# Brief Summary of Cosmology

See Jordan, *Am J Phys* **73**(7) 653-662 (July 2005) for much of this. by R. Klauber

| Central Point | Derivation/Remark |
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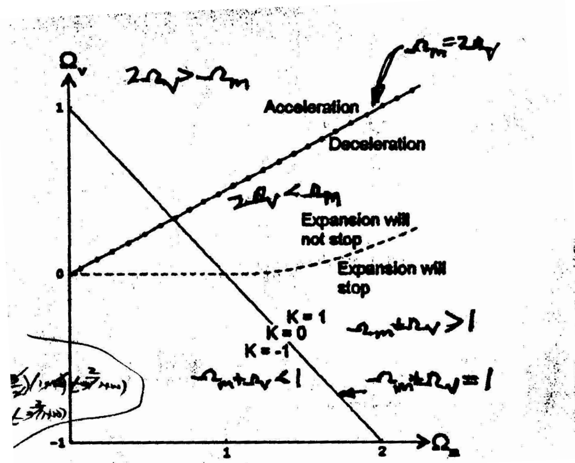
| **Conservation of Energy:** $d(\rho V) = -p dV$<br>Energy Density: $\dot{\rho} = -3(\rho + p)\dot{a}/a = -3(\rho + p)H$<br>$\rho(a)$ w/ $p=w\rho$: $\rho_w \propto 1/a^{3(1+w)}$<br>Rad $\rho_{w=1/3} \propto 1/a^4$ Matter $\rho_{w=0} \propto 1/a^3$ Vac $\rho_{w=-1} \propto$ const | 3 ways to Energy Density relation:<br>1) Divide top eq by $dt$, use $V=ka^3$<br>2) Freid metric in $T^{\mu\nu}{}_{;\nu} = 0$ for $\mu = 0$.<br>3) $T^{\hat{0}\hat{0}}$ & $T^{\hat{x}\hat{x}}$ field eqs combined.<br>$\rho(a)$: $\rho_w$ into top eq, w/ $V=ka^3$ & $p=w\rho$<br>$c^2\rho = \rho$ in all this ($c$ = 1) |
| **Basic Eqs: Freidman Universe** (homogeneous, isotropic)<br>**(A)** $\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - k\left(\frac{a_0}{a}\right)^2$ Freidman (initial val) Eq.<br>**(B)** $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}$ Dynamic eq. | Can rep $\Lambda$ in $\rho_{vac}$ and take $\Lambda = 0$ here.<br>Robertson-Walker metric. Einstein eq for $T^{00}$<br>Einstein eq for $T^{xx}$ plus (A), or energy density eq plus (A) |
| **Radiation:** $\lambda \propto a$ $\bar{E}_\gamma = 2.7kT \propto 1/\lambda \propto 1/a$<br>$n_\gamma \propto 1/a^3$ $\rho_\gamma = n_\gamma \bar{E}_\gamma = 2.7kn_\gamma T \propto 1/a^4$ also $p = \rho_\gamma/3$ | |
| ***k* = 0 Time Dependence ($\Lambda$ = 0):**<br>Rad: $a \propto t^{1/2}$ Matter: $a \propto t^{2/3}$<br>Vacuum: $a \propto e^{Ht}$ General: $a \propto t^{2/3(1+w)}$<br>(for $\rho_v = -p =$ const) | (A) $\rightarrow$ $\dot{a} = \sqrt{\frac{8\pi G\rho}{3}}a = Ha$ and use various values for $\rho(a)$ above. |
| **Critical Densty:** $\rho_c = \frac{3H^2}{8\pi G}$ $\Omega = \rho/\rho_c = \Omega_\gamma + \Omega_m + \Omega_v$ $\Omega_i = \rho_i/\rho$<br>Rad (early univ) $\rho \propto 1/a^4$, $1 - \frac{1}{\Omega} \propto a^2$ and $\rho > \rho_c$ grows fast<br>For inflation $\rho =$ const, $1 - \frac{1}{\Omega} \propto \frac{k}{a^2}$ and $\rho \rightarrow \rho_c$ fast | From (A), $\Omega = 1$, if $k = 0$.<br>(A) w/ $k$ and $\Lambda$ =0, rearranged gives this relation in $1/\Omega$.<br>If $k/a^2 \rightarrow 0$, then $\Omega \rightarrow 1$. If $\Omega$=1 at any time, then $k$=0 and $\Omega$=1 for all time |
| **Acceleration:** $\ddot{a} = -\frac{4\pi G}{3}(\rho_m - 2\rho_v)a$ $\ddot{a} > 0$ if $2\rho_v > \rho_m$<br> | (B) for mass and vacuum, assuming vac like cosmol const with w = -1.<br>More generally, in (B), if $p < -\rho/3$ (i.e., w < -1/3), where p and $\rho$ are total values, get acceleration. |

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| **Travel Time:** Basic principles: 1) $\Delta t = \int_a^{a_0} \frac{dt(a')}{da'} da'$<br>2) Find *da/dt* from (B) and invert | Ref Jordan.1) Looks flawed to me. Easier to start w basic princ in Hubble plot box<br>Integ (B). Here, result assumes $w_v = -1$. |
| Subtleties: $r = \frac{a}{a_0} \le 1 \quad \tau = \frac{t}{1/H_0} = H_0 t \quad r = \frac{\lambda}{\lambda_0} = \frac{1}{1+z}$ | For integ const, need to evaluate $ka_0^2$ term via Hubble relation now, i.e., |
| Result: $\Delta\tau\left(r = \frac{1}{1+z}\right) = \int_r^1 \frac{d\tau(r')}{dr'} dr'$<br>where $\frac{d\tau}{dr} = \left(\frac{\Omega_{m_0}}{r} + \Omega_{v_0} r^2 + 1 - \Omega_{m_0} - \Omega_{v_0}\right)^{-1/2}$<br>for vac with $w \neq -1$, get diff relation. | $\frac{da}{dt} = H_0 a_0 \rightarrow \frac{d(a/a_0)}{H_0 dt} = 1 = \frac{dr}{d\tau}$ now<br>This gives constant terms in last line in LHS box. |
| **Hubble Plots:** Def: $d_l$ = luminosity dist = distance to source if flat, static universe for the luminosity we see for standard candle. | Original Hubble plots, vel vs dist.<br>Now, $d_l$ vs $z$ |
| Basic principle: luminosity dist = $\int_r^1 \frac{c\,d\tau(r')}{dr'} dr'$ (see above) | = 13.7 bill lt-yrs for pg 6 example |
| Corrections:<br>1) expansion on dist $1/r'$ factor $\chi = \int_r^1 \frac{1}{r'} \frac{c\,d\tau(r')}{dr'} dr'$ | = 42 bill lt-yrs for pg 6 example |
| 2) curved space: $k=0$, $\sigma = \chi$; $k=1$, $r_c \sin\frac{\chi}{r_c}$; $k=-1$, $r_c \sinh\frac{\chi}{r_c}$ | $r_c = \frac{1}{\sqrt{1-\Omega_0}}$ |
| 3) expansion on photons: a) decrease intensity per $\gamma$ by $\lambda/\lambda_0 = r$<br>b) decrease $\gamma$/sec arriving by $r = \lambda/\lambda_0$<br>Thus: intensity $\propto 1/[4\pi(d_l)^2] \propto 1/[4\pi(\sigma)^2(1+z)^2]$ | |
| Subtlety: $d_l$ defined in terms of units of $H_0$, $d_l = (1+z)\frac{\sigma}{H_0}$<br>Finally: intensity $\propto \frac{1}{4\pi\left(\frac{\sigma}{H_0}\right)^2 (1+z)^2}$ | Value: $d_l$ vs z curve depends on $\Omega_{m_0}$ and $\Omega_{v_0}$ (for $w_v = -1$ case), so data helps determine them and from (B), $\ddot{a}$. For $w_v \neq -1$, will get diff $d\tau/dr$, so diff $\sigma$ and diff curve.<br>Supernovae (of known intrinsic brightness) shows accel and $w_v = -1$ const within 10% over most of hist. |