Branes and Open Bosonic Strings Summary

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Section, equation, and page numbers herein are with reference to Zwiebach, *A First Course in String Theory.*

1 Dd Branes (Space filling, $p = d$) - 1st row of Wholeness Chart 1 herein

$$X^{\mu}(\tau,\sigma) = X^{+}, X^{-}, \underset{(NN)}{X^{I}} \qquad I = 2,\dots,p \qquad p = d \quad d+1 = D \tag{1}$$

$$X^{\mu} = x_0^{\mu} + \underbrace{\sqrt{2\alpha'}\alpha_0^{\mu}}_{2\alpha' p^{\mu}}\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{\mu}e^{-in\tau}\cos n\sigma \qquad \text{(9.56) [186], } p^{\mu} \text{ from (9.52) [185]}$$

$$M^2 = \frac{1}{\alpha'}\left(a + \sum_{n=1} n a_n^{I\dagger}a_n^I\right) = \underbrace{\frac{1}{\alpha'}\left(\sum_{n=1} n a_n^{I\dagger}a_n^I - 1\right)}_{a=-1 \text{ and } d=25 \text{ for Lorentz invariance}} = \frac{1}{\alpha'}\left(N^{\perp}-1\right) \qquad \text{(12.108) [252] (12155), (12.156) [262]}$$

2 Dp branes and boundary conditions (Sect. 15.1)

Consider branes with spatial dimension p, where $p < 25$ embedded in the full $d = 25$ spatial dimension space. Strings vibrate in all 25 dimensions, but they have endpoints with Neumann B.C.s in the Dp brane and Dirichlet B.C.s with all dimensions outside the Dp brane. Take the coordinates μ to i, where i represents directions tangent to the Dp brane. For coordinates normal to the Dp brane, use coordinate symbol a for the D (Dirichlet) string coordinates normal to the Dp brane.

In the light cone gauge, the string coordinates are

$$X^{\mu}(\tau,\sigma) = X^{+}, X^{-}, \underset{(NN)}{X^{i}}, \underset{(DD)}{X^{a}} \qquad i = 2,\dots,p \qquad a = p+1,\dots,d \qquad p < d \qquad \text{(15.7) [333]}$$

3 Open Strings on Dp-branes (Sect. 15.2) - 2nd row of Wholeness Chart 1

X^i of (15.7) is simply (9.56) above with $\mu = i$ and the p value of (15.7). The coordinates normal to the Dp brane are

$$X^{a}(\tau,\sigma) = \tilde{x}^{a} + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{a}e^{-in\tau}\sin n\sigma\,, \qquad \text{(15.20) [335]}$$

where we note there is no $p^a\tau$ term, since the overall (average) momentum of the string in a DD direction is zero.

$$M^2 = \frac{1}{\alpha'}\left(-1 + \underbrace{\sum_{n}\sum_{i=2}^{p} n\, a_n^{i\dagger}a_n^i + \sum_{m}\sum_{a=p+1}^{d} m\, a_m^{a\dagger}a_m^a}_{N^{\perp}}\right) = \frac{1}{\alpha'}\left(N^{\perp}-1\right) \qquad \text{(15.27) [336]}$$

4 Open Strings between Parallel Dp-Branes (Sect 15.3) – 3rd row of Wholeness Chart 1

3rd row of Wholeness Chart 1 for visual image of two parallel branes showing different symbol meanings. [*ij*] represent string oriented from *i*th to *j*th brane. *i* and *j* called Chan-Paton indices. Strings with ends on same brane (symbols [11] and [22]) are like row 2 of Wholeness Chart 1.

Strings stretching from brane 1 to brane 2

$$X^{a}(\tau,\sigma) = \tilde{x}_1^{a} + \left(\tilde{x}_2^{a} - \tilde{x}_1^{a}\right)\frac{\sigma}{\pi} + \sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\alpha_n^{a}e^{-in\tau}\sin n\sigma \qquad \left(\alpha_n^a \text{ here different from } \alpha_n^a \text{ of single brane}\right) \qquad \text{(15.45) [340]}$$

$$M^2 = \left(\frac{\tilde{x}_2^{a} - \tilde{x}_1^{a}}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}\left(N^{\perp}-1\right) \qquad N^{\perp} \text{ same form as (15.27) above} \qquad \text{(15.11) [34]}$$

For two separate but coincident branes (i.e., $\tilde{x}_2^a - \tilde{x}_1^a = 0$ [343]), and $N^{\perp} = 1$, there are 4 massless gauge ($U(2)$ Yang-Mills) fields. For N coincident branes, there are N^2 massless gauge ($U(N)$ Yang-Mills) fields.

5 Strings between Parallel Dp and Dq Branes, $p > q$ (Sect. 15.4) – 4th row of Wholeness Chart 1

See the figure in 4th row of Wholeness Chart 1 for visual image of two parallel branes of different dimensions.

In the light-cone gauge, coordinates (note particular symbols) are

$$X^{\mu}(\tau,\sigma) = X^{+}, X^{-}, \underset{(NN)}{X^{i}}, \underset{(ND)}{X^{r}}, \underset{(DD)}{X^{a}} \qquad i = 2,\dots,q \quad r = q+1,\dots,p \quad a = p+1,\dots,d \qquad \text{(15.63) [347]}$$

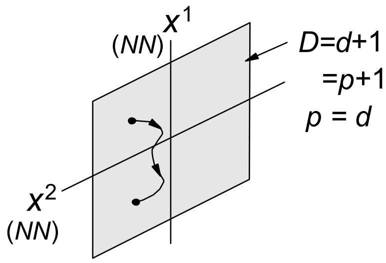
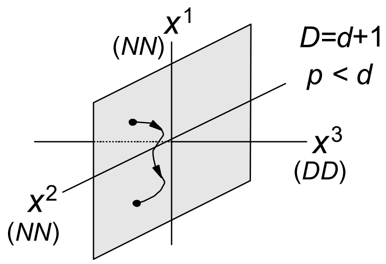
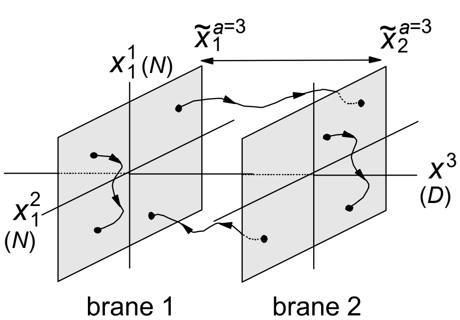
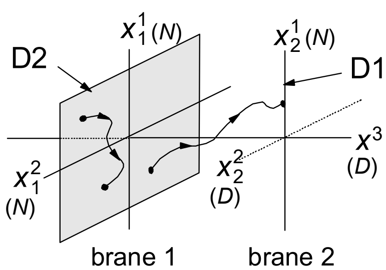
Coordinates for strings stretching from brane 1 to brane 2 in r (ND) direction are

$$X^r(\tau,\sigma) = \tilde{x}_2^r + i\sqrt{2\alpha'} \sum_{n\in\mathbb{Z}_{odd}} \frac{2}{n}\alpha^r_{n/2} e^{-i\frac{n}{2}\tau} \cos\frac{n\sigma}{2} \,. \qquad (15.72)\ [348]$$

$$M^2 = \left(\frac{\tilde{x}_2^a - \tilde{x}_1^a}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}\left(N^\perp - 1 + \frac{1}{16}(p-q)\right) \qquad (15.84)\ [350]$$

$$N^\perp = \underbrace{\sum_n \sum_{i=2}^{q} n\, a_n^{i\dagger} a_n^i}_{\text{tangent }(NN)} + \underbrace{\sum_{k\in\mathbb{Z}_{odd}} \sum_{r=q+1}^{p} \frac{k}{2} a_{k/2}^{r\dagger} a_{k/2}^r}_{\text{mixed }(ND)} + \underbrace{\sum_m \sum_{a=p+1}^{d} m\, a_m^{a\dagger} a_m^a}_{\text{normal }(DD)} \qquad (15.85)\ [350]$$

Wholeness Chart 1. Overview of Branes and Open Strings

	Visually in Low Dimensions	**Ground States**	**Tachyons**	**1 Tangent**	**1 Normal**
Dp brane in all d (p=d) spatial dimensions		$\lvert p^+, \vec{p}\rangle$ $\vec{p} = (p^2, \ldots, p^d)$ (12.159) [263]	$N^\perp = 0$ $n = 0$ For $D = 26$, $M^2 = -\frac{1}{\alpha'}$ Lorentz scalar	$N^\perp = 1$ $n = 0$ For $D = 26$ $M^2 = 0$ Maxwell field, 24 components	No such animal
Dp brane in d (p<d) spatial dimensions		$\lvert p^+, \vec{p}\rangle$ $\vec{p} = (p^2, \ldots, p^p)$ (15.82) [336]	$N^\perp = 0$ n=m=0 For $D = 26$ $M^2 = -\frac{1}{\alpha'}$ Lorentz scalar, like above	$N^\perp = 1$ $n = 1$, $m = 0$ For $D = 26$ $M^2 = 0$ Maxwell field, p-1 components	$N^\perp = 1$ $n = 0$, $m = 1$ For $D = 26$ $M^2 = 0$ Massless scalar each a direction
2 Dp branes in d (p<d) spatial dimensions	Figure 15.1 [339]	$\lvert p^+, \vec{p}; [11]\rangle$ $\lvert p^+, \vec{p}; [22]\rangle$ $\lvert p^+, \vec{p}; [12]\rangle$ $\lvert p^+, \vec{p}; [21]\rangle$ (15.54) [341] [11] and [22] like row above	[12] and [21] $N^\perp = 0$ n=m=0 $D = 26$, $M^2 =$ $\left(\frac{\tilde{x}_2^a - \tilde{x}_1^a}{2\pi\alpha'}\right)^2 - \frac{1}{\alpha'}$ M^2 neg, zero, or pos Lorentz scalar Tachyon if <0	[12] and [21] $N^\perp = 1$ $n = 1$, $m = 0$ For $D = 26$ $M^2 = \left(\frac{\tilde{x}_2^a - \tilde{x}_1^a}{2\pi\alpha'}\right)^2$ Massive vector (not Maxwell). One of scalars (at right) added for p components	[12] and [21] $N^\perp = 1$ $n = 0$, $m = 1$ For $D = 26$ $M^2 = \left(\frac{\tilde{x}_2^a - \tilde{x}_1^a}{2\pi\alpha'}\right)^2$ Massless scalars. Scalar pointing between branes added to vector at left. So d–p–1 scalars here.
Parallel Dp and Dq branes (p<d, q<d, p+q=d)	Figure 15.3 [346]	$\lvert p^+, \vec{p}; [11]\rangle$ $\lvert p^+, \vec{p}; [22]\rangle$ $\lvert p^+, \vec{p}; [12]\rangle$ $\lvert p^+, \vec{p}; [21]\rangle$ $\vec{p} = (p^2, \ldots, p^q)$ (15.86) [350] [11] and [22] like two rows above	[12] and [21] $N^\perp = 0$ n=k=m=0 $D = 26$, $M^2 =$ $\left(\frac{\tilde{x}_2^a - \tilde{x}_1^a}{2\pi\alpha'}\right)^2 - \frac{1}{\alpha'}$ M^2 neg, zero, or pos Lorentz scalar Tachyon if <0	[12] and [21] $N^\perp \geq 1$ For $D = 26$ $M^2 > 0$ since $p > q$. No massless gauge fields.	[12] and [21] As at left