Box 2-2 Correction to 1st Printing

You may wish to print this page, cut out the box, and tape it over Box 2-2 on pg. 23 in the 1st printing (March 2013) of *Student Friendly Quantum Field Theory*.

Box 2-2. Conjugate and Physical Momentum Densities

The relationship between physical momentum density and conjugate momentum density for fields is not so intuitive. It can be derived by assuming our physical 3-momentum density $\wp^i$ obeys the classical field variational relation of the RHS of (B2-2.1). (This can be intuited from (2-11), except that there we used a Cartesian system where $p_i = p^i$, and here we use the relativistic Minkowski metric system, where $p_i = -p^i$.) If we divide the particle relation by volume, we get a density relation.

$$p_i = \frac{\partial L}{\partial \dot{x}^i} \xrightarrow[\text{divide by particle volume}]{\text{for small particle in medium,}} \wp_i = \frac{\partial \mathcal{L}}{\partial \dot{x}^i}\,. \qquad \text{(B2-2.1)}$$

For continuous media like a fluid, $\dot{x}^i$ is the velocity of the medium (field) at the point where $\wp_i$ is measured. We note carefully that our x^i here is the position coordinate of a point fixed relative to the field (fluid particle in our example) and thus is time dependent. (It is different from the same x^i symbol we use in field theory, which is an independent variable that does not depend on time.) Further, the total derivative $\dot{x}^i = dx^i/dt$ equals the partial derivative with respect to time $\partial x^i/\partial t$, since $x^i(t)$ in the present case is only a function of time.

Now take the conjugate momentum density relation for relativistic fields (2-14),

$$\pi_r = \frac{\partial \mathcal{L}}{\partial \dot{\phi}^r}\,, \qquad \text{(B2-2.2)}$$

and divide the RHS of (B2-2.1) by (B2-2.2),

$$\frac{\wp_i}{\pi_r} = \frac{\partial \mathcal{L}/\partial \dot{x}^i}{\partial \mathcal{L}/\partial \dot{\phi}^r} = \frac{\partial \dot{\phi}^r}{\partial \dot{x}^i} = \frac{\partial \phi^r/\partial t}{\partial x^i/\partial t} = \frac{\partial \phi^r}{\partial x^i} \quad \rightarrow \quad \wp_i = \pi_r \frac{\partial \phi^r}{\partial x^i} \quad \rightarrow \quad \wp^i = -\pi_r \frac{\partial \phi^r}{\partial x^i}\,. \qquad \text{(B2-2.3)}$$

The partial derivative of ϕ^r with respect to either of our definitions of x^i (time dependent as the moving position of a point fixed to the field, or time independent as coordinates fixed in space) is the same because by definition, partial derivative means we hold everything else (specifically time here) constant. Thus, the above relation holds in field theory when we consider the x^i as independent variables (coordinates fixed in space).