final state), the inner product would be zero, because the two (different) states, represented by the bra and ket, would be orthogonal. Examples are

*Bracket of multiparticle state = 1 in QFT*

$${}_{x_1} \langle e^+ e^- | \underbrace{|e^+ e^+ \rangle_{x_1}}_{\substack{\text{two} \\ \text{positrons}}} = 0, \quad {}_{x_1} \langle e^+ e^- | \underbrace{|\mu^- \mu^+ \rangle_{x_1}}_{\substack{\text{muon and} \\ \text{anti-muon}}} = 0, \quad \text{and} \quad {}_{x_1} \langle e^+ e^- | \underbrace{|\gamma \rangle_{x_1}}_{\substack{\text{single} \\ \text{photon}}} = 0. \quad (1-5)$$

The whole process of Fig. 1-1 can be pictured as simply an evolution, or progression, of the original state, represented by the ket, to the final state, represented by the bra. At each step along the way, the operators act on the ket to change it into the next part of the progression. When we get to the point where the ket is the same as the bra, the full transition has been made, and the bracket then equals unity. What is left is our transition amplitude.

Finally, the probability of the interaction occurring turns out to be

$$\text{probability of interaction} = S_{Bhabha}^\dagger S_{Bhabha} = |S_{Bhabha}|^2. \quad (1-6)$$

*Probability = |Amplitude|<sup>2</sup>*

The quantity  $K_1 K_\gamma K_2 = S_{Bhabha}$  arising in (1-4) depends on particle momenta, spins, and rest masses, as well as the inherent strength of the electromagnetic interaction, all of which one would rightly expect to play a role in the probability of an interaction taking place.

From the interaction probability, scattering cross sections can be calculated.

### 1.4 From Whence Creation and Destruction Operators?

In ordinary QM, the solutions to the relevant wave equation, the Schroedinger equation, are states (particles or kets.) Surprisingly, the *solutions to the relevant wave equations in QFT are not states (not particles.)* In QFT, it turns out that these solutions are *actually operators that create and destroy states*. Different solutions exist that create or destroy every type of particle and antiparticle. In this unexpected (and, for students, often strange at first) twist lies the power of QFT.

*QFT wave equation solutions are operators*

### 1.5 Overview: The Structure of Physics and QFT's Place Therein

Students are often confused over the difference (and whether or not there is a difference) between relativistic quantum mechanics (RQM) and QFT. The following discussion, summarized below in Wholeness Chart 1-1, should help to distinguish them.

#### 1.5.1 Background: Poisson Brackets and Quantization

Classical particle theories contain rarely used entities call Poisson brackets, which, though it would be nice, are not necessary for you to completely understand at this point. (We will show their precise mathematical form in Chap. 2.) What you should realize now is that Poisson brackets are

mathematical manipulations of certain pairs of properties (dynamical variables like position and momentum) that bear a striking resemblance to commutators in quantum theories. For example, the Poisson bracket for position  $X$  (capital letters will designate Cartesian coordinates in this book) and momentum  $p_X$ , symbolically expressed herein as  $\{X, p_X\}$ , is non-zero (and equal to one), but the Poisson bracket for  $Y$  and  $p_X$  equals zero.

*Poisson brackets  
behavior parallels  
quantum  
commutators*

Shortly after NRQM theory had been worked out, theorists, led by Paul Dirac, realized that for each pair of quantum operators that had non-zero (zero) commutators, the corresponding pair of classical dynamical variables also had non-zero (zero) Poisson brackets. They had originally arrived at NRQM by taking classical dynamical variables as operators, and that led, in turn, to the non-zero commutation relations for certain operators (which result in other quantum phenomena such as uncertainty.) But it was soon recognized that one could do the reverse. One could, instead, take the classical Poisson brackets over into quantum commutation relations first, and because of that, the dynamical variables turn into operators. (Take my word for this now, but after reading the next section, do Prob. 6 at the end of this chapter.)

The process of extrapolating from classical theory to quantum theory became known as quantization. Apparently, for many, the specific process of starting with Poisson brackets and converting them to commutators was considered the more elegant way to quantize.

*Quantization:  
Poisson brackets  
become  
commutators*

### 1.5.2 First vs. Second Quantization

Classical mechanics has both a non-relativistic and a relativistic side, and each contains a theory of particles (localized entities, typically pointlike objects) and a theory of fields (entities extended over space and time). All of these are represented in the first two rows of Wholeness Chart 1-1. Properties (dynamical variables) of entities in classical particle theories are *total* values, such as object mass, charge, energy, momentum, etc. Properties in classical field theories are *density* values, such as mass density, charge density, field amplitude at a point, etc. Poisson brackets in field theories are just like those for particle theories, except they entail *densities* of the respective dynamical variables, instead of total values.

*Branches of  
classical  
mechanics*

With the success of quantization in NRQM, people soon thought of applying it to relativistic particle theory and found they could deduce RQM in the same way. Shortly thereafter they tried applying it to relativistic field theory, the result being QFT. The term first quantization came to be associated with *particle* theories. The term second quantization became associated with *field* theories.

*1<sup>st</sup> quantization is  
for particles; 2<sup>nd</sup> is  
for fields*

In quantizing, we also assume the classical Hamiltonian (total or density value) has the same quantum form. We can summarize all of this as follows.

#### First Quantization (Particle Theories)

- 1) Assume the quantum particle Hamiltonian has the same form as the classical particle Hamiltonian.
- 2) Replace the classical Poisson brackets for conjugate properties with commutator brackets (divided by  $i\hbar$ ), e.g.,

$$\{X_i, p_j\} = \delta_{ij} \Rightarrow [X_i, p_j] = i\hbar\delta_{ij} . \quad (1-7)$$

In doing (1-7), the classical properties (dynamical variables) of position and its conjugate 3-momentum become quantum non-commuting operators.

#### Second Quantization (Field Theories)

- 1) Assume the quantum field Hamiltonian density has the same form as the classical field Hamiltonian density.
- 2) Replace the classical Poisson brackets for conjugate property densities with commutator brackets (divided by  $i\hbar$ ), e.g.

$$\{\phi_r(\mathbf{x}, t), \pi_s(\mathbf{y}, t)\} = \delta_{rs} \delta(\mathbf{x} - \mathbf{y}) \Rightarrow [\phi_r(\mathbf{x}, t), \pi_s(\mathbf{y}, t)] = i\hbar \delta_{rs} \delta(\mathbf{x} - \mathbf{y}) , \quad (1-8)$$

where  $\pi_s$  is the conjugate momentum density of the field  $\phi_s$ , different values for  $r$  and  $s$  mean different fields, and  $\mathbf{x}$  and  $\mathbf{y}$  represent different 3D position vectors. In doing (1-8), the classical field dynamical variables become quantum field non-commuting operators (and this, as we will see, has major ramifications for QFT.)

Note that the specific quantization we are talking about here (both first and second) is called canonical quantization, because, in both the Poisson brackets and the commutators, we are using (in classical mechanics terminology) canonical variables. For example,  $p_x$  is called the *canonical momentum* of  $X$ . (It is sometimes also called the *conjugate momentum*, as we did above, or the *generalized momentum* of  $X$ .)

*Our approach here: canonical quantization*

This differs from the form of quantization used in the path integral approach (see Sect. 1.1 on page 1) to QFT, which is known as functional quantization, because the path integral approach employs mathematical quantities known as *functionals*. (See Chap. XXX at the end of the book for a brief introduction to this alternative method of doing QFT.)

*Path integral approach to QFT: functional quantization*

### 1.5.3 The Whole Physics Enchilada

All of the above two sections is summarized in Wholeness Chart 1-1. In using it, the reader should be aware that, depending on context, the term quantum mechanics (QM) can mean i) only non-relativistic (“ordinary”) quantum mechanics (NRQM), or ii) the entire realm of quantum theories including NRQM, RQM, and QFT. In the chart, we employ the second of these.

Note that because quantum field applications almost invariably involve photons or other relativistic particles, there is little real world application for a non-relativistic quantum field theory. Further, a relativistic theory will always suffice for any non-relativistic application. Hence, the QFT we restrict our study to is specifically relativistic.

As an aside, quantum theories of gravity such as superstring theory are not included in the chart, as QFT in its standard model form cannot accommodate gravity. Thus, the relativity in QFT is special, but not general, relativity.

**Wholeness Chart 1-1. The Overall Structure of Physics**

	<u>Non-relativistic</u>		<u>Relativistic</u>	
	<b>Particle</b>	<b>Field</b>	<b>Particle</b>	<b>Field</b>
<b>Classical (non-quantum) mechanics</b>	Newtonian particle theory	Newtonian field theory (continuum mechanics + gravity), $e/m$ (quasi-static)	Relativistic macro particle theory	Relativistic macro field theory (continuum mechanics + $e/m$ + gravity)
Properties (Dynamic variables) ↓ Operators	↓ 1 <sup>st</sup> quantization ↓		↓ 1 <sup>st</sup> quantization ↓	↓ 2 <sup>nd</sup> quantization ↓
<b>Quantum mechanics</b>	NRQM	No theory generally taught.	RQM	QFT (not gravity)

**Conclusions:** RQM is similar to NRQM in that both are particle theories. They differ in that RQM is relativistic. RQM and QFT are similar in that both are relativistic theories. They differ in that QFT is a field theory and RQM is a particle theory.

### 1.6 Comparison of Three Quantum Theories

NRQM employs the (non-relativistic) Schroedinger equation, whereas RQM and QFT must employ relativistic counterparts sometimes called relativistic Schroedinger equations. Students of

*Different spin types → different wave equations*

QFT soon learn that each spin type (spin 0, spin  $\frac{1}{2}$ , and spin 1) has a different relativistic Schroedinger equation. For a given spin type, that equation is the same in RQM and in QFT, and hence, both theories have the same form for the solutions to those equations.

The difference between RQM and QFT is in the meaning of those solutions. In RQM, the solutions are interpreted as states (particles, such as an electron), just as in NRQM. In QFT, though it may be initially disorienting to students previously acclimated to NRQM, the solutions turn out not to be states, but rather operators that create and destroy states. Thus, QFT can handle transmutation of particles from one kind into another (e.g., muons into electrons, by destroying the original muon and creating the final electron), whereas NRQM and RQM can not. Additionally, the problem of negative energy state solutions in RQM does not appear in QFT, because, as we will see, the creation and destruction operator solutions in QFT create and destroy both particles and anti-particles. Both of these have positive energies.

Additionally, while RQM (and NRQM) are amenable primarily to single particle states (with some exceptions), QFT more easily, and more compressively, accommodates multi-particle states.

In spite of the above, there are some contexts in which RQM and QFT may be considered more or less the same theory, in the sense that QFT encompasses RQM. By way of analogy, classical relativistic particle theory is inherent within classical relativistic field theory. For example, one could consider an extended continuum of matter which is very small spatially, integrate the mass density to get total mass, the force/unit volume to get total force, etc., resulting in an analysis of *particle* dynamics. The field theory contains within it, the particle theory.

In a somewhat similar way, QFT deals with relativistic states (kets), which are essentially the same states dealt with in RQM. QFT, however, is a more extensive theory and can be considered to encompass RQM within its structure.

And in both RQM and QFT (as well as NRQM), operators act on states in similar fashion. For example, the expected energy measurement is determined the same way in both theories, i.e.,

$$\bar{E} = \langle \phi | H | \phi \rangle, \quad (1-9)$$

with similar relations for other observables.

These similarities and differences, as well as others, are summarized in Wholeness Chart 1-2. Comparison of Three Theories. The chart is fairly self explanatory, though we augment it with a few comments. You may wish to follow along with the chart as you read them (below).

The different relativistic Schroedinger equations for each spin type are named after their founders (see names in chart.) We will cover each in depth. At this point, you have to simply accept that in QFT their solutions are operators that create and destroy states (particles). We will soon see how this results from the commutation relation assumption of 2<sup>nd</sup> quantization (1-8).

With regard to phenomena, I recall wondering, as a student, why some of the fundamental things I studied in NRQM seemed to disappear in QFT. One of these was bound state phenomena, such as the hydrogen atom. None of the introductory QFT texts I looked at even mentioned, let alone treated, it. It turns out that QFT can, indeed, handle bound states, but elementary courses typically don't go there. Neither will we, as time is precious, and other areas of study will turn out to be more fruitful. Those other areas comprise scattering (including inelastic scattering where particles transmute types), decay of elementary particles, and vacuum fluctuations.

I also once wondered why spherical solutions to the wave equations are not studied, as they play a big role in NRQM, in both scattering and bound state calculations. It turns out that scattering calculations in QFT can be obtained to high accuracy with the simpler plane wave solutions. So, for most applications in QFT, they suffice.

Wave packets, as well, seem no where to be found in QFT. Like the other things mentioned, they too can be incorporated into the theory, but simple sinusoids (of complex numbers) serve us well in almost all applications. So, wave packets, too, are ignored in introductory (and most advanced) courses.

The next group of blocks in the chart points out the scope of each theory with regard to the four fundamental forces. Nothing there should be too surprising.

*Solutions:  
RQM → states  
QFT → operators*

*QFT can be done  
without negative  
energies*

*QFT:  
multiparticle*

*RQM  
contained in  
QFT*

*Calculate  
expectation values  
same way*

*Phenomena in  
the 3 theories*

*QFT rarely  
uses spherical  
solutions*

*or wave  
packets*

*QFT handles  
various type  
interactions*

The final blocks note the similarities and differences between forces (interactions) in the different theories. As in classical theory, in all three quantum theories, interactions comprise forces that change the momentum and energy of particles. However, in QFT alone, interactions can also comprise changes in type of particle, such as shown in Fig. 1-1. At event  $x_2$ , the electron and positron are changed into a photon, and in the process energy and momentum is transferred to the photon.

There are other things from earlier studies that seem to have been lost, as well, and we will mention these as we cross paths with them.

**Wholeness Chart 1-2. Comparison of Three Theories**

	<u>NRQM</u>	<u>RQM</u>	<u>QFT</u>
<b>Wave equation</b>	Schrodinger	Klein-Gordon (spin 0) Dirac (spin 1/2) Proca (spin 1) Special case of Proca: Maxwell (spin 1 massless)	Same as RQM at left
<b>Solutions to wave equation</b>	States	States	Operators that create and destroy states
<b>Negative energy states?</b>	No	Yes	No
<b>Particles per state</b>	Single*	Single*	Multi-particle
<b>Expectation values</b>	$\bar{O} = \langle \phi   O   \phi \rangle$	As at left, but relativistic.	As at left in RQM.
<b>Phenomena:</b>			
1. bound states	Yes, non-relativistic	Yes, relativistic	Yes, but usually not studied in introductory courses
2. scattering			
a. elastic	a. Yes	a. Yes	a. Yes
b. inelastic (transmutation)	b. No	b. Yes and no. (i.e., cumbersome, but can be done, though only for particle/antiparticle creation & destruction.)	b. Yes
3. decay			
a. composite particles	a. Yes (tunneling)	a. Yes	a. Yes
b. elementary particles	b. No	b. No	b. Yes
4. vacuum fluctuations	No	Yes and no. (Only for simple particle/antiparticle loops.)	Yes

<b>Coordinates</b>			
1. Cartesian (plane waves)	Free, 1D potentials, particles in “boxes”	As at left	Used primarily for free particles, particles in “boxes”, and scattering.
2. Spherical (spherical waves)	Bound states and scattering.	As at left.	Not usually used in elementary courses.
<b>Wave Packets</b>	Yes	Yes	Yes, but rarely used. Not taught in intro courses.
<b>Interaction types</b>			
1. e/m	No, though can pseudo model for non relativistic cases	As at left	Yes
2. weak	No	No	Yes
3. strong	No*	No*	Yes
4. gravity	No	No	Not as of 2010.
<b>Interaction nature</b>			
Transfers energy & momentum?	Yes	Yes	Yes
Can change particle type?	No	No	Yes

\*Some caveats exist for this chart. For example, NRQM and RQM can handle certain multiparticle states (e.g. hydrogen atom), but QFT generally does it more easily and more extensively. And the strong force can be modeled in NRQM and RQM by assuming a Yukawa potential, though a truly meaningful handling of the interaction can only be achieved via QFT.

### 1.7 Major Components of QFT

There are four major components of QFT, and this book (after the first two foundational chapters) is divided into four major parts corresponding to them. These are:

*The four major  
parts of QFT*

#### 1. Free (non-interacting) fields/particles

The field equations (relativistic Schroedinger equations) have no interaction terms in them, i.e., no forces are involved. The solutions to the equations are free field solutions.

#### 2. Interacting fields/particles

In principle, one would simply add the interaction terms to the free field equations and find the solutions. As it turns out, however, doing this is intractable, at best (impossible, at least in closed form, is a more accurate word). The trick employed in interaction theory actually uses the free field solutions of 1 above, so those solutions end up being quite essential throughout all of QFT.

#### 3. Renormalization

If you are reading this text, you have almost certainly already heard of the problem with infinities popping up in the early, naïve QFT calculations. The calculations referred to here are specifically those of the transition amplitude (1-4), where some of the numeric factors, if calculated straightforwardly, turn out to be infinite. Renormalization is the mathematical means by which these infinities are tamed, and made finite.

#### 4. Scattering and decay calculations

The theory of parts 1, 2, and 3 above are put to practical use in determining scattering probabilities and cross sections, as well as decay probabilities (half lives, etc.) Particle decay is governed by the weak force, so we will not do anything with that in the present volume, which is devoted solely to quantum electrodynamics (QED), involving only the electromagnetic force.

#### ***1.8 Points to Keep in Mind***

When the word “field” is used classically, it refers to an entity, like fluid wave amplitude,  $\mathbf{E}$ , or  $\mathbf{B}$ , that is spread out in space, i.e., has different values at different places. By that definition, the wave function of ordinary QM, or even the particle state in QFT, is a field. But, it is important to realize that in quantum terminology, the word “field” means an operator field, which is the solution to the wave equations, and which creates and destroys particle states. States (= particles = wave functions = kets) are *not* considered fields in that context.

*Terminology*  
“field”=  
operator in  
QFT

In this text, the symbol  $e$ , representing the magnitude of charge on an electron or positron, is always positive. The charge on an electron is  $-e$ .

*Symbol  $e > 0$*

#### ***1.9 Big Picture of Our Goal***

The big picture of our goal is this. We want to understand Nature. To do so, we need to be able to predict the outcomes of particle accelerator scattering experiments and elementary particle half lives. To do this, we need to be able to calculate probabilities for scattering, and decay, to occur. To do that, we need to be able to calculate transition amplitudes for specific elementary particle interactions. And for that, we need first to master a fair amount of theory, based on the postulates of quantization.

*Our goal: predict scattering and decay seen in Nature*

We will work through the above steps in reverse. Thus, our immediate goal is to learn some theory in Parts 1 and 2. Then, how to calculate transition amplitudes, also in Part 2. Necessary refinements will take up Part 3, with scattering and decay in Part 4.

#### **Steps to our goal**

2<sup>nd</sup> quantization postulates → QFT theory → transition amplitude calculation → probability  
→ scattering and decay results → confirmation of QFT

*Steps to our goal*

#### ***1.10 Summary of the Chapter***

In this book, we will close each chapter with a summary, emphasizing its most salient aspects. However, the present chapter is actually a summary (in advance) of the entire book and all of QFT. So, you, the reader, can simply look back in this chapter to find appropriate summaries. These should include Sect. 1.1 (This Book’s Approach to QFT), the transition amplitude relations of Eqs. (1-1) through (1-6), Sect.1.5.2 (1<sup>st</sup> and 2<sup>nd</sup> Quantization), Wholeness Chart 1-1 (The Overall Structure of Physics), Wholeness Chart 1-2 (Comparison of Three Theories), and Sect. 9 (Big Picture of Our Goal).

#### ***1.11 Suggestions or Questions?***

In this and other chapters, if you have questions or suggestions to make the material easier to learn, or if you find any errors, please let me know at [rdklauber@quantumfieldtheory.info](mailto:rdklauber@quantumfieldtheory.info). I will post answers to frequently asked questions at that site, so if you have a question, check there first. I will also post suggestions/corrections that are used as well, and if yours is one, I will list your name as a contributor there and in the next edition.

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### 1.12 Problems

As there is not much in the way of mathematics in this chapter, for most of it, actual problems are not really feasible. However, you may wish to try answering the questions in 1 to 5 below without looking back in the chapter. Doing Prob. 6 can help a lot in understanding first quantization.

1. Draw a Feynman diagram for a muon and anti-muon annihilating one another to produce a virtual photon, which then produces an electron and a positron. Using simplified symbols to represent more complex mathematical quantities (that we haven't studied yet), show how the probability of this interaction would be calculated. Note that your destruction operators must be different than the example in the chapter in that they now destroy a muon and antimuon instead of an electron and positron.
2. Detail the basic aspects of first quantization. Detail the basic aspects of second quantization, then compare and contrast it to first quantization. In second quantization, what is analogous to position in first quantization? What is analogous to particle 3-momentum?
3. Construct a chart showing how non-relativistic theories, relativistic theories, particles, fields, classical theory, and quantum theory are interrelated.
4. For NRQM, RQM, and QFT, construct a chart showing i) which have states as solutions to their wave equations, ii) how to calculate expectation values in each, iii) which can handle bound states, inelastic scattering, elementary particle decay, and vacuum fluctuations, iv) which can treat the following interactions: i) e/m, ii) weak, iii) strong, and iv) gravity.
5. What are the four major areas of study making up QFT?
6. Using the corresponding Poisson bracket relation  $\{X, p_x\} = 1$ , we deduce, from first quantization postulate #2, that, quantum mechanically,  $[X, p_x] = i\hbar$ . For this commutator acting on a function  $\psi$ , i.e.,  $[X, p_x] \psi = i\hbar \psi$ , determine what form  $p_x$  must have. Is this an operator? Does it look like what you started with in elementary QM, and from which you then derived the commutator relation above? Can we go either way?

Then, take the eigenvalue problem  $E\psi = H\psi$ , and use the same form of the Hamiltonian  $H$  as used in classical mechanics (i.e.,  $p^2/2m + V$ ), with the operator form you found for  $p$  above. This last step is the other part of first quantization (see page 4).

Did you get the time independent Schrödinger equation? (You should have. 😊) Do you see how, by starting with the Poisson brackets and first quantization, you can derive the basic assumptions of NRQM, i.e., that dynamical variables become operators, the form of those operators, and even the time independent Schrödinger equation, itself? We won't do it here, but from that point, one could deduce the time dependent Schrödinger equation, as well.