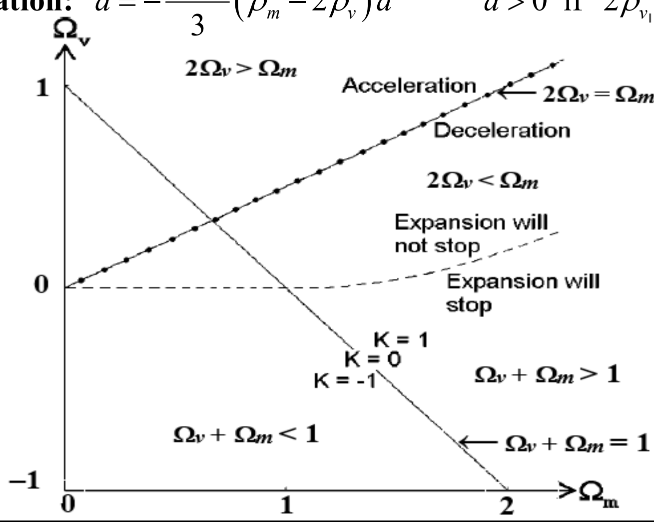
Brief Summary of Cosmology

See Jordan, *Am J Phys* **73**(7) 653-662 (July 2005) for much of this. by R. Klauber

Central Point	Derivation/Remark
Conservation of Energy: $d(\rho V) = -p dV$ Energy Density: $\dot{\rho} = -3(\rho + p)\dot{a}/a = -3(\rho + p)H$ $\rho(a)$ w/ $p=w\rho$: $\rho_w \propto 1/a^{3(1+w)}$ Rad $\rho_{w=1/3} \propto 1/a^4$ Matter $\rho_{w=0} \propto 1/a^3$ Vac $\rho_{w=-1} \propto$ const	3 ways to Energy Density relation: 1) Divide top eq by dt, use $V=ka^3$ 2) Freid metric in $T^{\mu\nu}{}_{;\nu} = 0$ for $\mu = 0$. 3) $T^{\hat{0}\hat{0}}$ & $T^{\hat{x}\hat{x}}$ field eqs combined. $\rho(a)$: ρ_w into top eq, w/ $V=ka^3$ & $p=w\rho$ $c^2\rho = \rho$ in all this ($c = 1$)
Basic Eqs: Friedmann Universe (homogeneous, isotropic) **(A)** $\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - k\left(\frac{a_0}{a}\right)^2$ Friedmann (initial val) Eq. **(B)** $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$ Dynamic eq.	Can rep Λ in ρ_{vac} and take $\Lambda = 0$ here. Robertson-Walker metric. Einstein eq for T^{00} Einstein eq for T^{xx} plus (A), or energy density eq plus (A)
Radiation: $\lambda \propto a$ $\bar{E}_\gamma = 2.7kT \propto 1/\lambda \propto 1/a$ $n_\gamma \propto 1/a^3$ $\rho_\gamma = n_\gamma \bar{E}_\gamma = 2.7kn_\gamma T \propto 1/a^4$ also $p = \rho_\gamma/3$	
***k* = 0 Time Dependence (Λ = 0):** Rad: $a \propto t^{1/2}$ Matter: $a \propto t^{2/3}$ Vacuum: $a \propto e^{Ht}$ General: $a \propto t^{2/3(1+w)}$ (for $\rho_v = -p =$ const)	(A) $\rightarrow$ $\dot{a} = \sqrt{\frac{8\pi G\rho}{3}}a = Ha$ and use various values for $\rho(a)$ above.
Critical Densty: $\rho_c = \frac{3H^2}{8\pi G}$ $\Omega = \rho/\rho_c = \Omega_\gamma + \Omega_m + \Omega_v$ $\Omega_i = \rho_i/\rho$ Rad (early univ) $\rho \propto 1/a^4$, $1 - \frac{1}{\Omega} \propto a^2$ and $\rho > \rho_c$ grows fast For inflation $\rho =$ const, $1 - \frac{1}{\Omega} \propto \frac{k}{a^2}$ and $\rho \rightarrow \rho_c$ fast	From (A), $\Omega = 1$, if $k = 0$. (A) w/ k and Λ =0, rearranged gives this relation in $1/\Omega$. If $k/a^2 \rightarrow 0$, then $\Omega \rightarrow 1$. If Ω=1 at any time, then k=0 and Ω=1 for all time
Acceleration: $\ddot{a} = -\frac{4\pi G}{3}(\rho_m - 2\rho_v)a$ $\ddot{a} > 0$ if $2\rho_{v_1} > \rho_m$ 	(B) for mass and vacuum, assuming vac like cosmol const with w = -1. More generally, in (B), if $p < -\rho/3$ (i.e., w < -1/3), where p and ρ are total values, get acceleration.

Travel Time: <u>Basic principles</u>: 1) $\Delta t = \int_a^{a_0} \frac{dt(a')}{da'} da'$ 2) Find *da/dt* from (B) and invert <u>Subtleties</u>: $r = \frac{a}{a_0} \le 1 \quad \tau = \frac{t}{1/H_0} = H_0 t \quad r = \frac{\lambda}{\lambda_0} = \frac{1}{1+z}$ <u>Result</u>: $\Delta\tau\left(r = \frac{1}{1+z}\right) = \int_r^1 \frac{d\tau(r')}{dr'} dr'$ where $\frac{d\tau}{dr} = \left(\frac{\Omega_{m_0}}{r} + \Omega_{v_0} r^2 + 1 - \Omega_{m_0} - \Omega_{v_0}\right)^{-1/2}$ for vac with w ≠ – 1, get diff relation.	Ref Jordan.1) Looks flawed to me. Easier to start w basic princ in Hubble plot box Integ (B). Here, result assumes $w_v = -1$. For integ const, need to evaluate ka_0^2 term via Hubble relation now, i.e., $\frac{da}{dt} = H_0 a_0 \rightarrow \frac{d(a/a_0)}{H_0 dt} = 1 = \frac{dr}{d\tau}$ now This gives constant terms in last line in LHS box.
Hubble Plots: <u>Def</u>: d_l = luminosity dist = distance to source if flat, static universe for the luminosity we see for standard candle. <u>Basic principle:</u> luminosity dist = $\int_r^1 \frac{c\,d\tau(r')}{dr'} dr'$ (see above) <u>Corrections</u>: 1) expansion on dist 1/*r'* factor $\chi = \int_r^1 \frac{1}{r'} \frac{c\,d\tau(r')}{dr'} dr'$ 2) curved space: k=0, $\sigma = \chi$; k=1, $r_c \sin\frac{\chi}{r_c}$; k=-1, $r_c \sinh\frac{\chi}{r_c}$ 3) expansion on photons: a) decrease intensity per γ by $\lambda/\lambda_0 = r$ b) decrease γ/sec arriving by $r = \lambda/\lambda_0$ <u>Thus</u>: intensity $\propto 1/[4\pi(d_l)^2] \propto 1/[4\pi(\sigma)^2(1+z)^2]$ <u>Subtlety</u>: d_l defined in terms of units of H_0, $d_l = (1+z)\frac{\sigma}{H_0}$ <u>Finally</u>: intensity $\propto \frac{1}{4\pi\left(\frac{\sigma}{H_0}\right)^2 (1+z)^2}$	Original Hubble plots, vel vs dist. Now, d_l vs z = 13.7 bill lt-yrs for pg 6 example = 42 bill lt-yrs for pg 6 example $r_c = \frac{1}{\sqrt{1-\Omega_0}}$ <u>Value:</u> d_l vs z curve depends on Ω_{m_0} and Ω_{v_0} (for $w_v = -1$ case), so data helps determine them and from (B), $\ddot{a}$. For $w_v \ne -1$, will get diff $d\tau/dr$, so diff σ and diff curve. Supernovae (of known intrinsic brightness) shows accel and $w_v = -1$ const within 10% over most of hist.